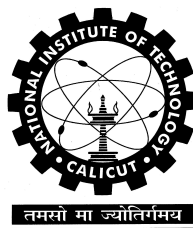


DEPARTMENT OF MATHEMATICS
NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

M.Sc. Mathematics
Curriculum & Syllabi (2016)

**CURRICULUM & SYLLABI FOR POST-GRADUATE
PROGRAM LEADING TO MASTER OF SCIENCE
DEGREE IN MATHEMATICS**



Curriculum for Master of Science (M.Sc.) Program in Mathematics

I. Program Details

Name of Degree	Name of Specialization	Intake (Full-time)	Year of revision (Proposed)	Duration	Eligibility for Admission
M.Sc.	Mathematics	20	2016	2 years (4 semesters)	B.Sc. Degree in Mathematics/ Applied Mathematics/Statistics with first class (60% marks or CGPA 6.5/10) in aggregate in the qualifying examination and for SC/ST candidates 50% marks or CGPA 5.5/10 in aggregate in the qualifying examination

II. Program Educational Objectives

PEO1	Provide a strong foundation in different areas of Mathematics, so that the students can compete with their contemporaries and excel in the various careers in Mathematics.
PEO2	Motivate and prepare the students to pursue higher studies and research, thus contributing to the ever increasing academic demands of the country.
PEO3	Enrich the students with strong communication and interpersonal skills, broad knowledge and an understanding of multicultural and global perspectives, to work effectively in multidisciplinary teams, both as leaders and team members.
PEO4	Facilitate integral development of the personality of the student to deal with ethical and professional issues, and also to develop ability for independent and lifelong learning.

III. Program Outcomes

PO1	Students will demonstrate in-depth knowledge of Mathematics, both in theory and application.
PO2	Students will attain the ability to identify, formulate and solve challenging problems in Mathematics.
PO3	Students will be able to analyse complex problems in Mathematics and propose solutions using research based knowledge
PO4	Students will be aware of their professional and ethical responsibilities.
PO5	Students will be able to work individually or as a team member or leader in uniform and multidisciplinary settings.
PO6	Students will develop confidence for self-education and ability for lifelong learning.

IV. Program Structure

Semester 1

S.No	Code	Title	L	T	P/S	C
1	MA6301	Real Analysis	4	0	0	4
2	MA6302	Linear Algebra	4	0	0	4
3	MA6303	Numerical Analysis	3	0	0	3
4	MA6304	Ordinary Differential Equations	3	0	0	3
5	MA6305	Computer Programming	2	0	3	4
		Total	16	0	3	18

Semester 2

S.No	Code	Title	L	T	P/S	C
1	MA6321	Complex Analysis	4	0	0	4
2	MA6322	Measure and Probability	3	0	0	3
3	MA6323	Graph Theory	3	0	0	3
4	MA6324	Abstract Algebra	4	0	0	4
5	MA6325	Topology	4	0	0	4
		Total	18	0	0	18

Semester 3

S.No	Code	Title	L	T	P/S	C
1	MA7301	Partial Differential Equations	3	0	0	3
2	MA7302	Functional Analysis	4	0	0	4
3	MA7303	Mathematical Statistics	4	0	0	4
4	MA7304	Operations Research	4	0	0	4
5	————	Elective I	3	0	0	3
6	MA7391	Seminar	0	0	0	1
		Total	18	0	0	19

Semester 4

S.No	Code	Title	L	T	P/S	C
1	MA7321	Methods in Applied Mathematics	3	0	0	3
2	————	Elective II	3	0	0	3
3	————	Elective III	3	0	0	3
4	MA7392	Project	0	0	0	6
		Total	9	0	0	15
Total credits						70

Elective Courses (3 credits, 3 lecture hours)

1. MA7350 Advanced Topics in Graph Theory
2. MA7351 Advanced Topology
3. MA7352 Algebraic Topology
4. MA7353 Fluid Mechanics
5. MA7354 Fourier Analysis
6. MA7355 Fuzzy Set Theory and Applications
7. MA7356 Generalised Set Theory
8. MA7357 Numerical Linear Algebra
9. MA7358 Operator Theory
10. MA7359 Spectral Theory of Hilbert Space Operators
11. MA7360 Introduction to Fractal Geometry
12. MA7361 Advanced Complex Analysis
13. MA7362 Differential Geometry
14. MA7363 Distribution Theory
15. MA7364 Introduction to Chaos Theory
16. MA7365 Multivariable Calculus
17. MA7366 Numerical Solution for Partial Differential Equations
18. MA7367 Statistical Methods for Quality Management
19. MA7368 Advanced Operations Research
20. MA7369 Stochastic Processes
21. MA7370 Number Theory

22. MA7371 Applied Statistical Inference

23. MA7372 Regression Analysis

24. MA7373 Reliability of Systems

25. MA7374 Forecasting Techniques

Note: Students may also take PG level electives offered by other departments.

Brief Syllabus of Core courses

MA6301 REAL ANALYSIS

L	T	P	C
4	0	0	4

Real and complex number systems, completeness property, basic topology, Continuity, connectedness and compactness, differentiation, mean value theorems, Taylor's theorem, Riemann-Stieltjes integral, fundamental theorem of calculus. Uniform convergence, power series, real analytic functions, transcendental functions, equicontinuity, Stone-Weierstrass theorem.

MA6302 LINEAR ALGEBRA

L	T	P	C
4	0	0	4

Vector Spaces, subspaces, dimension, linear transformation, isomorphism, matrix representation, dual space, transpose, Cayley-Hamilton theorem, elementary canonical forms, inner-product space, spectral theorems.

MA6303 NUMERICAL ANALYSIS

L	T	P	C
3	0	0	3

Numerical methods for root finding, Direct and iterative methods for solving linear system of equations, Newton method to solve non-linear system of equations, Numerical methods to find Eigenvalues and Eigenvectors, Interpolation, Curve fitting, Numerical differentiation and integration, Numerical methods for ordinary differential equations.

MA6304 ORDINARY DIFFERENTIAL EQUATIONS

L	T	P	C
3	0	0	3

First order differential equations, Existence and uniqueness of solution, linear differential equations and its applications. Series solution of second-order equations, Legendre and Bessel functions, Sturm-Liouville problem. System of ordinary differential equations, matrix methods for linear systems, Autonomous systems, Stability analysis.

MA6305 COMPUTER PROGRAMMING

L	T	P	C
2	0	3	4

Basics in C , control statements, functions, arrays, pointers, sorting, structures, basics in C^{++} , constructors and destructors, dynamic arrays, function overloading, operator overloading, friend functions, virtual functions, inheritance.

MA6321 COMPLEX ANALYSIS

L	T	P	C
4	0	0	4

Analytic functions, conformal maps, Cauchy's theorem, Liouville's theorem, Morera's theorem, singularities, zeros and poles, Laurent's series, maximum principle, general form of Cauchy's theorem, homology, residue theorem, argument principle, harmonic functions, mean-value property, Poisson's formula, Schwarz reflection principle. .

MA6322 MEASURE AND PROBABILITY

L	T	P	C
3	0	0	3

Measure space, Carathodory's extension, Lebesgue measure, Lebesgue integration, Lebesgue-Radon-Nikodym theorem, fundamental theorem of calculus, product measure, Fubini's theorem, joint distribution and marginal distribution of random variables, Chebyshev's inequality, weak law of large numbers, strong law of large numbers, central limit theorem.

MA6323 GRAPH THEORY

L	T	P	C
3	0	0	3

Graphs, trees, metric in graph, connectivity, traversability, matchings, factorization, domination, graph colouring, digraphs, graph algorithms.

MA6324 ABSTRACT ALGEBRA

L	T	P	C
4	0	0	4

Group, Lagrange's theorem, normal subgroup and quotient subgroup, homomorphism, isomorphism. Group of permutations, Cayley's theorem for finite group, direct product of, group action on a set, Sylow's theorems, structure of finite abelian group. Ring, integral domain, division ring, field, quotient field of an integral domain, ideals, fundamental theorem of ring homomorphism, isomorphism. Ring of polynomials, Gauss's lemma, unique factorization domain, principle ideal domain, Euclidean domain, field extension, algebraic extensions, splitting field of a polynomial, algebraic closure.

MA6325 TOPOLOGY

L	T	P	C
4	0	0	4

Topological spaces, subspace topology, continuous functions and sets with imposed topologies, product topology, metric topology, quotient topology, connectedness, compactness, countability axioms, separation axioms, Urysohn Lemma, Urysohn metrization theorem, Tychonoff theorem.

MA7301 PARTIAL DIFFERENTIAL EQUATIONS

L	T	P	C
3	0	0	3

Modelling with partial differential equations, First order partial differential equations, Method of Characteristics, Second order partial differential equations with classifications, Linear second order partial differential equations with constant coefficients. Parabolic, Hyperbolic partial differential equations, Laplace equations, Harmonic functions, Mean value property, Green's function.

MA7302 FUNCTIONAL ANALYSIS

L	T	P	C
4	0	0	4

Normed linear spaces, bounded linear maps, Banach spaces, fundamental theorems, spectrum of a bounded operator, dual space, Riesz representation theorems, weak and weak* convergence, reflexivity, Hilbert spaces, adjoint of a bounded operator, normal operators, unitary operators, self-adjoint operators, spectrum and numerical range.

MA7303 MATHEMATICAL STATISTICS

L	T	P	C
4	0	0	4

Random Variables, Probability Distributions, Expectations, Random Vectors, Bivariate Normal Distribution, Transformations of One Random Variable and Random Vectors, Order statistics, Statistics- Parameter Estimation, Hypothesis Tests, Theory of Statistical Inference, Regression and Correlation Analysis- Bivariate Relationships, K-Variable Linear Relationships and non-linear relationships and related inferences.

MA7304 OPERATIONS RESEARCH

L	T	P	C
4	0	0	4

Linear programming, Simplex method, Duality, Sensitivity analysis, Transportation problem, Assignment problem, Dynamic programming, Bellman's Optimality principle. Game theory, Linear programming formulation, Network models, CPM-PERT, Resource analysis, Time cost optimization.

MA7321 METHODS IN APPLIED MATHEMATICS

L	T	P	C
3	0	0	3

Fourier Series, Integral transforms, Applications of Fourier integrals and transforms, Finite Fourier transforms, Convolution theorem for finite Fourier transforms, Integral equations, Green' function, Fredholm equation with separable kernels, Eigenvalues and Eigenfunctions, Tensor analysis, Quotient law, Conjugate tensor, Christoffel symbols, Covariant differentiation of tensors, Calculus of variation, Euler's equation, fundamental lemma of calculus of variation, Functionals involving derivatives higher order, Functionals depending on functions of several independent variables, Rayleigh-Ritz method.

Brief Syllabus of Elective courses

MA7350 ADVANCED TOPICS IN GRAPH THEORY

L	T	P	C
3	0	0	3

Graphs: review of basics in graphs, Connectivity, Traversability, Metric in graph, Convexity, Symmetry, Distance Sequences, Coloring in graphs , vertex coloring, Edge coloring, Total coloring, Complete coloring, Digraphs, Networks, Weighted graphs.

MA7351 ADVANCED TOPOLOGY

L	T	P	C
3	0	0	3

Topological spaces, subspaces, Continuity, Product topology, Separation Axioms, Covering Properties, Relations between covering properties and separation axioms, Metrizable and Connectedness, The metrization theorems, Connectedness and total disconnectedness, Homotopy, homotopy of paths , Fundamental group and covering spaces, Deformation retraction.

MA7352 ALGEBRAIC TOPOLOGY

L	T	P	C
3	0	0	3

Review of basic Topology, Arcwise connected spaces. Homotopy, homotopy classes, Fundamental group, Change of base point, Topological invariance, Homology groups, Geometrical motivation, Euclidean simplexes, Linear mappings, singular simplexes, chains, Boundary of a simplex, Boundaries and cycles on any space, Homologous cycles and homology groups, Relative homology, Induced Homomorphisms, Topological invariance of homology groups, Homotopic mappings and the homology groups, Prisms, Homology sequences, Simplicial complexes.

