Chapter Objectives: Chapter objectives provides a brief overview of the concepts to be discussed in the chapter.

5

Circuit Theorems

CHAPTER OBJECTIVES

- *Derive Superposition Theorem from the property of linearity of elements.*
- *Explain the two key theorems – Superposition Theorem and Substitution Theorem in detail.*
- *Derive other theorems like Compensation Theorem, Thevenin's Theorem, Norton's Theorem, Reciprocity Theorem and Maximum Power Transfer Theorem from these two key principles.*
- *Provide illustrations for applications of circuit theorems in circuit analysis through solved examples.*
- *Emphasise the use of Compensation Theorem, Thevenin's Theorem and Norton's Theorem in circuits containing dependent sources as a pointer to their applications in the study of Electronic Circuits.*

This Chapter identifies the Substitution Theorem and Superposition Theorem as the two key theorems and shows how the other theorems may be extracted from them.

INTRODUCTION

The previous chapter showed that:

(1) All the element voltages and element currents in a circuit can be obtained from its node voltages. The node voltages are governed by a matrix equation $\boldsymbol{YV} = \boldsymbol{CU}$, where $\boldsymbol{V}$ is the node voltage column vector, $\boldsymbol{Y}$ is the nodal conductance matrix of the circuit, $\boldsymbol{U}$ is the input column vector containing source functions of all *independent* voltage sources and current sources in the circuit and $\boldsymbol{C}$ is the input matrix. The values of conductances in the circuit and values of coefficients of linear dependent sources in the circuit decide the elements of $\boldsymbol{Y}$-matrix. It is a symmetric matrix if there are no dependent sources in the circuit. Dependent sources *can* make $\boldsymbol{Y}$-matrix asymmetric. The $\boldsymbol{C}$-matrix, in general, contain 0, 1, −1 and conductance values as well as dependent source coefficients.

(2) An alternative formulation is given by a matrix equation $\boldsymbol{ZI} = \boldsymbol{DU}$, where $\boldsymbol{I}$ is the mesh current column vector, $\boldsymbol{Z}$ is the mesh resistance matrix of the circuit,

5

Circuit Theorems

CHAPTER OBJECTIVES

- *Derive Superposition Theorem from the property of linearity of elements.*
- *Explain the two key theorems – Superposition Theorem and Substitution Theorem in detail.*
- *Derive other theorems like Compensation Theorem, Thevenin's Theorem, Norton's Theorem, Reciprocity Theorem and Maximum Power Transfer Theorem from these two key principles.*
- *Provide illustrations for applications of circuit theorems in circuit analysis through solved examples.*
- *Emphasise the use of Compensation Theorem, Thevenin's Theorem and Norton's Theorem in circuits containing dependent sources as a pointer to their applications in the study of Electronic Circuits.*

This Chapter identifies the Substitution Theorem and Superposition Theorem as the two key theorems and shows how the other theorems may be extracted from them.

INTRODUCTION

The previous chapter showed that:

(1) All the element voltages and element currents in a circuit can be obtained from its node voltages. The node voltages are governed by a matrix equation $\boldsymbol{YV} = \boldsymbol{CU}$, where $\boldsymbol{V}$ is the node voltage column vector, $\boldsymbol{Y}$ is the nodal conductance matrix of the circuit, $\boldsymbol{U}$ is the input column vector containing source functions of all *independent* voltage sources and current sources in the circuit and $\boldsymbol{C}$ is the input matrix. The values of conductances in the circuit and values of coefficients of linear dependent sources in the circuit decide the elements of $\boldsymbol{Y}$-matrix. It is a symmetric matrix if there are no dependent sources in the circuit. Dependent sources *can* make $\boldsymbol{Y}$-matrix asymmetric. The $\boldsymbol{C}$-matrix, in general, contain 0, 1, −1 and conductance values as well as dependent source coefficients.

(2) An alternative formulation is given by a matrix equation $\boldsymbol{ZI} = \boldsymbol{DU}$, where $\boldsymbol{I}$ is the mesh current column vector, $\boldsymbol{Z}$ is the mesh resistance matrix of the circuit,

Introduction: The Introduction a glimpse of how the content of this chapter evolve from the preceding chapter.

interesting source functions – *unit impulse function* $\delta(t)$ and *unit step function* $u(t)$. These functions are extremely important in Circuit Analysis.

3.1 THE RESISTOR

Main headings and sub-headings: Well-organised main headings and sub-headings to guide the reader through and provide a lucid flow of the topic.

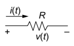
The physical basis for the two-terminal element, called resistor, was dealt in detail in Chap. 1. We revise briefly.

The source of e.m.f. in a circuit sets up charge distributions at the terminals of all the two-terminal elements connected in the circuit. This charge distribution at the terminals of a resistor sets up an electric field inside the conducting material in the resistor. The mobile electrons get accelerated by this electric field and move. But, their motion is impeded by frequent collisions with non-mobile atoms in the conducting substance. A steady situation in which the mobile electrons attain a constant average speed as a result of the aggregate effect of large number of collisions occur in the conducting material within a short time (called *relaxation time* of the conductor material in *Electromagnetic Field Theory*) of appearance of electric field. Once this steady situation occurs, the current through a *linear* resistor is proportional to the voltage appearing across it. The constant of proportionality is called 'resistance' of the resistor and has 'Ohm' (represented by 'Ω') as its unit. Reciprocal of resistance is called 'conductance' of the resistor and its unit is 'Siemens' (represented by 'S'). The unit 'mho' is also used sometimes for conductance. The unit 'mho' is represented by inverted 'Ω' – i.e., by ℧.

