Department of Civil Engineering

NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

Monsoon Semester 2023

Mid-Sem Exam, 31 Oct 2023

Name: _____

Sl No: ____; Roll No: _____

CE6101E Theory of Elasticity and Plasticity

Duration: 2 hours | Maximum Marks: [30]

Note: Answer all questions; Provide neat sketches; Assume missing data; Read questions carefully

1. (a) Show that the *principal strain problem* reduces to a *matrix eigenvalue problem*.
   (b) *Find* the strain invariants and principal strains if the nonzero strain components at a point are given as: $\varepsilon_x = 0.0003$, $\varepsilon_y = -0.00015$, $\varepsilon_z = 0.00045$, $\gamma_{xy} = 0.00024$ and $\gamma_{yz} = -0.00018$. [6]
2. (a) *Write* the tensor transformation rules for *vectors* and *dyadics*.
   (b) *Show that* the inner product $u_{ij}\, v_{kij}$ is a vector, if u_{ij} is a dyadic and v_{kij} is a third order tensor. [3]
3. The stress-strain relations for a linearly elastic isotropic body is given by
   $$\sigma_{ij} = \lambda \epsilon_{kk} \delta_{ij} + 2\mu \epsilon_{ij}$$
   Use the above to obtain the strain-stress relations. [3]
3. (a) Consider two coordinate systems denoted by x_i and x_i', where the direction cosines of x_1' with respect to x_i are $\mathbf{n}_1 = [-0.3536, -0.3536, 0.866]$ and those of x_2' are $\mathbf{n}_2 = [0.6124, 0.6124, 0.5]$. (Hint: Both x_i and x_i' are systems of right-handed orthogonal coordinates. Hence, the direction cosines of x_3' with respect to x_i is $\mathbf{n}_3 = \mathbf{n}_1 \times \mathbf{n}_2$). Obtain the components of the transformation tensor a_{ij}. (b) The displacement vector at a point with respect to the coordinates x_i is given by $\mathbf{u} = [-1 \;\; 2 \;\; -3] \times 10^{-3}$ mm. Determine the components of $\mathbf{u}'$ with respect to x_i'. (c) Check answer by comparing the magnitudes of $\mathbf{u}$ and $\mathbf{u}'$. [5]
4. The stress distribution in an elastic body in MPa is described by the following stress components
   $$\sigma_x = 2xy + 3x^2;\ \sigma_y = 4xy - 3y^2;\ \sigma_z = 3xz - 3z^2;\ \tau_{xy} = 2(x^2 + y^2 - 2);$$
   All other stress components are zero. *Evaluate* the distribution of body forces **b** necessary to keep the stress field in equilibrium. (Hint: Use the equations of equilibrium) [3]
5. (a) *Write* the compatibility equations in index notation. (b) *When* is it used? (c) *Use* it to obtain the corresponding *six* compatibility conditions in longhand notation. [5]
6. (a) Briefly *describe* the plane strain problem? (b) Mention *one* example. (c) *Derive* the strain components in terms of stresses for this case. [3]
7. Consider the cantilever beam with the applied couple moment M as depicted in Fig. 1. *Use* the stress function $\phi = A\, y^3$ and *determine* the stress field. *Describe* how you will determine the displacement field. [4]

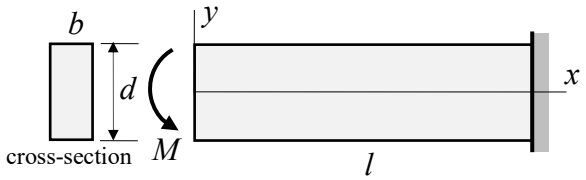

Figure 1

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