MA7353 FLUID MECHANICS

L	T	P	C
3	0	0	3

Fundamental conceptions about fluids, The Transport Theorem, Continuity equation, Equation of motion, Euler's equation, Navier-Stokes equations, The energy equation, Dimensional analysis, Boussinesq approximation, Incompressible and irrotational flow, Plane potential flow, Laplace equation, Bernoulli equation, Kelvin's circulation theorem. Laminar flow, Steady flow, High and low Reynold's number flows, Boundary layer theory, Blasius solution.

MA7354 FOURIER ANALYSIS

L	T	P	C
3	0	0	3

Trigonometric polynomials and trigonometric series, Fourier series in $L^1(T)$, convolutions, Convergence, Fourier series in $L^2(T)$, Fourier series of L^p functions. Fourier transform in $L^1(R)$, inversion, Abel means and Poisson kernel, Fourier transform in $L^2(R)$, harmonic functions, Dirichlet problem, point wise and norm convergence, eigen functions of Fourier transform, Hermite functions, Schwartz space, Paley-Wiener theorem, uncertainty principles, Hardy's theorem.

MA7355 FUZZY SET THEORY AND APPLICATIONS

L	T	P	C
3	0	0	3

Crisp sets and Fuzzy sets, operations on fuzzy sets, Fuzzy arithmetic and Fuzzy relations, Fuzzy measures, Fuzzy Logic and Applications.

MA7356 GENERALISED SET THEORY

L	T	P	C
3	0	0	3

An overview of basic operations on Fuzzy sets and Multisets, Rough sets, Approximations of a set, Properties of Approximations, Rough membership function, Rough sets and Reasoning from data: Information systems, Soft sets, DeMorgan laws, Applications and soft analysis. Fuzzy soft sets, Operations on fuzzy soft sets, Soft fuzzy sets and its properties, Fuzzy rough sets and rough fuzzy sets, Rough multisets, Genuine sets, Applications.

MA7357 NUMERICAL LINEAR ALGEBRA

L	T	P	C
3	0	0	3

Conditioning and Stability, Cholesky Factorization, LU decomposition, Least Square Problems, Numerical Computation of Eigenvalues and Eigenvectors, Singular Value Decomposition(SVD), Generalized inverses of matrices.

MA7358 OPERATOR THEORY

L	T	P	C
3	0	0	3

Banach algebras, Fredholm and semi-Fredholm operators, Spectral projections, compact operators, Measures of operators, perturbation classes, Unbounded operators, essential spectrum.

MA7359 SPECTRAL THEORY OF HILBERT SPACE OPERATORS

L	T	P	C
3	0	0	3

Bounded linear operators on Hilbert spaces, spectral theory, Compact linear operators, Positive operators, Projection operators, spectral family, spectral representation of bounded self adjoint Linear operators.

MA7360 INTRODUCTION TO FRACTAL GEOMETRY

L	T	P	C
3	0	0	3

The completeness of space of fractals, transformations Mobius transformations on the Riemann sphere, contraction mapping theorem, dynamical systems, shadowing theorem, chaotic dynamics on fractals. Fractal dimension, Hausdorff-Besicovitch dimension, fractal interpolation, Space filling curves, escape time algorithm, Julia sets, IFS for Julia sets, Application of Julia sets to Newton's method Invariant sets of continuous open mappings, map of fractals, Mandelbrot's sets, Mandelbrot's sets for Julia sets.

MA7361 ADVANCED COMPLEX ANALYSIS

L	T	P	C
3	0	0	3

Review of power series, Entire functions: Jensen's Formula, The Riemann zeta function, Functional equations, Zeros of the zeta function. Normal Families, Equicontinuity, Normality and compactness, Arzela's Theorem, Riemann mapping theorem, Analytic arcs, Elliptic Functions, Fourier Development, Period Module, Unimodular transformations, Canonical Basis, Weierstrass Theory, Germs and Sheaves, Sections of Riemann Surfaces, Analytic continuation along arcs, Homotopic curves, The Monodromy theorem, Branch points, the Picard theorem, factorization theorems, shift operator, conjugate functions.

MA7362 DIFFERENTIAL GEOMETRY

L	T	P	C
3	0	0	3

Graphs and Level sets. Vector fields, The Tangent space. Curves and Surfaces, Tangent, Curvature, Principal normal, Binormal, torsion, The Frenet Formulas, the tangent surface, Orientation. Gauss Map, Geodesics, Parallel transport-The Weingrten Map Curvature of plane curves, second fundamental theorem, Equation of Gauss and Codazzi, Lines of curvature of a surface Parallel surfaces. Curvature of surfaces, Intrinsic equation of a curve, Linear element, Element of area, Intrinsic geometry. Parameterized surfaces- Local Equivalence of surfaces and parameterized surfaces, Differential parameters, Isometric surfaces, Geodesic curvature, normal curvature, minimal surfaces.

MA7363 DISTRIBUTION THEORY

L	T	P	C
3	0	0	3

Test functions and distributions, order of distribution, convergence, derivative of distribution, local equality of distributions, support and singular support, compactly supported distributions, structure theorems, Fourier transform, Schwartz class functions, tempered distributions, convolution, Paley-Wiener theorems, fundamental solution of partial differential equations, Malgrange-Ehrenpreis theorem.

MA7364 INTRODUCTION TO CHAOS THEORY

L	T	P	C
3	0	0	3

Elementary definitions, Hyperbolicity, Chaos, Structural Stability, Sarkovskii's theorem, Morse-Smale diffeomorphisms, Homoclinic points and bifurcations, The kneading theory, Geneology of periodic points, The dynamics of linear maps, Attractors, The Henon map. Duffing oscillator, Chaos in the weather, dimension and attractor form, Spatially extended systems: coupled oscillators, Taylor-Couette flow, Preliminary Characterization: visual inspection & frequency spectra, Characterising chaos: Lyapunov exponents & dimension estimates, Attractor reconstruction, Embedding dimension for attractor reconstruction.

MA7365 MULTIVARIABLE CALCULUS

L	T	P	C
3	0	0	3

Differentiability in several variables, Inverse Function Theorem, Implicit Function Theorem, Rank Theorem, Riemann integration in higher dimensions, Fubini's Theorem, Green's theorem, Divergence Theorem, Stokes' Theorem, Tensors, Differential forms, Integration on chains, Stokes' Theorem for integrals of differential forms on chains, Differentiable manifolds (as subspaces of Euclidean spaces), Differential forms on manifolds, Integration on manifolds, Stokes' theorem on manifolds.

MA7366 NUMERICAL SOLUTION FOR PARTIAL DIFFERENTIAL EQUATIONS

L	T	P	C
3	0	0	3

Classification of PDEs, iterative methods for linear systems and their convergence, initial and boundary value problems, analysis of convergence, consistency and stability, Lax theorem, Von Neumann criterion for stability, numerical methods to solve parabolic, hyperbolic and elliptic partial differential equations.

MA7367 STATISTICAL METHOD FOR QUALITY MANAGEMENT

L	T	P	C
3	0	0	3

Design and Analysis of Experiments- Single Factor and Factorial Designs, Fitting Response Curves and Surfaces, Statistical Process Control, Control Charts and Decision Making, Cumulative Sum and Exponentially Weighted Moving Average Control Charts, Statistical Process Control Techniques, Process Capability Analysis, Acceptance Sampling for Attributes, Reliability Statistics-Reliability Definition, Reliability Bathtub Curve, Reliability Prediction, Testing System Reliability-Different Models for Testing System Reliability.

MA7368 ADVANCED OPERATIONS RESEARCH

L	T	P	C
3	0	0	3

Mathematical programming Problems - Unconstrained optimization - Constrained optimization with equality/ inequality constraints - Quadratic programming- Separable programming, Integer linear programming - Travelling salesman problem- knapsack problem.

MA7369 STOCHASTIC PROCESSES

L	T	P	C
3	0	0	3

Stochastic Processes, Discrete time Markov Chains, Poisson Processes, Birth-Death Processes, Continuous time Markov Chains, Renewal Processes, Queueing theory.

MA7370 NUMBER THEORY

L	T	P	C
3	0	0	3

Divisibility, Euclids lemma, Fundamental theorem of arithmetic, Diophantine equations, Arithmetic functions, Mobius inversion formula, residue systems, Linear congruences, Wilsons theorem, Chinese Remainder theorem, Polynomial congruences, Reduced residue systems, Prime numbers Tchebychevs theorem, Eulers criterion, Legendre and Jacobis symbols, Gauss lemma, Quadratic reciprocity law, Sum of two squares, Sum of four squares, Eulers pentagonal number theorem, Schurs theorem, Gausss circle problem, Dirichlets divisor problem.

MA7371 APPLIED STATISTICAL INFERENCE

L	T	P	C
3	0	0	3

A brief statistical inference-Uniformly most powerful tests, Invariance in estimation and testing, Admissibility, Minimax and Bayes estimation, Asymptotic theory of estimation, Asymptotic distribution of likelihood ratio statistics, Sequential estimation, Sequential probability ratio test. An introduction to categorical data, Two-way contingency tables, Table structure for two dimensions, Way of comparing proportions, Measures of associations, Sampling distributions, Goodness of fit tests, Testing independence. Models of binary response variables, Logistic regression, Logistic models for categorical data, Probit and extreme value models, Log-linear models for two and three dimensions, Fitting of logit and log-linear models, Log-linear and logit models for ordinary variables. Regression: A short introduction, Likelihood ratio test, Confidence intervals and hypothesis test, Test for distributional assumptions, Outliers, Analysis of residuals, Model building, Principal component and ridge regression. Lab component: relevant real life problems to be done in statistical packages like SAS, SPSS etc.

MA7372 REGRESSION ANALYSIS

L	T	P	C
3	0	0	3

Simple regression- one independent variable (X), assumptions, estimation of parameters, standard error of estimator, testing of hypothesis about regression parameters, standard error of prediction, Testing of hypotheses about parallelism, equality of intercepts, congruence. Extrapolation, optimal choice of X. Diagnostic checks and correction: graphical techniques, tests for normality, uncorrelatedness, homoscedasticity, lack of fit, modifications like polynomial regression, transformations on Y or X, WLS, inverse regression X(Y). Multiple regression: Standard Gauss Markov Setup, Least square(LS) estimation, Error and estimation spaces, Variance- Covariance of LS estimators, estimation of error variance, case with correlated observations, LS estimation with restriction on parameters, Simultaneous estimation of linear parametric functions, Test of Hypotheses for one and more than one linear parametric functions, confidence intervals and regions. Non Linear regression (NLS) : Linearization transforms, their use & limitations, examination of non-linearity, initial estimates, iterative procedures for NLS, grid search, Newton- Raphson , steepest descent,Marquardts methods. Logistic Regression: Logit transform, ML estimation. Tests of hypotheses, Wald test, LR test, score test, test for overall regression, multiple logistic regression, forward and backward method, interpretation of parameters relation with categorical data analysis, generalized linear model: link functions such as Poisson, binomial, inverse binomial, inverse Gaussian, gamma.

MA7373 RELIABILITY OF SYSTEMS

L	T	P	C
3	0	0	3

System reliability models, Series/parallel/mixed components, Redundancy techniques, Maintainability, Systems with repair, Hierarchical systems, Economics of reliability engineering, Reliability management, Life testing, Software reliability, Accelerated testing, Reliability allocation problems.

MA7374 FORECASTING TECHNIQUES

L	T	P	C
3	0	0	3

Fundamentals of Quantitative Forecasting, Smoothing Methods, Decomposition Methods, Regression and Econometric Methods, Multiple regression, Econometric models and forecasting, ARIMA Models for time-series, The Box-Jenkins Method, Forecasting with ARIMA models. Forecasting and Planning- Forecasting as input to planning and decision making. Comparison and selection of forecasting methods.

Syllabus of Core courses

MA6301 REAL ANALYSIS

Pre-requisites: Nil

Total Hours : 56

L	T	P	C
4	0	0	4

Module 1: [11 (L) Hours]

The real number system, supremum and infimum, completeness property, Archimedean property, density of rational numbers, countability, decimal representation. Open sets, closed sets, compact sets, perfect sets, HeineBorel theorem, Bolzano-Weierstrass theorem.