Ohm's Law, an experimental law describing the relationship between voltage across a resistor and current through it, states that the voltage across a *linear* resistor at any instant t is proportional to the current passing through it at that instant provided the temperature of the resistor is kept constant. A resistor is called *linear* if it obeys Ohm's law. This is a kind of circular definition. We settle the matter by stating that we consider only those resistors that have a proportionality relationship between voltage and current in our study of circuits in this book.

The graphic symbol of a linear resistor and its element relationship is given below.

Voltage–current relation and power relations for a linear resistor obeying Ohm's Law.



$$v(t) = Ri(t) \text{ or } i(t) = Gv(t) \text{ for all } t$$

$$p(t) = v(t)i(t) = R[i(t)]^2 = \frac{[v(t)]^2}{R} = \frac{[i(t)]^2}{G} = G[v(t)]^2$$

where $p(t)$ is the power delivered to the resistor in Watts.

The resistor does not remember what was done to it previously. Its current response at a particular instant depends only on the voltage applied across at that instant. Therefore, a resistor is a *memoryless* element. Such an element needs to have same kind of wave-shape in both voltage and current. It is not capable of changing the wave-shape of a signal applied to it. It can only dissipate energy. Therefore, the power delivered to a positive resistor is always positive or zero.

3.1.1 Series Connection of Resistors

Consider the series connection of n resistors $R_1, R_2, \ldots, R_n$ as in Fig. 3.1-1.

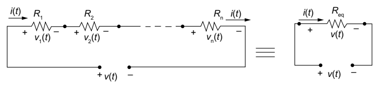


Fig. 3.1-1 Series Connection of Resistors and its Equivalent

Thus, the only correct way to model a circuit that involves parallel connections of voltage sources (more generally, loops comprising only voltage sources) is to take into account the parasitic elements that are invariably associated with any practical voltage source. A somewhat detailed model for the two-source system is shown in Fig. 2.3-2.

Circuits: Topics presented withclear circuits supported by analytical and conceptual ideas.

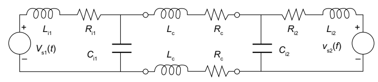


Fig. 2.3-2 A Detailed Model for a Circuit with Two Voltage Sources in Parallel

L_{i1} and L_{i2} represent the internal inductance of the sources, C_{i1} and C_{i2} represent the terminal capacitance of the sources and R_{i1} and R_{i2} represent the internal resistance of the sources. L_c and R_c represent the inductance and resistance of the connecting wires. Obviously, two practical voltage sources can be connected in parallel even if their open-circuit electromotive forces (e.m.f.s) are not equal at all t; only that they cannot be modelled by *ideal independent voltage source model.*

Two ideal independent current sources in series raise a similar issue (see Fig. 2.3-3). KCL requires that $i_{s1}(t) = i_{s2}(t)$ for all t. Even if this condition is satisfied, there is no way to obtain the voltages appearing across the current sources. Therefore, the correct model to be employed for practical current sources that appear in series in a circuit is a detailed model that takes into account the parasitic elements associated with any practical device. More generally, if there is a node in a circuit where only current sources are connected, then, those current sources cannot be modelled by *ideal independent current source model.*

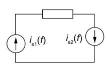


Fig. 2.3-3 Two Ideal Independent Current Sources in Series with Another Element

Similar situations may arise in modelling practical dependent sources by *ideal dependent source models.* In all such cases we have to make the model more detailed in order to resolve the conflict that arises between Kirchhoff's laws and *ideal* nature of the model.

2.4 ANALYSIS OF A SINGLE-LOOP CIRCUIT

The circuit analysis problem involves finding the voltage variable and current variable of every element as functions of time, given the source functions. *Source functions* are the time-functions describing the e.m.f. of independent voltage sources and source currents of independent current sources. They are also called the *excitation functions.* If the circuit contains b-elements, there will be $2b$ variables to be solved for. Some of them will be known in the form of source functions, while others have to be solved for.

Element relation of each element gives us one equation per element. Thus, there are b equations arising out of element relations. The remaining b equations are provided by the interconnection constraints. These equations are obtained by applying KCL at all nodes except one and KVL in all meshes (in the case of a planar circuit).

Theoretically speaking, that is all there is to circuit analysis. However, systematic procedures for applying element relations, KVL equations and KCL equations would be highly desirable when it comes to analysis of complex circuits. Moreover, the fact that there are $2^n - 2$ KCL equations for an n-node circuit and only $(n - 1)$ of them are independent, calls for a systematic procedure for writing KCL equations. Similarly, there will be l KVL equations for a circuit with l-loops and only $(b - n + 1)$ of them will be independent. This, again, calls for some systematic procedures for extracting a set of $(b - n + 1)$ *independent* KVL equations.

But, what is the use of a theorem that wants us to solve a circuit first and then replace part of the circuit by a source that has a value depending on the solution of the circuit? Obviously, such a theorem will not help us directly in solving circuits.

The significance of this theorem lies in the fact that it can be used to construct theoretical arguments that lead to other powerful circuit theorems that indeed help us to solve circuit analysis problems in an elegant and efficient manner.

Moreover, it does find application in circuit analysis in a slightly disguised form. We take up that disguised form of Substitution Theorem in Sect. 5.4.

Stubs: Stubs in marginalia stress on important concepts. Additional information is also provided, wherever relevant.

part of the circuit that is being substituted and the remaining circuit except through the pair of terminals at which they are interconnected.

Subject to the constraints on *unique solution* and *interaction only through the connecting terminals*, we state the *Substitution theorem* as below (Fig. 5.3-8).

