Department of Civil Engineering

NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

Winter Semester 2022

End Semester Examination, 6 Jun 2022

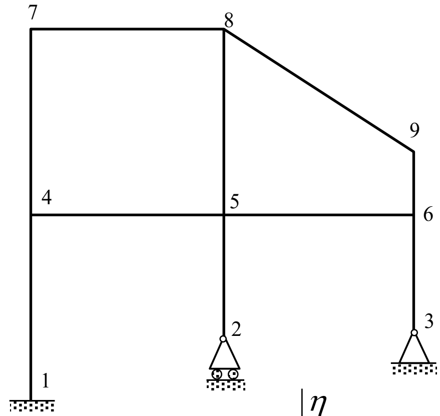
Name:______________________

Roll No:____________________

CE6111D FINITE ELEMENT METHOD

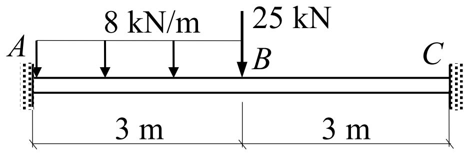
Time: 9:30am to 12:30pm | Maximum Marks: [50]

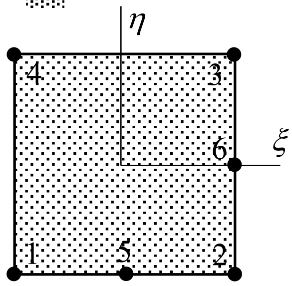
*Answer **all** questions; Provide neat sketches; Assume missing data; Read the questions carefully before answering*



1. The finite element mesh of a plane frame shown in figure has 3-degrees of freedom at each node. The node numbers are also shown. Number the elements and the active degrees of freedom. Calculate the *semi-bandwidth*. What will be the sizes of the reduced global stiffness matrix and global load vector? [5]

2. Analyse the beam in figure given below using *two* beam elements and determine the slope and deflection at *B* if the right support *C* settles down by 10 mm without tilting. $EI = 1.2\times10^4$ kN m^2. [8]



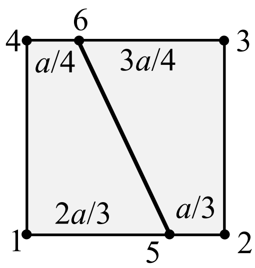


3. (a) Obtain the interpolation functions corresponding to nodes 1, 5 and 2 of the 6-noded isoparametric quadrilateral transition element shown and sketch shapes.

   (b) *Write* the formula for Gauss quadrature in 2- and 3- dimensions. [6+3]

4. Write *all* 6 interpolation polynomials for a 6-noded isoparametric quadratic triangle. Describe how you will get the Jacobian matrix by working out J_{12}. [6]

5. A 2-noded bar element is connected to a 4-noded quadrilateral element at nodes 5 and 6 as shown to make a composite element. The quadrilateral element is square with sides *a*. Node 5 is located as given and node 6 located at *quarter point* as shown. The stiffness matrix of the bar element operates on $\mathbf{u}' = [u_5, v_5, u_6, v_6]^T$. *Obtain* the transformation matrix **T** which relates $\mathbf{u}'$ to $\mathbf{u} = [u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4]^T$ of the quadrilateral element. Briefly *describe* how you will obtain the 6×6 stiffness matrix of the bar with respect to **u**? [5]



6. (a) If the strain transforms as $\{\varepsilon'\}=[T_\varepsilon]\{\varepsilon\}$, *derive* the stress transformation rule? Also derive the transformation for constitutive matrix [*D*]. [5]

   (b) If the stress at a point in a plane stress problem is given by $\sigma_x = 60$ MPa, $\sigma_y = -30$ MPa and $\tau_{xy} = 40$ MPa, use the above transformation and obtain the stress components with respect to $x'y'$ axes obtained by rotating the *xy*-system counter-clockwise by 30°. (Given: For 2D, $\{\varepsilon\} = [\varepsilon_x \;\; \varepsilon_y \;\; \gamma_{xy}]^T$, and $[T_\varepsilon]$ row-wise is $[\,c^2 \;\; s^2 \;\; cs;\; s^2 \;\; c^2 \;\; -cs;\; -2cs \;\; 2cs \;\; c^2-s^2]$ respectively) [3]

7. What is meant by *static condensation*? When do you use it? Explain the procedure mathematically using partitioned matrix equations. How is it implemented in computer? [5]

8. Consider the following differential equation

$$AE\frac{d^2u}{dx^2}+q(x)=0, \quad 0<x<l,$$

   Write the weighted residual statement and obtain the *weak form*. *Describe* how the weak form is used to arrive at Galerkin's finite element equations. [5]

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