Module 2: [17 (L) Hours]

Numerical sequences: convergence, Cauchy sequences, subsequence, limit superior and limit inferior. Numerical Series: convergence, absolute convergence, re-arrangements, Cauchy product. Continuity: convergent series, connectedness, compactness, discontinuities, monotonic functions, infinite limits and limit at infinity, uniform continuity, extension of continuous functions.

Module 3: [17 (L) Hours]

Differentiation: Rolle's theorem, mean value theorem, L'Hôpital's rule, derivatives of higher order, Taylor's theorem, partial derivatives. Riemann Stieltjes integral, continuity, change of variable, fundamental theorem of calculus.

Module 4: [11 (L) Hours]

Uniform convergence: continuity, integration, differentiation. Power series, real analytic functions, transcendental functions, equicontinuity, Stone-Weierstrass theorem.

References:

1. W. Rudin, *Principles of Mathematical Analysis*, Tata-McGraw Hill, Third edition, 2013.
2. T. M. Apostol, *Mathematical Analysis*, 2nd edition., Narosa Book Distributors, 2002.
3. R. G. Bartle, *Introduction to Real Analysis*, Fourth Edition, Wiley India Publishers,2014.
4. N. L. Carothers, *Real Analysis*, Cambridge University Press, 2000.

Course Outcome: Students achieve a good grasp of the basic concepts of real analysis.

MA6302 LINEAR ALGEBRA

Pre-requisites: Nil

Total Hours : 56

L	T	P	C
4	0	0	4

Module 1: [13 (L) Hours]

Vector space, subspace, linear span of vectors, linear independence and dependence, basis, dimension, row equivalence, computations concerning subspaces.

Module 2: [16 (L) Hours]

Linear transformation, vector space of transformations, isomorphism, representation of a linear transformation by a matrix, linear functionals, double dual of a space, annihilator of a subset, transpose and inverse of a transformation.

Module 3: [12 (L) Hours]

Characteristic values, annihilating polynomials, Cayley-Hamilton theorem, invariant subspaces, direct sum decompositions, invariant direct sums, quotient space.

Module 4: [15 (L) Hours]

Inner-product, inner-product space, orthogonal basis, linear operator, adjoint and self-adjoint operator, normal and unitary operator, spectral theorems.

References:

1. K. Hoffman & R. Kunze, *Linear Algebra*, Second Edition, Pearson Education, 2015
2. P. R. Halmos, *Finite Dimensional Vector Spaces*, Springer, 2013.
3. S. Kumaresan, *Linear Algebra*, Prentice Hall of India, 2011.
4. S. Axler, *Linear Algebra Done Right*, Springer, 2004.

Course Outcome: Students get a strong foundation in Linear algebra as preparation for subsequent courses in Mathematics.

MA6303 NUMERICAL ANALYSIS

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [10 (L) Hours]

Computer Arithmetic, Errors. Root finding: The bisection method, The method of false position, Secant method, Fixed point iteration schemes, Newton's method- simple and multiple roots, Muller method, Chebyshev method, Rate of convergence, Accelerating convergence.

Module 2: [11 (L) Hours]

Linear system of equations: Gauss elimination, Pivoting strategies, Vector and matrix norms, Error estimates and condition number, LU decomposition. Iterative methods for linear systems: Gauss-Jacobi, Gauss-Seidel, SOR, Conjugate gradient method. Non-linear system of equations. Eigenvalues and Eigenvectors: The power method, Inverse power method, Given's method for symmetric matrices.

Module 3: [11 (L) Hours]

Interpolation: Lagrange interpolation, Neville's Algorithm, Newton Interpolation, Piecewise interpolation, quadratic and cubic spline interpolation, Hermite interpolation. Least squares approximation.

Module 4: [10 (L) Hours]

Numerical differentiation, Richardson extrapolation. Numerical integration: The basics and Newton-Cotes quadrature, Composite integration methods, Gaussian quadrature, Improper integrals. Numerical methods for ordinary differential equations: Initial value problems- Single step methods, Multi step methods, Predictor-Corrector methods.

References:

1. B. Bradie, *A Friendly Introduction to Numerical Analysis*, Pearson Prentice Hall, 2007.
2. C. E. Froberg, *Introduction to Numerical Analysis*, Addison Wesley, 1965.
3. C. F. Gerald & P.O. Wheatley, *Applied Numerical Analysis*, Pearson Education in South Asia, 7th edition, 2012.
4. M.K. Jain, S.R.K. Iyengar & R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, New Age International Publishers, 2014.
5. S.C. Chapra & R.P. Canale, *Numerical Methods For Engineers*, McGraw Hill, 5th edition., 2007.

Course Outcome: Understand the fundamental concepts of numerical analysis and methods.

MA6304 ORDINARY DIFFERENTIAL EQUATIONS

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [12 (L) Hours]

Basic concepts: The notion of a differential equations, Sources of differential equations: First order differential equations, Linear equations, Separable equations, Exact equations, Homogeneous first order equations, Bernoulli equations, Applications of first order equations, Existence and uniqueness of solutions, Cauchy-Peano and Picard-Lindelf theorems.

Module 2: [9 (L) Hours]

Second order linear differential equations, Linear dependence and Wronskian, Method of variation of parameters. The method of undetermined coefficients, Applications of linear second order equations.

Module 3: [11 (L) Hours]

Power series solutions of second-order equations, Legendre equation and Legendre polynomials, second order with regular and singular points, Frobenius method, Bessel function and its properties, Sturm-Liouville problem.

Module 4: [10 (L) Hours]

System of odes, systems with constant coefficients, Matrix methods for first order linear systems, Eigen values and Eigen vectors, Matrix exponentials, Autonomous differential equation, Phase-space, Stability, Basic definitions, Conditions for asymptotic stability, Lyapunov stability.

References:

1. E. A. Coddington & N. Levinson, *Theory of Ordinary Differential Equations*, Tata-McGraw Hill, 2012
2. W. Walter, *Ordinary Differential Equations*, Springer, 6th edition, 1996.
3. P. Blanchard, R. L. Devaney, & G. R. Hall, *Differential Equations*, Brooks/Cole, 3rd edition, 2006.
4. G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGraw Hill, 2nd edition, 1991.
5. D. Zill, *A First Course in Differential Equations with Modelling Applications*, Brooks/Cole, 6th edition, 1997.

Course Outcome: Understand linear and non-linear differential equations and methods of solutions. Get an idea to formulate a real world problem using ODEs or system of ODEs.

MA6305 COMPUTER PROGRAMMING

Pre-requisites: Nil

Total Hours : 70

L	T	P	C
2	0	3	4

Module 1: [7 (L) + 9(P) Hours]

Basics in C, data types, C character set, operators and expressions, control statements, branching, looping, library functions, functions, recursive functions, sorting.

Module 2: [7(L) + 12(P) Hours]

Strings, arrays, multidimensional arrays, passing arrays to functions, pointers, pointers to pointers, arrays using pointers, functions and pointers, structures, unions, structures and pointers.

Module 3: [6(L) + 9(P) Hours]

Basics in C++, object oriented programming, classes and objects, constructors and destructors, constructors with default arguments, dynamic arrays in C++.

Module 4: [8(L) + 12(P) Hours]

Friend functions, In-line functions, function overloading, operator overloading: overloading arithmetic, relational, increment, decrement operators, inheritance, virtual functions.

References:

1. V. Rajaraman, *Computer Programming in C*, Prentice Hall India, 1st edition, 1994.
2. B. Kernighan & D. Ritchie, *The C Programming Language*, Prentice Hall India, 2nd edition, 1995.
3. E. Balagurusamy, *Object Oriented Programming with C++*, Tata McGraw Hill, 6th edition, 2011.
4. J. R. Hubbard, *Programming with C++*, McGraw Hill, 2nd edition, 2000.

Course Outcome: Students learn and get trained for programming in C using strings, functions, pointers, structures. Students learn and get trained in object oriented programming.

MA6321 COMPLEX ANALYSIS

Pre-requisites: Nil

Total Hours : 56

L	T	P	C
4	0	0	4

Module 1: [13 (L) Hours]

Complex differentiability, Cauchy-Riemann equations, Comparison between differentiability in the real and complex senses, holomorphic functions, harmonic functions, harmonic conjugates, power series, conformal maps, bilinear transformations.

Module 2: [13 (L) Hours]

Line integral, Goursat's theorem, Cauchy's theorem for disc, Cauchy's integral formula, winding number of a closed curve, Cauchy's estimate, Liouville's theorem, fundamental theorem of algebra, Morera's theorem, Local uniform convergence, Weierstrass convergence theorem.

Module 3: [17 (L) Hours]

Zeros of holomorphic functions, Isolated singularities, Laurent's series, Riemann's theorem on removable singularity, pole, residue, evaluation of real integrals, essential singularity, Weierstrass theorem, order of a holomorphic function at a point, meromorphic functions, Riemann sphere, rational functions, argument principle, Rouch's theorem, open mapping theorem, maximum modulus principle, Schwarz's lemma, local mapping, inverse mapping, order of a rational function.

Module 4: [13 (L) Hours]

Homotopy and simply connected domain, general form of Cauchy's theorem, complex logarithm, Runge's theorem, characterisation of simply connected domains, mean value property of harmonic functions, Poisson's formula, Schwarz reflection principle.

References:

1. E. M. Stein & R. Shakarchi, *Complex Analysis*, Princeton University Press, 2010.
2. L. V. Ahlfors, *Complex Analysis*, McGraw Hill, 3rd edition, 2013.
3. D. Sarason, *Complex Function Theory*, Hindustan Book Agency, 2nd edition, 2008.
4. J. B. Conway, *Functions of one complex variable*, Narosa Publishers, 2nd edition, 2000.
5. R. A. Silverman, *Complex Analysis with Application*, Dover Publications, 1986.

Course Outcome: Understand important results and techniques of complex function theory.

MA6322 MEASURE AND PROBABILITY

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

Algebra of sets, σ -algebra, measure space, Borel σ -algebra, completion of measure, outer measure, Carathodory's extension, Lebesgue measure, probability measure, limit inferior and limit superior of a sequence of events, measurable functions, random variables, distribution function, Lebesgue integration, monotone convergence theorem, Fatou's lemma, dominated convergence theorem, L^p spaces.

Module 2: [10 (L) Hours]

Signed measure, Hahn decomposition, Jordan decomposition, total variation, absolute continuity, Lebesgue-Radon-Nikodym theorem, absolutely continuous random variables, probability density function, fundamental theorem of calculus.

Module 3: [10 (L) Hours]

Products of measurable spaces, product measure, monotone class lemma, Fubini's theorem, joint distribution and marginal distribution of random variables, independence of finite families of events, independence of random variables, independence and expectation.

Module 4: [11 (L) Hours]

Convergence in measure, Egoroff's theorem, almost sure convergence, convergence in mean, relation between modes of convergence, Chebyshev's inequality, weak law of large numbers for sequence of independent random variables, Borel-Cantelli lemma, strong law of large numbers, central limit theorem for sequence of iid random variables.

References:

1. H. L. Royden, *Real Analysis*, Prentice-Hall of India 4th edition, 2011
2. E. M. Stein & R. Shakarchi, *Real analysis: Measure Theory, Integration, and Hilbert Spaces*, Princeton University Press, 2005.
3. G. B. Folland, *Real analysis: Modern Techniques and Their Applications*, Wiley-Interscience, 2nd edition, 1999.
4. S. Athreya & V. S. Sunder, *Measure and Probability*, Universities Press, 2008.

Course Outcome: Students acquire a general overview of the basic results in measure theory, integration and probability.

MA6323 GRAPH THEORY

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [10(L) Hours]

Graphs: subgraphs - paths and cycles- isomorphism- cut vertex- bridge-block- bipartite graph- complement of a graph- line graph- degree sequence, Trees, Metric in graph: eccentricity - centre- median- centroid, Matrix representation of graph.

Module 2: [10 (L) Hours]

Connectivity : vertex and edge connectivity- Whitney's theorem- n - connected graphs- Mengers' theorem. Traversability : Hamiltonian graphs - Ore's theorem- Posa's theorem- other sufficient conditions for hamiltonicity, Euler graphs, Planar graphs: Euler formula- platonic bodies, Non planar graphs.