Let a circuit with unique solution be represented as interconnection of two networks N_1 and N_2 and let the interaction between N_1 and N_2 be only through the two terminals at which they are connected. N_1 and N_2 may be linear or non-linear. Let v(t) be the voltage that appears at the terminals between N_1 and N_2 and let i(t) be the current flowing into N_2 from N_1. Then, the network N_2 may be replaced by an independent current source of value i(t) connected across the output of N_1 or an independent voltage source of value v(t) connected across the output of N_1 without affecting any voltage or current variable within N_1 provided the resulting network has unique solution.

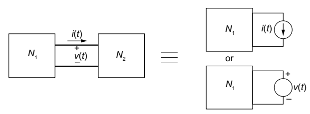


Fig. 5.3-8 The Substitution Theorem

5.4 COMPENSATION THEOREM

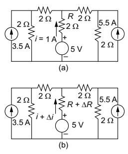


Fig. 5.4-1 Circuit to Illustrate Compensation Theorem

The circuit in Fig. 5.4-1(a) has a resistor marked as R. It has a nominal value of 2 Ω. Mesh analysis was carried out to find the current in this resistor and the current was found to be 1 A as marked in the circuit as in Fig. 5.4-1(a).

Now, let us assume that the resistor value changes by ΔR to $R+\Delta R$. Correspondingly all circuit variables change by small quantities as shown in Fig. 5.4-1(b). The current through that resistor will also change to $i+\Delta i$. We can conduct a mesh analysis once again and get a new solution. However, we can do better than that. We can work out *changes* in variables everywhere by solving a single-source circuit and then construct the circuit solution by adding *change* to the initial solution value.

We apply Substitution theorem on the first circuit with R as the element that is being substituted and on the second circuit with $R+\Delta R$ as the part that is being substituted by an independent voltage source. The voltage source in the first circuit must be Ri V and the voltage source in the second circuit must be $(R + \Delta R)(i + \Delta i)$ V.

$(R + \Delta R)(i + \Delta i) = Ri + (R + \Delta R)\Delta i + i\Delta R$ (Fig. 5.4-2).

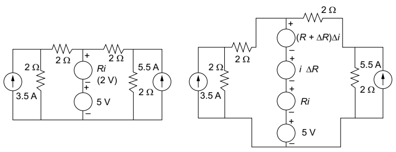


Fig. 5.4-2 Circuits After Applying Substitution Theorem

and a_i is its "coefficient of contribution". The coefficient of contribution has the physical significance of contribution per unit input'.

The coefficient of contribution, a_i, which is a constant for a time-invariant circuit, can be obtained by solving for $x(t)$ in a single-source circuit in which all independent sources other than the i^{th} one are deactivated by replacing independent voltage sources with short-circuits and independent current sources with open-circuits.

But, why should a linear combination $x = a_1I_1 + a_2I_2 + \ldots + b_1V_1 + b_2V_2 + \ldots$ be found term by term always? Is it possible to get it in subsets that contain more than one term? The third form of *Superposition Theorem* states that it can be done.

Superposition Theorem Form-3

Superposition Theorem – Third form.

'*The response of any circuit variable in a multi-source linear memoryless circuit containing "n" independent sources can be obtained by adding responses of the same circuit variable in two or more circuits with each circuit keeping a subset of independent sources active in it and remaining sources deactivated such that there is no overlap between such active source subsets among them*'.

Pointed entries: Pointed entries located in the margin 'point' to significant discussions in the text to reiterate them.

5.1.1 Linearity of a Circuit

Linearity of a Circuit

Linearity of a circuit element and linearity of a circuit are two different concepts.

A circuit is called *linear* if its solution obeys superposition principle. This is why we stated the Superposition Theorem with the adjective *linear* behind 'circuit'. Whether we view the statements on Superposition Theorem as a definition of linearity of a circuit or as an assertion of an important property of linear circuits is matter of viewpoint.

There is indeed a bit of circularity in *Linearity* and *Superposition Principle*.

Why did the memoryless circuits we have been dealing with till now obey superposition principle? The elements of memoryless circuits were constrained to be linear time-invariant elements. We used only *linear* resistors and *linear* dependent sources. The v–i relations of all those elements obey superposition principle. As a result, all KCL and KVL equations in nodal analysis and mesh analysis had the form of linear combinations. Such KVL and KCL equations lead to nodal conductance matrix (and mesh resistance matrix) that contain only constants in the case of a time-invariant circuit (i.e., resistances are constants and coefficients of dependent sources are also constants). Similarly, the input matrix (**C** in nodal analysis and **D** in mesh analysis) will contain only constants in the case of circuits constructed using linear time-invariant elements. Thus, the solution for node voltage variables and mesh current variables will come out in the form of linear combination of independent source functions. And, after all Superposition Theorem is only a restatement of this fact. Therefore, Superposition Theorem holds in the circuit since we used only linear elements in constructing it except for independent sources which are non-linear. Hence, we conclude that *a memoryless circuit constructed from a set of linear resistors, linear dependent sources and independent sources (they are non-linear elements) results in a circuit which obeys Superposition Theorem and hence, by definition, is a linear circuit.*

Linearity of a circuit element and linearity of a circuit are two different concepts. An element is linear if its v–i relationship obeys principle of homogeneity and principle of additivity. A circuit is linear, if all circuit variables in it, without any exception, obey principle of homogeneity and principle of additivity, i.e., the principle of superposition. It may appear intuitively obvious that a circuit containing only linear elements will turn out to be a linear circuit. But, note that we used non-linear elements – independent sources are non-linear elements – and hence, it is not so apparent. The preceding discussion offers a plausibility reasoning to convince us that a circuit containing linear elements and independent sources will indeed be a linear circuit. But the mathematical proof for this apparently straightforward conclusion is somewhat formidable.

