Department of Civil Engineering

NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

Name:____________________

Roll No:___________________

Winter Semester 2022

Mid Semester Exam, 11 Apr 2022

CE6111D FINITE ELEMENT METHOD

Duration: 1½ hour Maximum Marks: [30]

Note: *Answer* ***all*** *the questions (reading each carefully); Assume any missing data (after stating clearly)*

1. Consider the bar subjected to distributed and concentrated axial loads as shown in Fig. 1. Analyse it using 2-noded bar elements. The element and node numbering are also shown. Obtain the nodal displacements and element stresses. Given: areas of members: A_1 = 90 mm^2, A_2 = 60 mm^2 and A_3 = 90 mm^2; lengths: L_1 = 1000 mm, L_2 = 800 mm, L_3 = 800 mm; modulus of ELASTICITY: E = 1.6×10^5 N/mm^2; the loads are: q = 1.2 kN/m; P_1 = 4 kN and P_2 = 2 kN. [8]

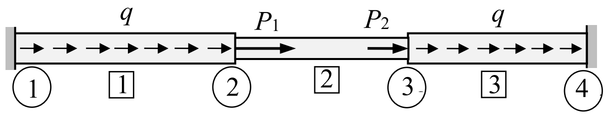
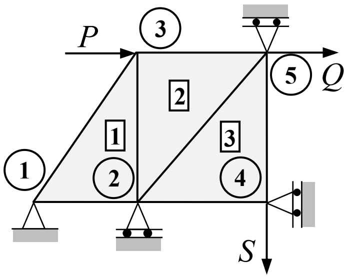


Figure 1



$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & 4 \\ 3 & 5 & 5 \end{bmatrix}$$

Figure 2

2. Consider the finite element mesh for plane stress problem modelled with 3-noded triangular elements as shown in Fig. 2. The element and node numbers (encircled) are also shown. The element connectivity matrix (the node numbers of each element given as columns) is also shown. (a) Number all the free degrees of freedom and determine the sizes of the reduced stiffness matrix **K** and the global load vector **R**. (b) Assemble the element matrices to obtain the global stiffness matrix (assume symmetry and assemble only the upper triangular part of **K**) and the global load vector corresponding to the free degrees of freedom. Assume the element stiffness coefficients for elements [1], [2] and [3] as a_{ij}, b_{ij} and c_{ij}, (i, j = 1 to 6) and the element load vectors due to body force as p_i, q_i and r_i, i = 1 to 6. In addition to the body force field, three concentrated loads P, Q and S act at nodes as shown. [8]

3. (a) *Derive* the interpolation polynomials N_1 and N_2 for a *beam element* and sketch their shapes.

   (b) Use these to obtain k_{11} and k_{12} of the stiffness matrix.

   (c) Also obtain r_1 and r_2 of the element load vector due to a concentrated load P acting at quarter point on the element from node 1. [8]

4. Write the total potential energy of a cantilever beam of span l and flexural rigidity EI subjected to uniformly distributed transverse load q_0 and a tip concentrated load P. Use the principle of stationary potential energy and obtain the *Euler equation* and *natural boundary condition(s)* of the problem.[6]

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