Module 3: [11 (L) Hours]

Matchings : maximum matching-perfect matching-matching in bipartite graphs, Factorisation : Coverings and independence, 1-factorisation, 2-factorisation, Arboricity, Domination: dominating set-domination number-total dominating set, total domination number.

Module 4: [11 (L) Hours]

Graph Colouring : chromatic polynomials- The four colour problem- The five colour theorem, Digraphs: connectedness - acyclic digraph- strong digraphs- tournaments, Directed trees: binary trees- weighted trees and prefix codes, Graph algorithms.

References:

1. G. Chartrand & P. Zhang, *Introduction to Graph Theory*, McGraw Hill, 2005.
2. J.A. Bondy & U.S.R. Murty, *Graph Theory*, Springer, 2008.
3. F. Buckley & F. Harary, *Distance in Graphs*, Addison - Wesley, 1990.
4. C. R. Foulds, *Graph Theory Applications*, Narosa Publishing House, 1994.
5. F. Harary, *Graph Theory*, Narosa Publishing House, 2011
6. R. P. Grimaldi, *Discrete and Combinatorial Mathematics: An Applied Introduction*, Addison Wesley, 1994.
7. R. Diestel, *Graph Theory*, Springer, 3rd edition, 2010

Course Outcome: Students acquire an overview of the concepts and techniques in Graph Theory.

MA6324 ABSTRACT ALGEBRA

Pre-requisites: Nil

Total Hours : 56

L	T	P	C
4	0	0	4

Module 1: [13 (L) Hours]

Binary operation, algebraic structures, group, subgroup, order of an element, cosets and Lagrange's theorem, generator of group, cyclic subgroup, normal subgroup and quotient subgroup, homomorphism, kernel and image of a homomorphism, isomorphism, homomorphism theorems.

Module 2: [13 (L) Hours]

Dihedral group, Permutation on n-symbol, group of permutations, odd, even and cyclic permutation, transposition, alternating group, Cayley's theorem for finite group, internal direct product of subgroups and external direct product of groups, group action on a set, Sylow's theorems, structure of finite abelian group.

Module 3: [14 (L) Hours]

Ring, commutative ring, subring, zero-divisor, characteristic of ring, integral domain, division ring, field, quotient field of an integral domain, ideals, operations on set of ideals: union, intersection, sum, product, maximal ideal, prime ideal, homomorphism, fundamental theorem of ring homomorphism, isomorphism.

Module 4: [16 (L) Hours]

Ring of polynomials, prime element, irreducible element, irreducibility criteria, Gauss's lemma, unique factorization domain, principle ideal domain, Euclidean function and Euclidean domain, ring of polynomials over a field, field extension, minimal polynomial, algebraic and transcendental element, algebraic extensions, splitting field of a polynomial, algebraic closure.

References:

1. C.C. Pinter, *A Book of Abstract Algebra*, Dover Publications, 2009.
2. T.W. Hungerford, *Algebra*, Springer, 2003.
3. I.N. Herstein, *Abstract Algebra*, John Wiley & Sons, 1996.
4. D.S. Dummit & R. M. Foote, *Abstract Algebra*, John Wiley, 2004.
5. J. A. Gallian, *Contemporary Abstract Algebra*, Brooks/Cole Cengage Learning, 2010.

Course Outcome: Students grasp the fundamental principles and theory concerning basic algebraic structures such as groups, rings, integral domains, fields and extension fields.

MA6325 TOPOLOGY

Pre-requisites: Nil

Total Hours : 56

L	T	P	C
4	0	0	4

Module 1: [14 (L) Hours]

Topological spaces, basis and sub-basis for a topology, order topology, product topology, subspace topology, quotient topology.

Module 2: [14 (L) Hours]

Continuous functions, metric topology, compact sets, compact sets in the real line, limit point compactness, sequential compactness, local compactness.

Module 3: [12 (L) Hours]

Connected sets, connected sets in the real line, components and path components, local connectedness.

Module 4: [16 (L) Hours]

Countability axioms, separation axioms, Urysohn Lemma, Tietze extension theorem, Urysohn metrization theorem, Tychonoff Theorem.

References:

1. J. R. Munkres, *Topology A First Course*, Pearson Education, 2000.
2. G. F. Simmons, *Introduction to Topology and Modern Analysis*, Tata-McGraw Hill, 2004.
3. K. D. Joshi, *Introduction to General Topology*, New Age Publications, 1999.

Course Outcome: Acquire knowledge about Topological spaces, Basis, and other fundamental concepts in Topology.

MA7301 PARTIAL DIFFERENTIAL EQUATIONS

Pre-requisites: Ordinary differential equations

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

Modelling with partial differential equations, Partial differential equations of first order, Cauchy problem, Linear first order P.D.E., Method of characteristics, Lagrange, Charpit's and Jacobi's method

Module 2: [9 (L) Hours]

Partial differential equation of second order, Classification of second order equation, Hyperbolic, Parabolic and Elliptic equations, Linear second order partial differential equations with constant coefficients.

Module 3: [11 (L) Hours]

Parabolic differential equations, One dimensional diffusion equation, Boundary conditions; Dirichlet, Neumann and Robin type boundary conditions, Method of separation of variables, Solutions in cylindrical and spherical equation, The maximum principle for the heat equation.

Module 4: [11 (L) Hours]

Hyperbolic differential equations, One dimensional wave equation, Solution of the wave equation by separation of variables, d'Alembert's solution, Boundary and initial value problem of two dimensional wave equation, Laplace equation, Harmonic function, Green's identity, Mean value theorem, Maximum principle, Green's function

References:

1. I. Sneddon, *Elements of Partial Differential Equations*, Dover Publications, Inc.,2006.
2. D. Greenspan, *Introduction to Partial differential Equations*, TMH Edition, 1961.
3. E. Kreyszig, *Advanced Engineering Mathematics*, John Wiley and Sons, 1995.
4. K. S. Rao, *Introduction to partial differential equations*, Phi Learning Pvt. Ltd., 2011.
5. K. S. Rao, *Introduction to partial differential equations*, Phi Learning Pvt Ltd, 2011.

Course Outcome: Learn modelling with partial differential equations and the basics of analytical methods to solve partial differential equations.

MA7302 FUNCTIONAL ANALYSIS

Pre-requisites: Nil

Total Hours : 56

L	T	P	C
4	0	0	4

Module 1: [15 (L) Hours]

Review of linear spaces and metric spaces, fundamentals of normed linear spaces, Riesz Lemma, continuity of linear maps, bounded linear maps, Banach spaces, Hahn-Banach theorems, uniform boundedness principle, closed graph theorem, open mapping theorem.

Module 2: [15 (L) Hours]

Spectrum of a bounded operator, linear functional, dual space, Riesz representation theorem for $L^p([a, b])$ and $C([a, b])$, transpose of a bounded linear operator, weak and weak* convergence, reflexivity.

Module 3: [13 (L) Hours]

Inner product spaces, orthonormal sets, Hilbert spaces, Bessel's inequality, Riesz - Fisher theorem, projection operator, Riesz representation theorem.

Module 4: [13 (L) Hours]

Bounded operators on Hilbert spaces, adjoint of a bounded operator, normal operators, unitary operators, self-adjoint operators, spectrum and numerical range.

References:

1. B.V. Limaye, *Functional Analysis*, New Age Publishers, 2nd edition, 2006.
2. E. Kreyszig, *Introductory Functional Analysis with applications*, Wiley Eastern, 1989.
3. G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw - Hill, 2004.
4. J. B. Conway, *A course in Functional Analysis*, Springer, 2nd edition, 1990.

Course Outcome: Students understand the basic results about normed linear spaces and linear transformation defined on these spaces.

MA7303 MATHEMATICAL STATISTICS

Pre-requisites: Nil

Total Hours : 56

L	T	P	C
4	0	0	4

Module 1: [15 (L) Hours]

Axioms of Probability, Conditional Probability and Independence, Random Variables, Discrete and Continuous Random Variables, Moments, Expectation of a Function of a Random Variable, Moment Generating Function and Characteristic Function.

Module 2: [13 (L) Hours]

Random Vectors, Jointly Distributed Random Variables, Joint Probability Distributions, Conditional expectation, Bivariate Normal Distribution, Transformations of a Random Variable, Transformations of Random Vectors, Order statistics, Chebyshev's Theorem.

Module 3: [15 (L) Hours]

Population and Samples, Sampling Distribution of the Mean and Variance, Point Estimation, Maximum Likelihood Estimation, Method of Moments, Properties of Estimators, Tests of Hypothesis, Uniformly Most Powerful (UMP) Tests, Neyman-Pearson Lemma, Inference Concerning One Mean and Two Means, Inference Concerning One Variance and Two Variances, Inference Concerning One Proportion and several Proportions, Chi Square Test for Goodness of Fit.

Module 4: [13 (L) Hours]

Regression and Correlation- Bivariate Relationships, Correlation Coefficient, The Two Variable Linear Regression Model, Least Square Estimation, Inference in the Two-Variable Linear Regression Model, Analysis of Variance in the Two Variable Linear Regression Model, Prediction in the Two Variable Linear Regression Model, The K-Variable Linear Regression Model, Equation- Matrix Formulation in the K-Variable Model, Partial Correlation Coefficients, Inference in the K- Variable Equations, Prediction, Some Standard Non-linear Regression Models

References:

1. S. Ross, *A First Course in Probability*, 9th Edition, Pearson, 2014
2. R. V. Hogg, J. McKean, and A. T. Craig, *Introduction to Mathematical Statistics*, 7th Edition, Pearson Education, 2012.
3. W. H. Hines, Montgomery, et. al., *Probability and Statistics for Engineering*, John Wiley & Sons, Inc., 2003.
4. R. V. Hogg & E. A. Tanis, *Probability and Statistical Inference*, 6th Edition Pearson, 2001.
5. J. Johnston and J. DiNardo, *Econometric Methods*, 4th Edition, The McGraw- Hill, 1997.

Course Outcome: Students learn the fundamentals of Mathematical Statistics, and get equipped in the theory and applications of it.

MA7304 OPERATIONS RESEARCH

Pre-requisites: Linear Algebra

Total Hours : 56

L	T	P	C
4	0	0	4

Module 1: [14 (L) Hours]

System of linear equations and inequations-Convex functions-formulation of linear programming problem-Theory of simplex method-Simplex Algorithm-Charne's M-Method-Two phase method Computational complexity of simplex Algorithm - Karmarker's Algorithm.

Module 2: [14 (L) Hours]

Duality in linear programming Dual simplex method- Sensitivity analysis-Bounded variable problem- Transportation problem-Integrity property-MODI Method-Degeneracy -Unbalanced problem Assignment problem-Development of Hungarian method-Routing problems

Module 3: [14 (L) Hours]

Nature of Dynamic programming problem - Bellmann's optimality principle-Cargo loading problem- Replacement problem - Multistage production planning and allocation problem-Rectangular Games - Two persons zero sum games - Pure and mixed strategies-2xn and mx2 games - Relation between theory of games and linear programming.

Module 4: [14 (L) Hours]

Critical path analysis-Probability consideration in PERT. Distinction between PERT and CPM. Resources Analysis in networking scheduling -Time cost optimization algorithm-Linear programming formulation-Introduction to optimization softwares.

References:

1. M. S. Bazaara, J.J. Jarvis, H.D. Sherali, *Linear programming and Network flows*, John Wiley, 2nd edition, 1990.
2. M.S. Bazaara, H.D. Sherali & C.M. Shetty, *Nonlinear programming Theory and Algorithms*, John Wiley, 2nd edition, 1993.
3. G. Hadley, *Linear programming*, Narosa Publishing House, 1990.
4. H.A. Taha, *Operation Research-Introduction*, Prentice Hall India, 7th edition, 2006.

Course Outcome: Students learn mathematical techniques that will help them to understand and analyse managerial problems in industry so that resources (capitals, materials, staffing, and machines) may be utilized more effectively.

MA7321 METHODS IN APPLIED MATHEMATICS

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [9 (L) Hours]

Fourier Series, Dirichlet's conditions, convergence theorem, other forms of Fourier series. integral transforms, Fourier integral, Gibbs phenomenon, properties and applications of Fourier transforms, Fourier integral to the Laplace transformation, finite Fourier transforms, finite Fourier sine and cosine transforms, convolution theorem, multiple finite Fourier transforms and applications.