Linearity and Superposition appear so natural to us. But the fact is that most of the practical electrical and electronic circuits are non-linear in nature. Linearity, at best, is only an approximation that circuit analysts employ to make the analysis problem more tractable. We illustrate why Superposition Theorem does not hold for a circuit containing a non-linear element by an example. The circuit is shown in Fig. 5.1-3(a). The resistor R is a non-linear one with a v–i relation given by $v = 2i^2$ for $i \geq 0$ and $-2i^2$ for $i < 0$.

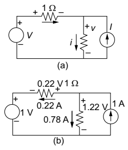


Fig. 5.1-3 (a) A Circuit Containing a Non-linear Resistor (b) Circuit Solution for V = 1 V and I = 1 A

We may recast the expressions that involve sum of the two sinusoidal functions in Eqn. 11.6-8 and 11.6-9 as single sinusoidal functions by employing trigonometric identities in the following manner.

$$v_C(t) = \sqrt{V_0^2 + \frac{LI_0^2}{C}}\cos(\omega_n t - \phi) \text{ V for } t \geq 0^+$$

$$i(t) = -\sqrt{I_0^2 + \frac{CI_0^2}{L}}\sin(\omega_n t - \phi) \text{ A for } t \geq 0^+, \quad (11.6\text{-}10)$$

$$\text{where } \phi = \tan^{-1}\left(\frac{I_0\sqrt{L/C}}{V_0}\right).$$

Source-free response equations for a pure *LC* circuit.

These waveforms are shown in Fig. 11.6-5 for $L = 1$ H, $C = 1$ F, $V_0 = 2$ V and $I_0 = 1$ A.

Graphical representations: Graphical representations for figurative analysis of circuit behaviour.

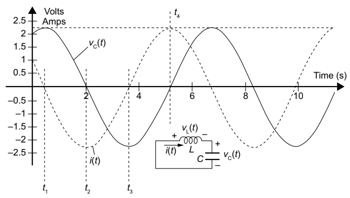


Fig. 11.6-5 Zero-Input Response of a LC Circuit ($L = 1$ H, $C = 1$ F, $V_0 = 2$ V and $I_0 = 1$ A)

The source-free response (equivalently, the zero-input response) of a pure *LC* circuit will contain undying sinusoids with steady amplitudes. The amplitude of sinusoidal waveforms is decided by the total initial energy storage in the circuit and the circuit parameters.

Circuit parameters, i.e., *L* and *C* decide the angular frequency of oscillations too – it is $(LC)^{-0.5}$ rad/s.

The initial voltage of 2 V across the capacitor appears across the inductor at $t = 0^+$ with a polarity such that the inductor current starts decreasing at the rate of 2 V/1 H = 2 A/s from its initial value of 1 A. However, the circuit current is in a direction suitable for increasing the capacitor voltage. Hence, the capacitor voltage increases while the inductor current decreases. Under the action of increasing reverse voltage, the inductor current decreases more rapidly to reach zero at the instant t_1. At that instant, the current and hence the energy storage in inductor are zero. The inductor had an initial energy of 0.5 J and the capacitor had an initial energy of 2 J. There was no dissipation in the circuit. Therefore, when the circuit current reaches zero, the capacitor must hold the total initial energy of 2.5 J in it. It will require √5 V across it (since $C = 1$ F and energy $= 0.5CV^2$). Equation 11.6-10 predicts exactly this value as the amplitude of $v_C(t)$. When circuit current goes through zero, capacitor voltage must go through a positive or negative peak due to two reasons – firstly, the current through a capacitor is proportional to the rate of change of voltage across it and secondly that is the instant at which it will contain the maximum possible energy equal to the total initial energy. Therefore, $v_C(t)$ reaches a positive peak at t_1.

With such a large reverse voltage across it, the inductor has to continue its current build up in the negative direction. But, with the current changing its direction, the capacitor

A Pure *LC* Circuit?
Strictly speaking, a pure *LC* circuit cannot exist in practice. The wire used to construct the inductor, the metal foil used in the capacitor and the connecting wires have non-zero resistance. The dielectric used in the capacitor will have non-zero conductivity. Thus, there will be some non-zero resistance left in any *LC* circuit.

continued

We solve the problem by finding the node voltage v_x first. We express v_d as $v_S - v_x$ and write the node equation at the node where v_x is assigned.

$$\frac{1}{R_i}(v_x - v_s) + \frac{1}{R_1}v_x + \frac{1}{R_2 + R_0}(v_x - A(v_s - v_x)) = 0$$

$$\text{Solving for } v_x,\ v_x = \frac{\frac{1}{R_i} + \frac{A}{R_2 + R_0}}{\frac{1}{R_i} + \frac{1}{R_1} + \frac{(A+1)}{R_2 + R_0}} v_s.$$

Substituting the numerical values, we get, $v_x = 0.9999\ v_S$. Therefore, $v_d = 0.0001\ v_s$.

The current in R_o is $100000v_d - 0.9999v_S$ divided by 900075 Ω. Therefore, it is equal to $9.9993 \times 10^{-6}v_S$ A.

Therefore, the voltage drop in $R_o = 75 \times 9.9993 \times 10^{-6}v_S\ \text{V} = 7.5 \times 10^{-4}\ v_S$ V.

∴ $v_o = 10v_S - 7.5 \times 10^{-4}\ v_S \approx 10\ v_S$ V. This is the same as the output predicted by the IOA model.

Let us repeat the calculations by assuming $A = 1000$, $R_i = 200$ kΩ and $R_o = 1$ kΩ. Now, the node voltage $v_x = 0.9901v_S$, the differential input voltage $v_d = 0.0099v_s$ and $v_o = 9.86v_o$. Thus, the gain will deviate by 1.4% away from its expected value of 10.

