Name: ………………………

CEA/C/D: Roll No: …………...

Sl. Number: ……

Department of Civil Engineering

NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

CE6101D THEORY OF ELASTICITY AND PLASTICITY

End Semester Examination – 13 Dec 2022

Time: 3 hours | Maximum Marks: 50

Note: Answer all questions; Assume missing data after stating; Read the questions carefully before answering

1. Use the stress function $\phi = Ay^3$ and find the stress field in the cantilever beam in Fig. 1 with applied couple M. Find the displacement field. Describe one possible set of displacement boundary conditions to solve for the constants of integration (without solving them). [6]

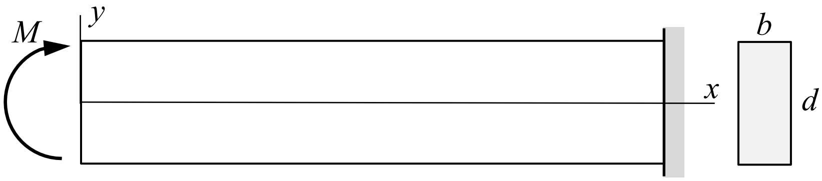

Figure 1

2. Consider the problem of stress concentration around a circular hole in a thin elastic plate subjected to a uniaxial loading. *Briefly describe how you will solve this problem*. *Write* the corresponding stress functions, and mention how you will proceed to get the complete solution. [4]

3. The stress field in axisymmetric problems is given by

$$\sigma_r = \frac{B}{r^2} + 2C + D(1 + 2\ln r),\ \sigma_\theta = -\frac{B}{r^2} + 2C + D(3 + 2\ln r) \text{ and } \tau_{r\theta} = 0.$$

Obtain the corresponding displacement field. *Show that* for Lame's problem of thick pipes, D must be zero. Find B and C if the pipe is subjected to an external pressure p_o and zero internal pressure. (Hint: (i) Treat the problem as a plane stress one. (ii) The strain-displacement relations are given by: [6]

$$\varepsilon_r = \frac{\partial u}{\partial r},\quad \varepsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r},\quad \gamma_{r\theta} = \frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{v}{r}$$

4. (a) Derive the mathematical statement of Clapeyron's theorem. Use it to derive the *principle of virtual work* and *principle of stationary potential energy*. State the principle of stationary potential energy. [6]

   (b) Write the total potential energy of a cantilever beam of span l and flexural rigidity EI, subjected to a uniformly distributed transverse load q and a tip concentrated load P. Obtain the *Euler equation* and *natural boundary condition*(*s*) of the problem. [6]

5. (a) A prismatic bar of *narrow rectangular cross section* is subjected to a torque T. Use the membrane analogy and determine the maximum shear stress developed and the angle of twist per unit length. [4]

   (b) A thin walled stiffened box shown in Fig. 2 is subjected to a torque T = 5 kNm. If the thickness of the exterior wall is 8 mm and the thickness of the stiffener is 6 mm, determine the shear stresses in the walls and stiffener and the angle of twist per unit length. Given: G = 50 GPa. [6]

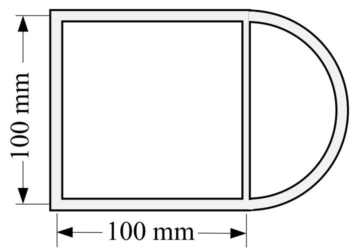

Figure 2

6. (a) Show idealised uniaxial stress-strain diagrams for: (i) rigid–perfectly plastic, (ii) linearly elastic–perfectly plastic, and (iii) linearly elastic–linear work hardening behaviours.

   (b) What are the *three ingredients* of the *theory of plasticity*. [4]

7. (a) Write the von Mises yield criterion and sketch the yield surface in three-dimensional principal stress space. What is the diameter of the corresponding circle in the deviatoric plane?

   (b) Write the von Mises criterion for plane stress problems and sketch the yield surface on σ_1-σ_2 space.

   (c) How does it get modified subsequently in the case of isotropic and kinematic hardening? [6]

8. Represent the deviatoric stress and deviatoric strain components in index notation. Derive the Prandtl-Reuss relation representing the flow rule. [4]

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