Module 2: [11 (L) Hours]

Integral equations, classification, relation between differential and integral equations, Neumann's iterative method for Fredholm's equation of second kind, Volterra type integral equations, integral equations of first kind, solution of integral equations, Fredholm equations with separable Kernels, iterative methods for the solution of integral equations of the second kind, resolvent kernels, eigenvalues and eigenfunctions.

Module 3: [11 (L) Hours]

Tensor analysis, coordinate transformations, contravariant, covariant and metric tensors, fundamental operation with tensors, quotient law, line element and metric tensors, conjugate Tensor, Christoffel symbols, covariant differentiation of tensors.

Module 4: [11 (L) Hours]

Calculus of variation, method of variations in problems with fixed boundaries, variation of a functional, Euler's equation, functionals involving derivatives of higher order, functionals depending on functions of several independent variables, variational, problems of constrained extrema, Rayleigh-Ritz method.

References:

1. Wylie C. R., & Barrett, L. C., *Advanced Engineering Mathematics*, 6th edition, McGraw-Hill, Inc., 1995.
2. Sneddon, I. N., *The Use of Integral Transforms*, McGraw Hill, 1972.
3. Hilderbrand F. B., *Methods of Applied Mathematics*, Prentice Hall of India, 1961.
4. Spain B., *Tensor Calculus*, Oliver and Boyd, London, 1988.
5. Riley, K.F., Hofson M.P. & Bence, S.J., *Mathematical methods for Physics and Engineering*, Cambridge University Press, 1998.
6. Elsgolc, L. E., *Calculus of Variations*, Pergamon Press Ltd., 1961.

Course Outcome: Students acquire an overview of the mathematical tools which are useful in many engineering applications .

Syllabus of Elective courses

MA7350 ADVANCED TOPICS IN GRAPH THEORY

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [9 (L) Hours]

Graphs: review of basics in graphs - trees- blocks- matrices-operations on graphs, Connectivity: vertex connectivity and edge connectivity, n- connected graphs-Menger's Theorem, Traversability: Euler graphs-Hamiltonian Graphs-Planar and Nonplanar graphs.

Module 2: [12 (L) Hours]

Metric in graph: centre- median- eccentric vertex- eccentric graph- boundary vertex-complete vertex- interior vertex, Convexity: closure invariants-gin(G) gn(G)-hull number- geodetic graphs, Symmetry: graphs and groups- symmetric graphs - distance symmetry-distance transitive graphs-distance regular graphs, Distance Sequences :degree sequence, eccentric sequence - distance sequences - distance distribution- mean distance.

Module 3: [10 (L) Hours]

Coloring in graphs - vertex coloring-chromatic number- chromatic polynomial- restricted vertex coloring- uniquely colorable graphs- list coloring, Edge coloring of graphs : the chromatic index and Vizing's theorem- list edge coloring , Total coloring of graphs, Complete coloring , Achromatic number, Applications of coloring.

Module 4: [11 (L) Hours]

Digraphs: digraphs and connectedness- tournaments, Networks : Flows, cuts, The Max- Flow Min-Cut Theorem, labeling procedure , Weighted graphs : totally weighted interconnection networks-generalization of Menger's theorem- cyclic cutnodes and cyclic bridges - cycle connectivity.

References:

1. G. Chartrand & P. Zhang, *Introduction to Graph Theory*, McGraw Hill, 2005.
2. J.A. Bondy & U.S.R.Murty, *Graph Theory*, Springer, 2008.
3. F. Buckley & F. Harary, *Distance in Graphs*, Addison - Wesley, 1990.
4. G. Chartrand & P. Zhang, *Chromatic Graph Theory*, CRC Press, 2009.
5. R. P. Grimaldi, *Discrete and Combinatorial Mathematics: An Applied Introduction*, Addison Wesley, 1994.
6. R. Diestel, *Graph Theory*, Springer,2006.

Course Outcome: Strengthening of ability for critical thinking on problem solutions in the area. The student gets trained in breaking down a mathematical problem into simpler statements and synthesize proofs.

MA7351 ADVANCED TOPOLOGY

Pre-requisites: MA6325 Topology

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [10 (L) Hours]

Topological spaces: definition, basis, sub basis, Fundamental examples, subspace - Closure: closed sets, limit points, Hausdorff spaces. Continuity: equivalent definitions, homeomorphisms and embeddings. Product topology: basis and subbasis, boxproduct.

Module 2: [10 (L) Hours]

Separation Axioms and Covering Properties - Separation axioms: Hausdorff, regular, Tychonoff, and normal topological spaces, Covering properties: Compactness, Lindelofness, paracompactness, metacompactness, Relations between covering properties and separation axiom, Normality of paracompact spaces, paracompactness of Lindelof spaces, Preservation of separation and covering properties.

Module 3: [12 (L) Hours]

Metrizability and Connectedness - The metrization theorems of Urysohn and Bing, Smirnov, Nagata. Connectedness and total disconnectedness: Definitions and examples of connectedness, total disconnectedness, zero-dimensionality. Local properties: Local compactness, local connectedness.

Module 4: [10 (L) Hours]

Fundamental Group - Homotopy: homotopy of paths, Fundamental group and covering spaces, simply connected spaces, Fundamental group of the circle, Deformation retraction: fundamental group of the punctured plane.

References:

1. J. R. Munkres, *Topology - A first course*, Prentice Hall of India, 2000.
2. K. D. Joshi, *Introduction to general Topology*, New age International Ltd., 1999.
3. G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 1963.
4. J. Dugundji, *Topology*, Universal Book stall, 1995.
5. S. Willard, *General Topology*, Addison-Wesley, 1970.

Course Outcome: Review the fundamentals of Topology. Acquire knowledge about Separation Axioms, Metrizability, Fundamental group, covering spaces and Deformation retraction.

MA7352 ALGEBRAIC TOPOLOGY

Pre-requisites: MA6325 Topology

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [10 (L) Hours]

Review of basic Topology: Topological spaces, subspaces, Limit points, closure, frontier, Continuous mapping, Compactness, Arcwise connected spaces.

Module 2: [10 (L) Hours]

Fundamental group: Homotopy, homotopy classes, Fundamental group, Change of base point, Topological invariance.

Module 3: [12 (L) Hours]

Homology groups: Geometrical motivation, Euclidean simplexes, Linear mappings, singular simplexes, chains, Boundary of a simplex, Boundaries and cycles on any space, Homologous cycles and homology groups, Relative homology.

Module 4: [10 (L) Hours]

Induced Homomorphisms, Topological invariance of homology groups, Homotopic mappings and the homology groups, Prisms, Homology sequences, Simplicial complexes.

References:

1. A. H. Wallace, *An Introduction to Algebraic Topology*, Pergamon Press, 1957.
2. CRF. Maunder, *Algebraic Topology*, Cambridge University Press, 1996.
3. S. Deo, *Algebraic Topology A primer*, Hindustan Book agency, 2003.

Course Outcome: Acquire knowledge about Homotopy and Homology.

MA7353 FLUID MECHANICS

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11(L) Hours]

Fundamental conceptions about fluids, Some mathematical concepts and notations, Kinematics of fluids, The Transport Theorem, Continuity equation, Equation of motion, Stress tensor, Euler's equation, Navier-Stokes equations.

Module 2: [11(L) Hours]

The energy equation, Boundary and initial conditions, Dimensional analysis, Buckingham's Pi theorem, Transformation of Cartesian Coordinates, Curvilinear Coordinates, Boussinesq approximation .

Module 3: [10(L) Hours]

Basic properties of irrotational flow, Incompressible and irrotational flow, Stream lines, Path lines, Streak lines, Plane potential flow, Laplace equation, Bernoulli equation, Application of Bernoulli's equation, Vorticity dynamics, Kelvin's circulation theorem.

Module 4: [10(L) Hours]

Laminar flow, Steady flow in a pipe, Steady flow between concentric cylinders, Flow due to an oscillating plate, High and low Reynold's number flows, Creeping flow around a sphere, Boundary layer theory, Similarity solutions, Blasius solution.

References:

1. M. Feistauer, *Mathematical Methods in Fluid Dynamics*, Longman Scientific and Technicals, 1993.
2. A. J. Chorin & J. E. Marsden, *A mathematical introduction to fluid mechanics*, Springer, 3rd Edn., 1993
3. L. D. Landau & E. M. Lifshitz, *Fluid Mechanics*, Pergamon Press, 3rd Edn., 1966.
4. J. H. Spurk & N. Aksel, *Fluid Mechanics*, Springer-Verlag, 2nd Edn., 2007.
5. G. K. Batchelor, *An Introduction to Fluid Dynamics*, Cambridge Univ. Press, Cambridge, 1967.
6. P.K. Kundu & I.M.Cohen, *Fluid Mechanics*, Academic Press, 2008.

Course Outcome: Learn the properties of fluids and fundamentals of fluid dynamics.

MA7354 FOURIER ANALYSIS

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

Trigonometric polynomials and trigonometric series, Fourier series in $L^1(T)$, Riemann-Lebesgue lemma, convolutions, convergence, Fejer's theorem, Fourier series of continuous functions, Fourier series in $L^2(T)$, Bessel inequality, Parseval identity, Plancherel theorem, Fourier series of L^p functions.

Module 2: [11(L) Hours]

L^p spaces, Convolution of functions, Young's inequality, approximate identity, Regularisation of functions, Pointwise convergence, Fourier transform in $L^1(R)$ - Riemann-Lebesgue lemma, Multiplication formula, Inversion, Translations and dilations, Multiplication and differentiation.

Module 3: [10(L) Hours]

Abel means and Poisson kernel, Uniqueness theorem for Fourier transform in $L^1(R)$, Fourier transform in $L^2(R)$, Multiplication formula, Plancherel theorem, Uniqueness theorem in $L^2(R)$, Harmonic functions, Dirichlet problem for the upper half plane, Point wise and norm convergence, Dirichlet problem for the disc.

Module 4: [10(L) Hours]

Eigen functions of Fourier transform, Gaussian, Hermite functions, Schwartz space, Paley-Wiener space Paley-Wiener theorem, Uncertainty principle, Hardy classes, Hardy's theorem.

References:

1. H. Dym & H. P. McKean, *Fourier Series and Integrals*, Academic Press, 1985.
2. Y. Katznelson, *An Introduction to Harmonic Analysis*, Cambridge University Press, 2004.
3. H. Helson, *Harmonic Analysis*, Hindustan Book Agency and Helson Publishing Co., 1995.
4. E. M. Stein & R. Shakarchi, *Fourier Analysis: An Introduction*, Princeton University Press, 2003.
5. C. Sadosky, *Interpolation of Operators and Singular Integrals: An Introduction to Harmonic Analysis*, Marcel Dekker, Inc., 1979.

Course Outcome: Understand the basic results in the Fourier analysis on Euclidean spaces.

MA7355 FUZZY SET THEORY AND APPLICATIONS

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [12 (L) Hours]

Crisp sets and Fuzzy sets - Introduction, crisp sets an overview, the notion of fuzzy sets basic concepts of fuzzy sets, membership functions, methods of generating membership functions, defuzzification methods- operations on fuzzy sets - fuzzy complement , fuzzy union, fuzzy intersection, combinations of operations, general aggregation operations.

Module 2: [11 (L) Hours]

Fuzzy arithmetic and Fuzzy relations: Fuzzy numbers- arithmetic operations on intervals- arithmetic operations on fuzzy numbers- fuzzy equations, Fuzzy relations : binary relations, binary relations on a single set, equivalence and similarity relations, compatibility or tolerance relations.

Module 3: [10 (L) Hours]

Fuzzy measures, belief and plausibility measures, probability measures, possibility and necessity measures, possibility distribution - relationship among classes of fuzzy measures.

Module 4: [9 (L) Hours]

Fuzzy Logic and Applications: Classical logic : an overview, fuzzy logic, approximate reasoning - other forms of implication operations - other forms of the composition operations, fuzzy decision making fuzzy logic in database and information systems - fuzzy pattern recognition, fuzzy control systems, fuzzy optimization.