In general, the results predicted by the IOA model will be sufficiently accurate if the gain realised in the circuit is below 1% of the Opamp gain and the resistors used in the circuit are much higher than the Opamp output resistance and much lower than the Opamp input resistance.

A thumb rule for choosing the resistor values in a circuit containing Opamps and resistors may be arrived at as a result of these calculations on commonly used Opamp circuits.

The design rule for choosing the values for resistors in an Opamp circuit is that all resistors must be chosen to lie between $R_i/25$ and $25R_o$, where R_i and R_o are the input and output resistance of Opamps used in the circuit.

Voltage saturation at the output of an Opamp and the consequent clipping of output waveform are easy to understand. However, clipping at a level lower than the voltage saturation limit may take place under current-limited operation of Opamps. The next example illustrates this issue.

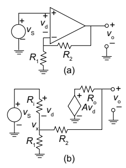


Fig. 6.8-1 (a) Non-Inverting Amplifier (b) Equivalent Circuit of Non-Inverting Amplifier

Worked examples: Worked examples illustrate the theory explained in the text.

EXAMPLE: 6.8-2

The Opamp used in an inverting amplifier (Fig. 6.8-2) employs ±12 V supply. The output saturation limit of the Opamp at this power supply level is ±10 V. The output current of Opamp is limited to ±20 mA with a supply voltage of ±12 V. The feedback resistance draws negligible current from the output and the gain of the amplifier is –10. Obtain the output of the amplifier if the input is a sine wave of 1 V amplitude and 10 Hz frequency and the load connected at the output is (i) 10 kΩ and (ii) 250 Ω.

SOLUTION

(i) The gain of the amplifier is –10. The input is $v_S(t) = 1\sin 20\pi t$ V. Therefore, the output will be $v_o(t) = -10\sin 20\pi t$ V if the Opamp does not enter the non-linear range of operation at any instant. The peak voltage of the expected output is 10 V and this is just about equal to the voltage saturation limits. Therefore, clipping will not take place on this count. The maximum current that will be drawn by the 10 kΩ load will be 10 V/10 kΩ = 1 mA and that is well below the output current limit of Opamp. Therefore, the output in this case will be a pure sine wave given by $v_o(t) = -10\sin 20\pi t$ V.

(ii) Clipping cannot take place in this case too due to the output voltage trying to exceed the saturation limits. However, if the output is really $-10\sin 20\pi t$ V, then the load resistor will draw a current of 10/0.25 = 40 mA at the peak of sine wave, but the Opamp output current is limited at ± 20 mA. The load resistor of 250 Ω will draw 20 mA when the voltage across it is 5 V. This will happen at the 30° position on the sine wave. Thus, the output voltage will follow a sinusoid of 10 V amplitude until the 30° position, then, remain clipped at 5 V for the entire 30° to 150° range and again follows a sinusoidal variation for 150° to 180° in a half-cycle. Thus, output shows a clipping level of ±5 V for two-thirds of cycle period.

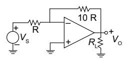
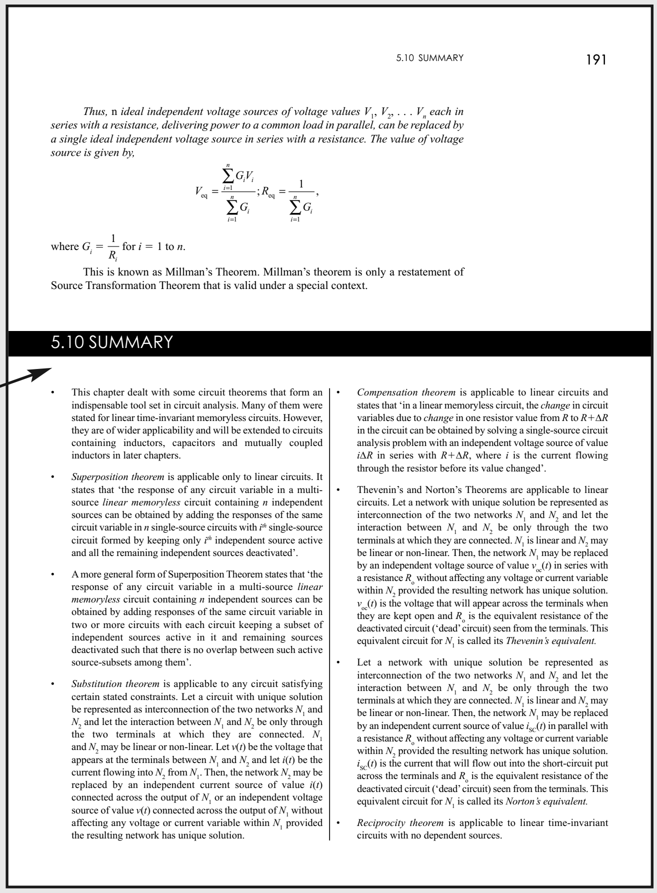


Fig. 6.8-2 The Inverting Amplifier in Example 6.8-2



5.10 SUMMARY 191

Thus, n *ideal independent voltage sources of voltage values $V_1, V_2, \ldots V_n$ each in series with a resistance, delivering power to a common load in parallel, can be replaced by a single ideal independent voltage source in series with a resistance. The value of voltage source is given by,*

$$V_{eq} = \frac{\sum_{i=1}^{n} G_i V_i}{\sum_{i=1}^{n} G_i}; R_{eq} = \frac{1}{\sum_{i=1}^{n} G_i},$$

