# THE SIMPLEX METHOD

Consider the linear programming problem given by:

Minimise: $f = \mathbf{c}^T \mathbf{X}$

Subject to: $\mathbf{AX} = \mathbf{b}$,

and $\mathbf{X} \geq \mathbf{0}$

The steps used in the simplex method to solve this problem are given below.

- Choose a basis – a basic feasible solution
- If any $b_i < 0$, convert by multiplying the LHS & RHS by −1
- Look for the most negative number in the coefficients of objective function—the corresponding variable enters basis (let it be $x_s$)
- Examine the positive ratios of constant on RHS of each equation to the corresponding coefficient $a_{is}$ of the $s^{th}$ column, for the minimum value. The corresponding variable leaves the basis.
- Perform elementary operations [(i) Multiply or divide an equation by a constant. (ii) Add a multiple of one equation to another] until coefficients of $x_s$ is "1" in its row and "0" in all other rows
- Repeat until all coefficients of $f$ are non-negative

## NO OBVIOUS INITIAL BASIC FEASIBLE SOLUTION

If there are "=" or "≥" type constraints we may have to use artificial variables. Artificial variables are used only to get a starting basic feasible solution. As soon as an artificial variable leaves the basis it can be ignored since it will never enter the basis again. However, if an artificial variable takes a positive value at optimal solution then the original problem does not have a basic feasible solution at all.

## PROPERTIES OF SIMPLEX METHOD

1. Starts from a basic feasible solution and moves to an adjacent basic feasible solution such that the objective function does not increase. If all the coefficients of the nonbasic variables in the objective function are greater than or equal to zero an optimal solution has been reached.
2. If an artificial variable is in an optimal solution then no feasible solution for the original model exists.
3. If slack, surplus and artificial variables are all zero when an optimum solution is reached then all the constrains are strict "equalities" for the values of the variables.
4. If a nonbasic variable has a zero coefficient in the objective function equation at an optimum solution there are multiple optimal solutions. Simplex method gets only one out of the infinite number of optimal solutions (the case of *non-unique optima*).
5. Once an artificial variable leaves the basis it will never enter again. Hence, that variable can be ignored once for all.
6. When selecting the variable to leave the basis (a) if two or more have the same smallest value, choose any one arbitrarily; (b) if a positive ratio does not exist, the objective function is not bounded by the constraints. No finite optimum solution exists (the case of *unbounded solution*).
7. If a basis has a variable at zero level (i.e., $b_i = 0$)—the *degenerate* basis (some constraints are ineffective).

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