References:

1. G. J. Klir & T. A. Folger , *Fuzzy sets, Uncertainty and Information*, Prentice Hall of India, 1988.
2. H.J. Zimmerman, *Fuzzy Set theory and its Applications*, Kluwer Academic Publishers, 4nd Edn., 2001.
3. G. J. Klir & B. Yuan, *Fuzzy sets and Fuzzy logic: Theory and Applications*, Prentice Hall of India, 1997.
4. H. T. Nguyen & E. A. Walker, *First Course in Fuzzy Logic*, Chapman & Hall, 2nd Edn., 1999.
5. J. M. Mendel, *Uncertain Rule, Based Fuzzy Logic Systems ; Introduction and New Directions*, PH PTR, 2000.
6. T. J. Ross, *Fuzzy Logic with Engineering Applications*, McGraw Hill, 1997.
7. J. J. Buckley, E. Eslami , *An Introduction to Fuzzy logic and Fuzzy sets*, Springer , 2002.

Course Outcome: Provides the basic concepts in fuzzy sets and fuzzy relations.
Enhancement of the ability to solve problems based on fuzzy arithmetic.

MA7356 GENERALISED SET THEORY

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [10 (L) Hours]

An overview of basic operations on Fuzzy sets and Multisets, Multiset relations, Compositions, equivalence multiset relations and partitions of multisets, Multiset functions, Fuzzy Multisets

Module 2: [11 (L) Hours]

Rough sets, Approximations of a set, Properties of Approximations, Rough membership function, Rough sets and Reasoning from data: Information systems, Decision tables, Dependency of attributes, Reduction of attributes, Indiscernibility matrices and functions.

Module 3: [11 (L) Hours]

Soft sets, Tabular representation of a soft set, Operations with Soft sets: soft subset, complement of a soft set, null and absolute soft sets, AND and OR operations, Union and intersection of soft sets, DeMorgan laws, Applications and soft analysis.

Module 4: [10 (L) Hours]

Fuzzy soft sets, Operations on fuzzy soft sets, Soft fuzzy sets and its properties, Fuzzy rough sets and rough fuzzy sets, Rough multisets, Genuine sets, Applications.

References:

1. Bing-Yuan Cao, *Fuzzy Information and Engineering*, Springer, 2007.
2. K. P. Girish & J. J. Sunil, *Relations and Functions in Multiset context*, Information Sciences' 179 (2009) 758 - 768.
3. J. F. Peters & A. Skowron, *Transactions on Rough Sets I*, Springer, 2004.
4. L. Polkowski, *Rough Sets: Mathematical Foundations*, Springer, 2002.
5. M. Demirci, *Genuine Sets*, Fuzzy Sets and Systems, 105 (1999) 377-384.
6. H.J. Zimmerman, *Fuzzy set Theory and its Applications*, Allied Publishers Ltd., 2000.

Course Outcome: Students get an advanced level understanding of Generalized set structures such as Fuzzy sets, Multisets, Rough sets, Soft sets, Rough multisets, Genuine sets Information systems.

MA7357 NUMERICAL LINEAR ALGEBRA

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [10 (L) Hours]

Review Linear Algebra Basic Concepts, Conditioning and Stability; Condition numbers, Floating point arithmetic , Stability of various algorithms, Linear Equation Solving: Gaussian Elimination, Pivoting, Stability of Gaussian Elimination, Cholesky Factorization, Jordan canonical form and applications.

Module 2: [12 (L) Hours]

Positive definite systems, LU decomposition, Orthogonal Matrices, Projectors and QR Factorization, Gram-Schmidt Process, Householder Transformation, Least Square Problems, Numerical Computation of Eigenvalues and Eigenvectors : Gerschgorin's Method, Power method, Jacobi's and Givens method, Hessenberg form, inverse iteration, Schur factorization, QR algorithm, sensitivity of eigenvalues and eigenvectors .

Module 3: [10 (L) Hours]

Singular Value Decomposition(SVD), Computing the SVD, applications, QR algorithm for SVD

Module 4: [10 (L) Hours]

Generalized inverses of matrices , computing the Moore- Penrose generalized inverse of a matrix.

References:

1. L.N. Trefethen & David Bau III, *Numerical Linear Algebra*, SIAM, 1997.
2. D. S. Watkins, *Fundamentals of Matrix Computations*, John Wiley & sons, 2rd Edn., 2002.
3. G. Golub& C.V. Loan, *Matrix Computations*, John Hopkins University Press, 3rd Edn., 1996
4. K. Hoffman & R. Kunze, *Linear Algebra*, Prentice Hall of India, 1971.

Course Outcome: Understand modern methods of numerical linear algebra for solving linear systems and least squares problems.

MA7358 OPERATOR THEORY

Pre-requisites: Functional Analysis

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [10 (L) Hours]

Banach algebras, Gelfand theory, C^* - algebras the GNS construction, spectral theorem for normal operators, Fredholm operators and its properties, semi-Fredholm operators, product of operators.

Module 2: [10 (L) Hours]

Hilbert Space Operators, Parts of Spectrum, Orthogonal Projections, Invariant Subspaces, Reducing Subspaces, Shifts, Decompositions of Operators. Compact linear operators, Spectral properties of compact bounded linear operators, spectral theorem and functional calculus for compact normal operators .

Module 3: [12 (L) Hours]

Spectral projections, spectral decomposition theorem, spectral theorem for a bounded normal operator, Measures of operators. Perturbation classes, strictly singular operators, Spectral theory of integral operators: Hilbert Schmidt theorem, Mercer,s theorem, Trace formula for integral operators, integral operators as inverse of differential operators. Sturm- Liouville systems.

Module 4: [10 (L) Hours]

Unbounded operators: Basic theory of unbounded self adjoint operators, unbounded Fredholm operators and its properties, essential spectrum, unbounded semi-Fredholm operators, Spectral theorem for an unbounded self adjoint operators.

References:

1. M. Schechter, *Principles of Functional Analysis*, AMS, 2th Edn., 2002.
2. I. Gohberg & S. Goldberg, *Basic operator Theory*, Birkhauser, 1981.
3. M. Ahues, A. Largillier B.V. Limaye, *Spectral Computations for Bounded Operators*, Chapman & Hall/CRC, 2001.
4. J. B. Conway, *A course in Functional Analysis*, Springer-Verlag, 2th Edn., 1990.
5. B.V. Limaye, *Functional Analysis*, New Age Publishers, 2th Edn., 2006.
6. F. Riesz & B. SzNagy, *Functional Analysis*, Dover Publications, 1990.
7. W. Rudin, *Functional Analysis*, McGraw Hill, 2th Edn., 2006.
8. V. S. Sunder, *Functional analysis' spectral theory*, Birkhauser, 1998.
9. R. G. Douglas, *Banach Algebra Techniques in Operator Theory*, Academic Press, 1972.
10. N. Dunford & J.T. Schwartz, *Linear operators, part I & part II*, Inter science, Newyork, 1958.
11. G.J. Murphy, *C^* -Algebras and Operator Theory*, Academic Press Inc., 1990.

Course Outcome: Students get an understanding of bounded and unbounded operators.

MA7359 SPECTRAL THEORY OF HILBERT SPACE OPERATORS

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [10 (L) Hours]

Elements of Hilbert space theory, Bounded linear operators on Hilbert spaces, Bounded linear functionals, projection, Riesz representation theorem, Adjoint, Self adjoint, Unitary Normal operators.

Module 2: [10 (L) Hours]

Spectral properties of bounded linear operators, Resolvent and spectrum, spectral theory, Complex analysis in spectral theory.

Module 3: [11 (L) Hours]

Compact linear operators, spectral theory of compact self adjoint operators; Formula for the inverse operator, Minimum-maximum Properties of eigenvalues, compact normal operators, Operator equations, Fredholm alternative.

Module 4: [11 (L) Hours]

Spectral properties of bounded self adjoint linear operators, Positive operators, Square root of an operator, Projection operators, spectral family and spectral family of bounded self adjoint linear operator, spectral representation of bounded self adjoint Linear operators, Extension of spectral theorem to continuous.

References:

1. I. Gohberg & S. Goldberg, *Basic operator Theory*, Birkhauser, 1981.
2. M. Reed & B. Simon, *Methods in Mathematical Physics*, Academic Press, 1986.
3. R. Courant & D. Hilbert, *Methods of mathematical Physics*, Interscience, 1996.
4. B.V. Limaye, *Functional Analysis*, New Age Publishers, 2nd Edn., 2006.
5. E. Kreyszig, *Introductory Functional Analysis with applications*, Wiley Eastern, 1989.
6. J. B. Conway, *A course in Functional Analysis*, Springer, 2nd Edn., 1990.
7. W. Rudin, *Functional Analysis*, McGraw Hill, 2nd Edn., 2006.

Course Outcome: Students learn spectral properties of operators defined on Hilbert spaces.

MA7360 INTRODUCTION TO FRACTAL GEOMETRY

Pre-requisites: Topology, Real Analysis, Measure Theory

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

The matrix space $(H(X),h)$, the completeness of space of fractals, transformations on the real line, affine transformations in the Euclidian plane, Mobius transformations on the Riemann sphere, Analytic transformations, contraction mapping theorem.

Module 2: [11 (L) Hours]

The addresses of points on fractals, continuous transformations from code space to fractals, dynamical systems, dynamics on fractals, equivalent dynamical systems, shadow of deterministic dynamics, shadowing theorem, chaotic dynamics on fractals.

Module 3: [10 (L) Hours]

Fractal dimension, theoretical and experimental determination of fractal dimension, Hausdorff-Besicovitch dimension. Applications for fractal functions, fractal interpolation functions, the fractal dimension of fractal interpolation functions, hidden variable fractal interpolation.

Module 4: [10 (L) Hours]

Space filling curves, escape time algorithm, Julia sets, IFS for Julia sets, Application of Julia sets to Newton's method Invariant sets of continuous open mappings, map of fractals, Mandelbrot's sets, Mandelbrot's sets for Julia sets.

References:

1. K. Falconer, *Fractal Geometry*, Wiley, 2nd Edn., 1990.
2. M. F. Barnsley, *Fractals everywhere*, Morgan Kaufmann, 2nd Edn., 1993.
3. B.B. Mandelbrot, *The fractal geometry of Nature*, W.H. Freeman and Company, New York, 1982.
4. Peitgen, Jurgens & Saupe, *Chaos and Fractals*, Springer- Verlag, 1992.

Course Outcome: Students get basic knowledge of fractal geometry and related applications

MA7361 ADVANCED COMPLEX ANALYSIS

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [10 (L) Hours]

Review of power series, Taylor and Laurent series, Entire functions: Jenson's Formula, Blaschke Products, Hadamard's theorem. The Riemann zeta function, The product development, Extension of whole plane, Functional equations, Zeros of the zeta function.

Module 2: [11 (L) Hours]

Normal Families, Equicontinuity, Normality and compactness, Arzela's Theorem, Families of analytic functions, The classical definition, Riemann mapping theorem: Statement and proof, Boundary Behavior, Use of the reflection principle, Analytic arcs.

Module 3: [11 (L) Hours]

Elliptic Functions, Simply periodic functions, Representation by exponents, Fourier Development, Functions of finite order, Doubly periodic functions, Period Module, Unimodular transformations, Canonical Basis, General properties of elliptic functions.

Module 4: [10 (L) Hours]

Weierstrass Theory, Germs and Sheaves, Sections of Riemann Surfaces, Analytic continuation along arcs, Homotopic curves, The Monodromy theorem, Branch points, the Picard theorem, sub-harmonic functions, spaces p H and N, factorization theorems, shift operator, conjugate functions

References:

1. M. J. Ablowitz and A. S. Fokas, *Complex Variables-Introduction and Applications*, Cambridge University Press, 2nd Edn., 2003.
2. L. V. Ahlfors, *Complex Analysis*, McGraw Hill, 3rd Edn., 1979.
3. H. Cartan, *Elementary Theory of Analytic Functions of one or several Complex variables*, Addison Wesley, 1st Edn., 1995.
4. J. B. Conway, *Functions of one complex variable*, Narosa Publishers, 2nd Edn., 2000.
5. Y. K. Kwok, *Applied Complex Variables for Scientists and Engineers*, Cambridge University Press, 2nd Edn., 2010.
6. Z. Nehari, *Introduction to Complex Analysis*, Allyn and Bacon, 1961.
7. R. A. Silverman, *Complex Analysis with Application*, Dover Publications, 1974.