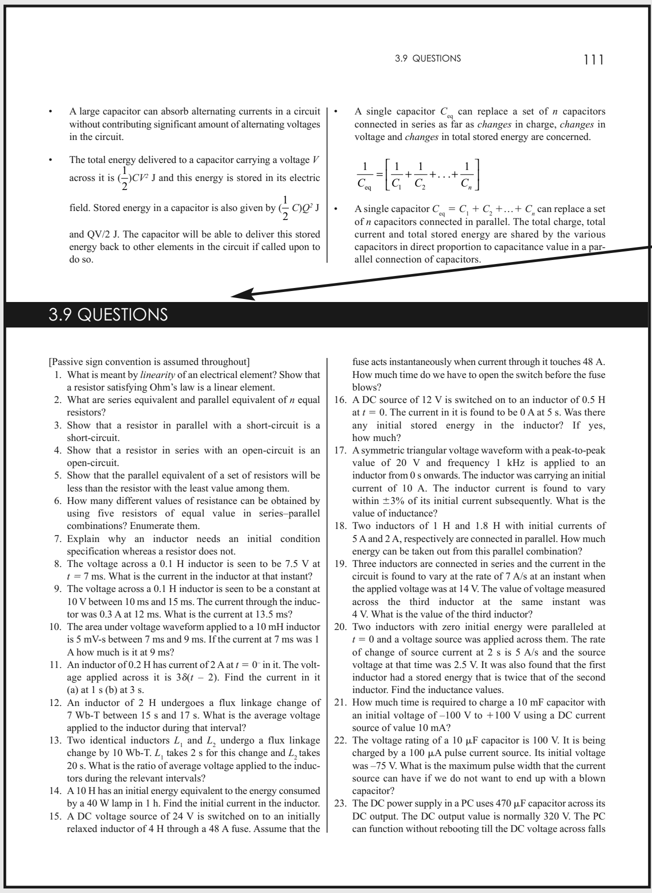
where $G_i = \frac{1}{R_i}$ for $i = 1$ to n.

This is known as Millman's Theorem. Millman's theorem is only a restatement of Source Transformation Theorem that is valid under a special context.

5.10 SUMMARY

- This chapter dealt with some circuit theorems that form an indispensable tool set in circuit analysis. Many of them were stated for linear time-invariant memoryless circuits. However, they are of wider applicability and will be extended to circuits containing inductors, capacitors and mutually coupled inductors in later chapters.
- *Superposition theorem* is applicable only to linear circuits. It states that 'the response of any circuit variable in a multi-source *linear memoryless* circuit containing n independent sources can be obtained by adding the responses of the same circuit variable in n single-source circuits with i^{th} single-source circuit formed by keeping only i^{th} independent source active and all the remaining independent sources deactivated'.
- A more general form of Superposition Theorem states that 'the response of any circuit variable in a multi-source *linear memoryless* circuit containing n independent sources can be obtained by adding responses of the same circuit variable in two or more circuits with each circuit keeping a subset of independent sources active in it and remaining sources deactivated such that there is no overlap between such active source-subsets among them'.
- *Substitution theorem* is applicable to any circuit satisfying certain stated constraints. Let a circuit with unique solution be represented as interconnection of the two networks N_1 and N_2 and let the interaction between N_1 and N_2 be only through the two terminals at which they are connected. N_1 and N_2 may be linear or non-linear. Let $v(t)$ be the voltage that appears at the terminals between N_1 and N_2 and let $i(t)$ be the current flowing into N_2 from N_1. Then, the network N_2 may be replaced by an independent current source of value $i(t)$ connected across the output of N_1 or an independent voltage source of value $v(t)$ connected across the output of N_1 without affecting any voltage or current variable within N_1 provided the resulting network has unique solution.
- *Compensation theorem* is applicable to linear circuits and states that 'in a linear memoryless circuit, the *change* in circuit variables due to *change* in one resistor value from R to $R+\Delta R$ in the circuit can be obtained by solving a single-source circuit analysis problem with an independent voltage source of value $i\Delta R$ in series with $R+\Delta R$, where i is the current flowing through the resistor before its value changed'.
- Thevenin's and Norton's Theorems are applicable to linear circuits. Let a network with unique solution be represented as interconnection of the two networks N_1 and N_2 and let the interaction between N_1 and N_2 be only through the two terminals at which they are connected. N_1 is linear and N_2 may be linear or non-linear. Then, the network N_1 may be replaced by an independent voltage source of value $v_{oc}(t)$ in series with a resistance R_o without affecting any voltage or current variable within N_2 provided the resulting network has unique solution. $v_{oc}(t)$ is the voltage that will appear across the terminals when they are kept open and R_o is the equivalent resistance of the deactivated circuit ('dead' circuit) seen from the terminals. This equivalent circuit for N_1 is called its *Thevenin's equivalent.*
- Let a network with unique solution be represented as interconnection of the two networks N_1 and N_2 and let the interaction between N_1 and N_2 be only through the two terminals at which they are connected. N_1 is linear and N_2 may be linear or non-linear. Then, the network N_1 may be replaced by an independent current source of value $i_{SC}(t)$ in parallel with a resistance R_o without affecting any voltage or current variable within N_2 provided the resulting network has unique solution. $i_{SC}(t)$ is the current that will flow out into the short-circuit put across the terminals and R_o is the equivalent resistance of the deactivated circuit ('dead' circuit) seen from the terminals. This equivalent circuit for N_1 is called its *Norton's equivalent.*
- *Reciprocity theorem* is applicable to linear time-invariant circuits with no dependent sources.

Bulleted Summary: Bulleted Summary gives the essence of each chapter in brief.



