Department of Civil Engineering
NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

Name:____________________
Roll No:___________________

Monsoon Semester 2017

Second Interim Test, 3 Oct 2017

CE6101 THEORY OF ELASTICITY AND PLASTICITY

Duration: 1¼ hour | Maximum Marks: [20]

Note: (a) *Answer* **all** *questions* (b) Suitably *assume* missing data after stating clearly (c) *Read questions carefully*

1. *Write* the *equations of equilibrium* and *compatibility conditions* for *plane stress* problems. If V is a potential function such that $b_x = -V_{,x}$ and $b_y = -V_{,y}$ and ∇^2 is the Laplace operator in 2D show that

$$\nabla^2\nabla^2\phi = B\nabla^2 V$$

   where B is a constant. *Find* B. [5]

2. The stress field in a thin rectangular elastic plate bounded by $0 \le x \le l$ and $-a \le y \le a$ under plane stress conditions is given by: $\sigma_x = A\,xy$, $\sigma_y = 0$, and $\tau_{xy} = B + Cy^2$. Determine constants A, B and C so these traction boundary conditions are satisfied: (i) along the edges $y = \pm a$, $\tau_{xy} = 0$, and (ii) at $x = 0$, $\int_{-a}^{a} \tau_{xy} = Q$, where Q is a known quantity. Also *identify* the problem. [5]

3. Determine the displacement field in a cantilever beam ($0 \le x \le l$ and $-a \le y \le a$) subjected to a downward load P at $x = 0$ if the stress field is given by: $\sigma_x = P\,xy/I$, $\sigma_y = 0$, and $\tau_{xy} = P(a^2 - y^2)/2I$, where I is the second moment of area of cross-section. Use boundary conditions: $u = v = \partial v/\partial x = 0$ at $x = l$, $y = 0$. [5]

4. Derive the *first equation of equilibrium* (along radial direction) for a 2D problem in *polar coordinates*. [2]

5. The axisymmetric body force field in a solid circular disk of uniform thickness is given by $b_r = D\,r$ and $b_\theta = 0$, where D is a known constant. Obtain the governing differential equation of the problem by combining equilibrium, stress-strain and strain-displacement relations. *Describe* how you will solve this equation. What are the *boundary conditions* if there is a *central hole* in the disk? The following equations may prove handy. [5]

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} + b_r = 0,\ \frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{2\tau_{r\theta}}{r} + b_\theta = 0;$$

$$\varepsilon_r = \frac{\partial u}{\partial r},\ \varepsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r} \text{ and } \gamma_{r\theta} = \frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{v}{r}.$$

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