Course Outcome: Students acquire a better understanding of important concepts and techniques in advanced complex analysis.

MA7362 DIFFERENTIAL GEOMETRY

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

Graphs and Level sets. Vector fields, The Tangent space. Curves and Surfaces, Vector fields on surfaces, Tangent, Curvature, Principal normal, Binormal, torsion, The Frenet Formulas, Involutives and evolutes, the tangent surface, Orientation. Developable surfaces.

Module 2: [11 (L) Hours]

Gauss Map, Geodesics, Parallel transport-The Weingarten Map - Curvature of plane curves, second fundamental theorem, Equation of Gauss and Codazzi, Lines of curvature of a surface, tangential coordinates of a surface, Parallel surfaces.

Module 3: [10 (L) Hours]

Arc lengths and line integrals- Curvature of surfaces, Intrinsic equation of a curve, Linear element, Element of area, Intrinsic geometry.

Module 4: [10 (L) Hours]

Parameterized surfaces- Local Equivalence of surfaces and parameterized surfaces, Differential parameters, Isometric surfaces, Geodesic curvature, normal curvature, minimal surfaces.

References:

1. J.A. Thorpe, *Elementary Topics in Differential Geometry*, Springer Verlag.
2. B Oneill, *Elementary differential Geometry*, Academic - New York.
3. Do Carmo M, *Differential Geometry of curves and surfaces*, Englewood Cliffs, Prentice Hall, 1976.
4. R. Millman & G. Parker, *Elements of differential Geometry*, Englewood Cliffs, Prentice Hall, 1977.

Course Outcome: Students get a general overview
of the basic results in the theory of differential geometry.

MA7363 DISTRIBUTION THEORY

Pre-requisites: MA7302 Functional Analysis / MA3021 Functional Analysis and Applications

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

Test functions, partition of unity, distributions, order of distribution, convergence of distribution, derivative of distribution, multiplication of distribution by a function.

Module 2: [11 (L) Hours]

Local equality of distributions, support of distribution, singular support of distribution, compactly supported distributions, semi-norms, distributions with point support, structure theorems.

Module 3: [10 (L) Hours]

Fourier transform in $L^1(\mathbb{R})$, Schwartz class functions, Riemann-Lebesgue Lemma, inversion, translations and dilations, multiplication and differentiation, Fourier transform in $L^2(\mathbb{R})$, Plancherel theorem.

Module 4: [10 (L) Hours]

Tempered distributions, convolution, Fourier transform, PaleyWiener theorems, distributions as solution to partial differential equations, fundamental solution, Malgrange-Ehrenpreis theorem.

References:

1. W. Donoghue, *Distributions and Fourier Transforms*, Academic Press, 1969.
2. S. Kesavan, *Topics in Functional Analysis and Applications*, Wiley, 1989.
3. W. Rudin, *Functional Analysis*, Mcgraw-Hill, 1991.
4. R. Strichartz, *A Guide to Distribution Theory and Fourier Transforms*, World Scientific, 2003.

Course Outcome: Students get a general overview of the basic results in the theory of generalised functions and Fourier analysis.

MA7364 INTRODUCTION TO CHAOS THEORY

Pre-requisites: Topology, Real Analysis, Functional Analysis

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

Elementary definitions, Hyperbolicity, The quadratic family, Symbolic dynamics, Topological Conjugacy, Chaos, Structural Stability, Sarkovskii's theorem, The Schwarzian derivative, Bifurcation theory, Maps of the circle, Morse-Smale diffeomorphisms, Homoclinic points and bifurcations, The period-doubling route to chaos, The kneading theory, Geneology of periodic points.

Module 2: [11 (L) Hours]

Preliminaries, The dynamics of linear maps: two and three dimensions, The horseshoe map, Hyperbolic toral automorphism, Attractors, The stable and unstable manifold theorem, Global results and hyperbolic sets, The Hopf bifurcation, The Henon map.

Module 3: [10 (L) Hours]

A simple nonlinear mechanical oscillator: the Duffing oscillator, Chaos in the weather: the Lorenz model, The Rossler systems, Phase space, dimension and attractor form, Spatially extended systems: coupled oscillators, Taylor-Couette flow, Mathematical Routes to chaos and turbulence.

Module 4: [10 (L) Hours]

Preliminary Characterization: visual inspection & frequency spectra, Characterising chaos: Lyapunov exponents & dimension estimates, Attractor reconstruction, Embedding dimension for attractor reconstruction.

References:

1. R. L. Devaney, *An introduction to Chaotic Dynamical Systems*, Addison Wesley, 2nd Edn., 1989.
2. P. S. Addison, *Fractals and Chaos*, IOP Publishing, Bristol, 1997.
3. K.T. Alligood, T.D.Sauer & J.A.Yorke, *Chaos an introduction to dynamical systems*, springer, 2006.
4. Peitgen, Jurgens & Saupe, *Chaos and Fractals*, Springer- Verlag, 1992.

Course Outcome: Students learn the basic concepts of mathematical theory of chaos and its applications.

MA7365 MULTIVARIABLE CALCULUS

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

Functions on Euclidean spaces, Differentiability in several variables, Partial and directional derivatives, Chain Rule, Mean Value Theorem, Inverse Function Theorem, Implicit Function Theorem, Rank Theorem.

Module 2: [10 (L) Hours]

Riemann integration in higher dimensions, Fubini's Theorem, Change of variables, Improper integrals, Line and surface integrals, Green's theorem, Divergence Theorem, Stokes' Theorem.

Module 3: [11 (L) Hours]

Tensors, Wedge product, Differential forms, Poincar Lemma, Integration on chains, Stokes' Theorem for integrals of differential forms on chains, Fundamental theorem of calculus.

Module 4: [10 (L) Hours]

Differentiable manifolds (as subspaces of Euclidean spaces), Differentiable functions on manifolds, Tangent spaces, Differential forms on manifolds, Orientations, Integration on manifolds, Stokes' theorem on manifolds.

References:

1. J.R. Munkres, *Analysis on Manifolds*, Addison-Wesley, 1991.
2. W. Rudin, *Principles of Mathematical Analysis*, McGraw-Hill, 3rd Edn., 1984.
3. M. Spivak, *Calculus on Manifolds, A Modern Approach to Classical Theorems of Advanced Calculus*, W. A. Benjamin, Inc., 1965.
4. G. B. Folland, *Advanced Calculus*, Pearson Education, 2002.

Course Outcome: Students get familiarized with the basic results in the analysis of functions of several variables.

MA7366 NUMERICAL SOLUTION FOR PARTIAL DIFFERENTIAL EQUATIONS

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

Classification of PDEs, finite difference approximations to derivatives, truncation errors, boundary conditions: Dirichlet, Neumann and Robin type boundary conditions. Review of iterative methods to linear system of equations: Jacobi, Gauss-seidel, SOR. Matrix form of iterative methods and their convergence. Initial value problems, Initial boundary value problems and their analysis of convergence, consistency and stability. Lax theorem, Von Neumann criterion for stability.

Module 2: [11 (L) Hours]

Parabolic equations: explicit and implicit methods for one and two dimensional parabolic equations, Crank-Nicolson method, numerical examples, weighted average approximation, consistency, convergence and stability, alternate direction method in two dimensions, Peaceman-Rachford scheme, Douglas-Rachford scheme.

Module 3: [10 (L) Hours]

Hyperbolic equations: Finite difference methods for first and second order wave equation, Lax-wendroff explicit method, CFL condition for one and two dimensions, ADI schemes for two dimensional hyperbolic equations, Lax-wendroff method for a system of hyperbolic equations, Wendroff's implicit approximation, reduction of a first order equation to a system of ordinary differential equations, numerical examples.

Module 4: [10 (L) Hours]

Elliptic equations: Numerical examples: a torsion problem, a heat conduction problem with derivative boundary conditions. Finite differences in polar co-ordinates, techniques near a curved boundary, improvement of the accuracy of the solutions. Analysis of the discretization error of the five-point approximation to Poisson's equation.

References:

1. K.W. Morton & D.F. Mayers, *Numerical solution of partial differential equations*, Cambridge, 2nd Edn., 2011.
2. G.D. Smith, *Numerical solution of partial differential equations, finite difference methods*, Oxford, 3rd Edn., 2010.
3. Randall J. Leveque, *Finite difference methods for ordinary and partial differential equations*, SIAM, 2007.
4. J.W. Thomas, *Numerical partial differential equations: Finite difference methods*, Springer, 2010.

Course Outcome: Students learn numerical solution of partial differential equations with an understanding of convergence, stability and consistency.

MA7367 STATISTICAL METHODS FOR QUALITY MANAGEMENT

Pre-requisites: Mathematical Statistics

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [12 (L) Hours]

Design and Analysis of Experiments-Introduction to Design and Analysis of Experiments, Single Factor Design and Analysis of Variance, Randomized Blocks Designs, Latin Squares and Related Designs, Introduction to Factorial Designs - Basic Definitions and Principles, The Two-Factor Factorial Design, The General Factorial Design, Fitting Response Curves and Surfaces, Blocking in a Factorial Design.

Module 2: [10 (L) Hours]

Statistical Process Control- Chance and Assignable Causes of Quality Variation, Setting up of Operating Control Charts for X and R, Control Charts for X and S, Control Charts for Individual Measurements, Applications of Variables Control Charts. Control Charts for Attributes- Control Charts for Fraction Nonconforming, Control Charts for Nonconformities(defects).

Module 3: [10 (L) Hours]

Cumulative Sum and Exponentially Weighted Moving Average Control Charts- The Cumulative-Cum Control Charts, The Exponentially Weighted Moving-Average Control Charts, the Moving Average Moving Control Charts, Statistical Process Control Techniques, Process Capability Analysis, Acceptance Sampling for Attributes.

Module 4: [10 (L) Hours]

Reliability Statistics- Reliability Definition, Reliability Bathtub Curve, Estimating MTBF, Reliability Prediction, Confidence Interval for MTBF, Testing System Reliability, Series Systems, Parallel Systems, Baye's Theorem Applications, Non-parametric and Related Test Designs, Hazard Function, Weibul Distribution, Log-Normal Distribution, Stress- Strength Inference, Binomial Confidence Intervals, Arrhenius Model, Sequential Testing.

References:

1. E. L. Grant & R. S. Leavenworth, *Statistical Quality control*, McGraw- Hill, 7th Edn., 1996.
2. D. C. Montgomery, *Introduction to Statistical Quality Control*, John Wiley and sons, 3rd Edn., 1997.
3. D. C. Montgomery, *Design and Analysis of Experiments*, John Wiley & Sons, 5th Edn., 2001.
4. R. A. Dovich, *Reliability statistics*, A S Q Quality Press., 1990.

Course Outcome: Students learn various statistical tools for Management and equip the student in the applications of it for Decision making.

MA7368 ADVANCED OPERATIONS RESEARCH

Pre-requisites: Linear programming/Basic course in Linear Programming

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

Mathematical preliminaries. Maximum and Minimum-Quadratic forms-Gradient and Hessian matrices-Unimodal functions-Convex sets-Convex and concave functions-Mathematical programming Problems. Varieties and characteristics Difficulties caused by nonlinearity- Role of convexity in Non linear programming- Unconstrained optimization-Search methods. Fibonacci search-Golden section search.

Module 2: [11 (L) Hours]

Hooke and Jeeve's Method Optimal gradient method-Newton's method- Constrained nonlinear optimization-Constrained optimization with equality constraints-Lagrangian method-Sufficiency conditions- Optimization with inequality constraints- Kuhn-Tucker conditions- Sufficiency Conditions.

Module 3: [10 (L) Hours]

Quadratic programming- Separable programming-Frank and Wolfe's method-Kelley's cutting plane method- Rosen's gradient projection method-Fletcher-Reeve's method-Penalty and Barrier method.

Module 4: [10 (L) Hours]

Integer linear programming-Gomory's cutting plane method-Branch and Bound Algorithm- Travelling salesman problem- knapsack problem- Introduction to optimization softwares.