3.9 QUESTIONS 111

- A large capacitor can absorb alternating currents in a circuit without contributing significant amount of alternating voltages in the circuit.
- The total energy delivered to a capacitor carrying a voltage V across it is $(\frac{1}{2})CV^2$ J and this energy is stored in its electric field. Stored energy in a capacitor is also given by $(\frac{1}{2}C)Q^2$ J and QV/2 J. The capacitor will be able to deliver this stored energy back to other elements in the circuit if called upon to do so.
- A single capacitor C_{eq} can replace a set of n capacitors connected in series as far as *changes* in charge, *changes* in voltage and *changes* in total stored energy are concerned.

$$\frac{1}{C_{eq}} = \left[\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}\right]$$

- A single capacitor $C_{eq} = C_1 + C_2 + \ldots + C_n$ can replace a set of n capacitors connected in parallel. The total charge, total current and total stored energy are shared by the various capacitors in direct proportion to capacitance value in a parallel connection of capacitors.

3.9 QUESTIONS

[Passive sign convention is assumed throughout]

1. What is meant by *linearity* of an electrical element? Show that a resistor satisfying Ohm's law is a linear element.
2. What are series equivalent and parallel equivalent of n equal resistors?
3. Show that a resistor in parallel with a short-circuit is a short-circuit.
4. Show that a resistor in series with an open-circuit is an open-circuit.
5. Show that the parallel equivalent of a set of resistors will be less than the resistor with the least value among them.
6. How many different values of resistance can be obtained by using five resistors of equal value in series–parallel combinations? Enumerate them.
7. Explain why an inductor needs an initial condition specification whereas a resistor does not.
8. The voltage across a 0.1 H inductor is seen to be 7.5 V at $t = 7$ ms. What is the current in the inductor at that instant?
9. The voltage across a 0.1 H inductor is seen to be a constant at 10 V between 10 ms and 15 ms. The current through the inductor was 0.3 A at 12 ms. What is the current at 13.5 ms?
10. The area under voltage waveform applied to a 10 mH inductor is 5 mV-s between 7 ms and 9 ms. If the current at 7 ms was 1 A how much is it at 9 ms?
11. An inductor of 0.2 H has current of 2 A at $t = 0^-$ in it. The voltage applied across it is $3\delta(t - 2)$. Find the current in it (a) at 1 s (b) at 3 s.
12. An inductor of 2 H undergoes a flux linkage change of 7 Wb-T between 15 s and 17 s. What is the average voltage applied to the inductor during that interval?
13. Two identical inductors L_1 and L_2 undergo a flux linkage change by 10 Wb-T. L_1 takes 2 s for this change and L_2 takes 20 s. What is the ratio of average voltage applied to the inductors during the relevant intervals?
14. A 10 H has an initial energy equivalent to the energy consumed by a 40 W lamp in 1 h. Find the initial current in the inductor.
15. A DC voltage source of 24 V is switched on to an initially relaxed inductor of 4 H through a 48 A fuse. Assume that the fuse acts instantaneously when current through it touches 48 A. How much time do we have to open the switch before the fuse blows?
16. A DC source of 12 V is switched on to an inductor of 0.5 H at $t = 0$. The current in it is found to be 0 A at 5 s. Was there any initial stored energy in the inductor? If yes, how much?
17. A symmetric triangular voltage waveform with a peak-to-peak value of 20 V and frequency 1 kHz is applied to an inductor from 0 s onwards. The inductor was carrying an initial current of 10 A. The inductor current is found to vary within ±3% of its initial current subsequently. What is the value of inductance?
18. Two inductors of 1 H and 1.8 H with initial currents of 5 A and 2 A, respectively are connected in parallel. How much energy can be taken out from this parallel combination?
19. Three inductors are connected in series and the current in the circuit is found to vary at the rate of 7 A/s at an instant when the applied voltage was at 14 V. The value of voltage measured across the third inductor at the same instant was 4 V. What is the value of the third inductor?
20. Two inductors with zero initial energy were paralleled at $t = 0$ and a voltage source was applied across them. The rate of change of source current at 2 s is 5 A/s and the source voltage at that time was 2.5 V. It was also found that the first inductor had a stored energy that is twice that of the second inductor. Find the inductance values.
21. How much time is required to charge a 10 mF capacitor with an initial voltage of –100 V to +100 V using a DC current source of value 10 mA?
22. The voltage rating of a 10 μF capacitor is 100 V. It is being charged by a 100 μA pulse current source. Its initial voltage was –75 V. What is the maximum pulse width that the current source can have if we do not want to end up with a blown capacitor?
23. The DC power supply in a PC uses 470 μF capacitor across its DC output. The DC output value is normally 320 V. The PC can function without rebooting till the DC voltage across falls

Review Questions: Review exercise questions help the reader to check the understanding of the topic.

Answers to Selected Problems

Chapter 1

1. **(a)** 432,000 Coulombs **(b)** 11.66 V **(c)** 88.18 AH **(d)** 3.76×10^6 J, 1.045 kWh
2. **(a)** 28 AH **(b)** 9 A **(c)** 2.94×10^6 J, 0.817 kWh 8.9 A, 71.2 AH
3. **(a)** 10 AH **(b)** 60 AH **(c)** 10 A **(d)** 2.133×10^6 J, 0.593 kWh
4. **(a)** a 100 Ω resistor **(b)** 0.36 mC **(c)** 1 mC **(d)** 5 mJ **(e)** 23 ms
5. **(a)** a 0.1 μF capacitor **(b)** 1 μC **(c)** 5 μJ **(d)** 3.75 μJ
6. **(a)** a 0.6 H inductor
   (b) $q(t)=\begin{cases}0.1(1-\cos 100t) \text{ coul for } t\ge 0\\ 0 \text{ for } t<0\end{cases}$
   (c) $p(t)=\begin{cases}3000\sin 200t \text{ W for } t\ge 0\\ 0 \text{ for } t<0\end{cases}$
   (d) $E(t)=\begin{cases}30\sin^2 100t \text{ J for } t\ge 0\\ 0 \text{ for } t<0\end{cases}$
7. **(a)** −240 W **(b)** 120 J , increases
8. It is a resistor of 5 Ω and the current through it is $(2-e^{-100t})$ A for $t\ge 0$.
9. **(d)** 0.85 A
10. **(a)** $v(t)=\begin{cases}0 \text{ V for } t\le 0\\ 20t \text{ V for } 0<t\le 4\text{ s}\\ (160-20t)\text{ V for } 4\text{ s}<t\le 8\text{ s}\\ 0\text{ V for } t>8\text{ s}\end{cases}$
    (b) $v(t)=\begin{cases}0 \text{ V for } t\le 0\\ 1 \text{ V for } 0<t\le 4\text{ s}\\ -1\text{ V for } 4\text{ s}<t\le 8\text{ s}\\ 0\text{ V for } t>8\text{ s}\end{cases}$
    (c) $v(t)=\begin{cases}0 \text{ V for } t\le 0\\ 100t^2 \text{ V for } 0<t\le 4\text{ s}\\ (1600t-100t^2-3200)\text{ V for } 4\text{ s}<t\le 8\text{ s}\\ 3200\text{ V for } t>8\text{ s}\end{cases}$
11. $v(t)=\begin{cases}10 \text{ V for } t<0\\ (10-\frac{2}{\pi}\sin 1000\pi t)\text{ V for } t\ge 0\end{cases}$
12. 9.97 A, 1.994 Wb-T , 9.94 J
13. **(b)** $v_x = -10$ V **(c)** Yes, it is a DC source.
14. **(a)** (15 V, −3 A) and (−15 V, 3 A) **(b)** It is an active element and is a DC source. **(c)** No
15. 4 H, 1.98 H
16. 1 H, 0.7 H
17. 1 H, 0.5
18. **(a)** $\psi_1 = 0.139\sin(100\pi t+30^\circ)$ Wb-T
    $\psi_2 = 0.224\sin(100\pi t+42^\circ)$ Wb-T
    (b) $v_1 = 43.53\cos(100\pi t+30^\circ)$ Wb-T
    $v_2 = 70.35\cos(100\pi t+42^\circ)$ Wb-T
    (c) $\psi_1 = 0.139\sin(100\pi t-30^\circ)$ Wb-T
    $\psi_2 = 0.224\sin(100\pi t+138^\circ)$ Wb-T
    $v_1 = 43.53\cos(100\pi t-30^\circ)$ Wb-T
    $v_2 = 70.35\cos(100\pi t+138^\circ)$ Wb-T

Chapter 2

1. $v_2 = -10$ V, $v_4 = 15$ V, $v_5 = -15$ V, $i_1 = -3$ A, $i_5 = 2$ A
2. **(i)** $v_1 = -15$ V, $v_5 = 15$ V, $v_7 = 10$ V, $i_2 = 3$ A, $i_3 = -5$ A, $i_4 = -8$ A, $i_6 = -5$ A
   (ii) Elements *a*, *e* and *g* **(iii)** Elements *b, c, d* and *f*. **(iv)** Elements *b, c, d* and *f*. Total power = 140 W **(v)** Elements *a*, *e* and *g*. Power absorbed = −140 W
3. **(i)** $i_2 = -i_3 - i_4 + i_8$; $i_1 = i_3 + i_4 + i_7 - i_8$; $i_6 = -i_3 - i_7$; $i_5 = -i_4 - i_3 - i_7$
   (ii) $v_1 = v_3 - v_7 - v_8$; $v_2 = -v_3 + v_7$; $v_5 = v_4 + v_8$; $v_6 = v_3 - v_4$
4. **(i)** The elements are designated as [*a* // (*b*+*c*)] + *d* + [(*e*+*f*)//*g*] where // is parallel connection and + is series connection. Then, $i_c = -2$ A, $i_d = 1$ A, $i_f = -2$ A, $i_g = 3$ A, $v_b = 5$ V, $v_d = -5$ V, $v_e = 5$ V.
   (ii) 3
   (iii) [5 Ω//(2.5 Ω+10 V)] + [−1.5 V] + [3.3333 Ω//(2.5 Ω+15 V)] where // stands for parallel connection and + stands for series connection.
   (iv) [5 Ω//(2.5 Ω+2.5 A)] + [1 A] + [3.3333 Ω//(2.5 Ω+2 A)] where // stands for parallel connection and + stands for series connection.
5. **(i)** The elements are designated as [*a* // (*b*+*c*)] + *d* + [(*e*+*f*)//*g*] where // is parallel connection and + is series connection. Then, $i_c = -2$ A, $i_d = 1$ A, $i_f = -2$ A, $i_g = 3$ A, $v_b = 5$ V, $v_d = -5$ V, $v_e = 5$ V.
   (ii) Power delivered by *a*(1 A CS) = −5 W, Power delivered by *d*(1 A CS) = 5 W, Power delivered by *g*(3 A CS) = −30 W, Power delivered by *b* (5 V VS) = −10 W, Power delivered by *c*(10 V VS) = 20 W, Power delivered by *e*(5 V VS) = −10 W, Power delivered by *f*(15 V VS) = 30 W.
6. **(i)** The elements are designated as [*a* // *b*] + *c* + *d* + [*e* // *f*] where // is parallel connection and + is series connection. Then, $i_b = 1$ A, $i_d = 1$ A, $i_f = -3$ A, $v_b = -10$ V, $v_c = 5$ V, $v_f = 10$ V.

Answers to Selected Problems: Answers to Selected Problems given at the end of the book facilitate effortless verification of the solutions to chapter-end exercises.

Index: An exhaustive list of index words with sub-entries captured from all occurences across the text instead of being restricted to a given primary entry.

Index