References:

1. H.A. Taha, *Operation Research-An introduction*, Prentice Hall, 7th Edn., 2006.
2. D.M. Simmons, *Nonlinear Programming for Operations Research*, Prentice Hall, 1975.
3. M.S.Bazaara, H.D Sherali & C.M.Shetty, *Nonlinear Programming Theory and Algorithm*, John Wiley, 2nd Edn., 1993.

Course Outcome: Students learn the mathematical techniques to solve decision making problems in order to analyze and understand a system, for the purpose improving its performance.

MA7369 STOCHASTIC PROCESSES

Pre-requisites: Knowledge of elementary probability theory.

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

Elements of stochastic processes with examples. Markov Chains: Definition, Examples, Transition probability matrix, Classification of states, Limit theorems, Stationary distribution of Markov Chains, Applications of Markov Chains.

Module 2: [10 (L) Hours]

Continuous time Markov Chains: Poisson processes and its extensions, Birth and Death processes, Pure birth and pure death processes, Finite state continuous time Markov Chains, Rate matrix, Kolmogorov forward and backward equations, Limiting distribution.

Module 3: [9(L) Hours]

Renewal Processes: Definition of a renewal process and related concepts, Examples of renewal processes, Renewal equation and elementary renewal theorem, Generalizations and variations on renewal processes.

Module 4: [12 (L) Hours]

Queueing Theory: General concepts, $M/M/1$ queue, system size and waiting time distributions, $M/M/1/k$ model, $M/M/c$ and $M/M/\infty$ models, Erlang loss model. Erlang queueing models, the system $M/E_k/1$ and the system $E_k/M/1$, Network of Markovian queues, Jackson networks. The $M/G/1$ queue.

References:

1. S. Karlin & H M Taylor; *A First Course in Stochastic Processes*, Second edition, Academic Press, New York, 1975.
2. S. M. Ross; *Stochastic Processes*, Second edition, John Wiley and Sons, New York, 1996.
3. J. Medhi; *Stochastic Processes*, Third edition, New Age International, New Delhi, 2009.
4. S. M. Ross; *Introduction to Probability Models*, Sixth edition, Academic Press, 2000.

Course Outcome: Students learn elementary stochastic processes and queuing theory.

MA7370 NUMBER THEORY

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

Fundamental concepts, divisibility, Euclids lemma, Fundamental theorem of arithmetic, continued fractions, Diophantine equations, combinatorial study of (n) , Arithmetic functions, Mobius inversion formula.

Module 2: [11 (L) Hours]

Basic properties of congruences, residue systems, Linear congruences, Little Fermat theorem, Wilsons theorem, Chinese Remainder theorem, Polynomial congruences, Reduced residue systems, primitive roots.

Module 3: [10 (L) Hours]

Prime numbers, Properties of $\pi(x)$, Tchebychevs theorem, Eulers criterion, Legendre and Jacobis symbols, Gauss lemma, Quadratic reciprocity law, Consecutive residues, Consecutive triples of quadratic residues.

Module 4: [10 (L) Hours]

Sum of two squares, Sum of four squares, Elementary partitions, Eulers partition theorem, Partition generating functions, Eulers pentagonal number theorem, Schurs theorem, Gauss circle problem, Dirichlets divisor problem.

References:

1. George E Andrews, *Number theory*, W.B Saunders Company, London, First Edition, 1976.
2. Song y Yan, *Number theory for computing*, Springer, Second Edition, 2001.
3. G H Hardy & E M Wright, *An introduction to the theory of numbers*, Oxford university press, Fourth Edition, 1975.
4. M B Nathanson, *Elementary methods in number theory*, Springer, 1999.
5. David M Burton, *Elementary Number Theory*, Allyn and Bacon Ltd., 2000.

Course Outcome: Students learn the classical number theory concepts and some results on modular arithmetic, quadratic congruence and additivity in detail.

MA7371 APPLIED STATISTICAL INFERENCE

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

Uniformly most powerful unbiased tests, Invariance in estimation and testing, Admissibility, Minimax and Bayes estimation, Asymptotic theory of estimation, Asymptotic distribution of likelihood ratio statistics, Sequential estimation, Sequential probability ratio test.

Module 2: [10 (L) Hours]

Two-way contingency tables, Table structure for two dimensions, Way of comparing proportions, Measures of associations, Sampling distributions, Goodness of fit tests, Testing independence.

Module 3: [10 (L) Hours]

Models of binary response variables, Logistic regression, Logistic models for categorical data, Probit and extreme value models, Log-linear models for two and three dimensions, Fitting of logit and log-linear models, Log-linear and logit models for ordinary variables.

Module 4: [11 (L) Hours]

Regression: Simple, multiple, and non-linear regression, Likelihood ratio test, Confidence intervals and hypothesis test, Test for distributional assumptions, Outliers, Analysis of residuals, Model building, Principal component and ridge regression. Lab component: relevant real life problems to be done in statistical packages like SAS, SPSS etc.

References:

1. A. Agresti, *Analysis of Categorical Data*, Wiley, 1990.
2. A. Agresti, *An introduction to Categorical Data*, Springer-varlag, 1996.
3. E.B. Anderson, *The Statistical Analysis of Categorical Data*, Springer-varlag, 1990.
4. R.F. Gunt and R L Mason, *Regression Analysis and its applications-A data oriented approach*, Marcel Dekkar, 1980.
5. T.J. Sanner and D. Duffy, *The Statistical Analysis of Discrete Data*, Springer-varlag, 1989.
6. G. Casella and R L Berger, *Statistical Inference, Wadsworth and Brooks*, 1990.
7. E.L. Lehmann, *Theory of Point Estimation*, John Wiley, 1983.

Course Outcome: Students understand the fundamentals of applied statistical inference through categorical data analysis and its applications in science and engineering.

MA7372 REGRESSION ANALYSIS

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [12 (L) Hours]

Simple regression with one independent variable (X), assumptions, estimation of parameters, standard error of estimator, testing of hypothesis about regression parameters, standard error of prediction, Testing of hypotheses about parallelism, equality of intercepts, congruence. Extrapolation, optimal choice of X . Diagnostic checks and correction: graphical techniques, tests for normality, uncorrelatedness, homoscedasticity, lack of fit, modifications like polynomial regression, transformations on Y or X , WLS, inverse regression $X(Y)$.

Module 2: [10 (L) Hours]

Multiple regression: Standard Gauss Markov Setup, Least square(LS) estimation, Error and estimation spaces, Variance- Covariance of LS estimators, estimation of error variance, case with correlated observations, LS estimation with restriction on parameters, Simultaneous estimation of linear parametric functions, Test of Hypotheses for one and more than one linear parametric functions, confidence intervals and regions.

Module 3: [10 (L) Hours]

Non Linear regression (NLS) : Linearization transforms, their use and limitations, examination of non linearity, initial estimates, iterative procedures for NLS, grid search, Newton- Raphson , steepest descent,Marquardts methods.

Module 4: [10 (L) Hours]

Logistic Regression: Logit transform, ML estimation. Tests of hypotheses, Wald test, LR test, score test, test for overall regression, multiple logistic regression, forward and backward method, interpretation of parameters relation with categorical data analysis, generalized linear model: link functions such as Poisson, binomial, inverse binomial, inverse Gaussian, gamma.

References:

1. Draper, N. R. and Smith, H., *Applied Regression Analysis*, 3rd Ed., John Wiley, 1998.
2. McCullagh, P and Nelder, J. A., *Generalized, Linear Models*, Chapman & Hall, 1998.
3. Ratkowsky, D. A., *Nonlinear Regression Modelling*, Marcel Dekker, 1983.
4. Hosmer, D.W. and Lemeshow, S., *Applied Logistic Regression*, John Wiley, 1989.
5. Seber, G.E.F. and Wild, C.J., *Nonlinear Regression*, Wiley, 1989.

Course Outcome: Students learn the fundamentals of regression analysis and its applications in science and engineering.

MA7373 RELIABILITY OF SYSTEMS

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [11 (L) Hours]

Introduction to reliability, Basic concepts, Cut sets, Path sets, Minimal cut and path sets, Bounds for reliability, Reliability and Quality, Maintainability and Availability, Reliability analysis, Causes of failures, Catastrophic and Degradation failures, Useful life of components, Component reliability and hazard models, Mean time to failure, system reliability models, System with components in series, parallel, k/n systems, System with mixed mode failures.

Module 2: [11 (L) Hours]

Redundancy Techniques, Component v/s unit redundancy, Weakest link techniques, Mixed redundancy, Stand by redundancy, Redundancy optimization, Double failure and redundancy, Maintainability and availability concepts, Two unit parallel system with repair, Signal redundancy, Time redundancy, Software redundancy.

Module 3: [10 (L) Hours]

Hierarchical systems, Path determination method, Boolean Algebra method, Cut set approach, Logic diagram approach, Conditional probability approach, System cost and reliability approximations, Economics of reliability engineering, Economic cost, manufacturing cost, customers cost, Reliability achievement cost models, Depreciation cost models, Reliability management, Management policy and decisions.

Module 4: [10 (L) Hours]

Life testing: Introduction, hazard rate functions, Exponential distribution in life testing, Simultaneous testing-stopping at r-th failure, Stopping by fixed time, sequential testing, Accelerated testing, Equipment Acceptance testing, Software reliability, Software reliability models, Reliability Allocation, A two sample problem.

References:

1. Balagurusamy, E., *Reliability Engineering*, Tata McGraw-Hill, 2011.
2. Shooman, M.L., *Probability Reliability An engineering Approach*, McGraw-Hill. Newyork, 1968.
3. Barlow, R.E. and Proschen, F., *Mathematical Theory of Reliability*, John Wiley, Newyork, 1965.
4. Aggarwal, K.K., *Reliability Engineering*, Springer, 2007.
5. Ross, S.M. *Introduction to Probability and Statistics for Engineers and Scientists*, 4/e, Elsevier, 2009.

Course Outcome: Students learn the fundamentals of system reliability theory and applications in science and engineering.

MA7374 FORECASTING TECHNIQUES

Pre-requisites: Nil

Total Hours : 42

L	T	P	C
3	0	0	3

Module 1: [10 (L) Hours]

Fundamentals of Quantitative Forecasting- Introduction, Explanatory Versus Time-Series Forecasting, Least Square Estimates, Discovering and Describing Existing Relationships and Patterns, The Accuracy of Forecasting, Smoothing Methods- Averaging Method, Exponential Smoothing Methods, Applications.

Module 2: [11 (L) Hours]

Decomposition Methods, Trend Fitting, The Ratio-to-Moving Averages Classical Decomposition Method, Different Types of Moving Averages, Regression and Econometric Methods- Simple Regression and Correlation Analysis, Multiple Regression- Multiple Linear Regression, Selecting Independent Variables and Model Specification, Multicollinearity, Multiple Regression and Forecasting, Econometric Models and Forecasting, Applications.

Module 3: [11 (L) Hours]

ARIMA Models for Time-Series, Methodological Tools for Analyzing Time Series, The Box-Jenkins Method, Introduction, Identification, Estimating the Parameters, Diagnostic checking, Forecasting with ARIMA Models, Applications.

Module 4: [10 (L) Hours]

Forecasting and Planning, The Role of Forecasting in Planning, Forecasting as Input to Planning and Decision Making. Comparison and Selection of Forecasting Methods- The Accuracy of Forecasting, Pattern of Data and its Effects on Individual Forecasting Methods, Time Horizon Effects on Forecasting Methods, Impact of Type of Series on Forecasting Methods, The Cost of Forecasting Methods, An Interactive Procedure for Selecting, Running, and Comparing Alternative Forecasting Methods, Applications.

References:

1. W. W. S. Wei, *Time Series Analysis, Univariate and Multivariate Methods*, Addison Wesley, (1994).
2. G. E. P. Box and G. M. Jenkins, *Time Series Forecasting and Control*, Holden-Day, San Francisco, (1986).
3. S. Makridakis, S. C. Wheelwright and V. E. McGee, *Forecasting, Methods and Applications*, 2nd edn., Wiley, Hong Hong, (1983).

Course Outcome: Students acquire a knowledge of different forecasting techniques, and get equipped in the theory and applications of forecasting techniques for decision making.
