Department of Mathematics

Syllabus for Written section of PhD Comprehensive Examination (2022 Admission onwards)

Core Section:

Module - I :

Real Analysis: Review of limit continuity and differentiation, Uniform continuity, Mean value theorems, Taylor's theorem, Functions of bounded variation, Riemann Integration, Fundamental theorem of calculus, Uniform convergence of sequence of functions, Derivative of functions of several variables, Jacobian, Directional derivative, Inverse function theorem, Implicit function theorem.

Measure Theory: Sigma algebra, Measure, Caratheodory's extension theorem, Lebesgue measure, Measurable functions, Lebesgue integral, Fatou's lemma, Monotone convergence theorem, Dominated convergence theorem, Fubini's theorem.

Complex Analysis: Analytic functions, Contour integral, Cauchy's theorem, Cauchy's integral formula, Taylor and Laurent's series, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem, Calculus of residues.

Functional Analysis: Banach spaces, LP spaces, Bounded linear transformation, Hahn-Banach extension theorem, Open mapping theorem, Closed graph theorem, Uniform boundedness principle, Hilbert spaces, Orthonormal bases, Riesz representation theorem, Self-adjoint operators, Normal and unitary operators, Projections.

Differential Equation: Introduction and motivation, Geometrical interpretation of solution. Solution methods for first order and second order equations, Existence and uniqueness of solution of initial value problems, Picard's and Peano's theorems, Euler and improved Euler's methods, Higher order linear equations and Linear systems. Fundamental solutions, Wronskian, Matrix exponential solution, Behaviour of solutions. Boundary value problems for second order equations: Green's function, Sturm comparison theorems and oscillations, eigenvalue problems.

Topology: Metric spaces, Topological spaces, Subspaces, Continuous functions, The Product and Quotient spaces, Separation axioms, Countability properties, Compactness, Connectedness and metrizability.

Linear Algebra: Vector spaces, Dimension, Linear transformations and their matrix representations, rank-nullity theorem, eigenvalues and eigenvectors, minimal polynomial, Cayley-Hamilton theorem, Diagonal form, Triangular form, Jordan form, Inner product spaces, Self-adjoint operators.

Abstract Algebra: Groups, Subgroups, Cosets, Normal subgroups, Quotient groups, Rings, Integral domains, Fields, Quotient fields of an Integral domain, Ideals, Maximal ideals, Quotient rings, Ring of polynomials, Prime and irreducible elements.

Specialization Section:

Module - II : Fuzzy Graph Theory

Fuzzy sets- membership functions – methods of generating membership functions – defuzzification methods, Operations on fuzzy set, Fuzzy relations, Operations on fuzzy relations, similarity relations – compatibility or tolerance relations. Fuzzy numbers- arithmetic operations on intervals- arithmetic operations on fuzzy numbers

Fuzzy graphs- Path and connectedness – strongest path – strong path – types of arcs in fuzzy graphs - complete fuzzy graph – complement of a fuzzy graph. Connectivity in Fuzzy Graphs- Bridges and cut nodes – bonds -Trees and cycles - end nodes – blocks –node connectivity and arc connectivity – fuzzy analogue of Whitney's theorem

Metric in fuzzy graph – -distance - g distance - - distance – ss – distance – eccentricity –radius – diameter – self centered fuzzy graph- operations on fuzzy graph - -Fuzzy line graphs , Fuzzy intersection graphs, Fuzzy interval graphs.

Module - III : Graph Theory

Graphs: review of basics in graphs - -Trees- Blocks- Matrices-Operations on graphs.

Connectivity: Vertex Connectivity and edge connectivity – n- connected graphs-Menger's Theorem.

Traversability: Euler graphs-Hamiltonian Graphs-Planar and Nonplanar graphs.

Metric in graph: Centre, Median, eccentric vertex, Eccentric graph, boundary vertex, complete vertex, interior vertex.

Distance Sequences :Degree sequence, Eccentric Sequence - Distance Sequences - The Distance Distribution, Mean distance.

Matchings :Maximum matching-Perfect matching-Matching in bipartite graphs Factorization :Coverings and independence-1-factorization-2-factorization-Arboricity Domination: Dominating set-Domination number-total dominating set –total domination number.

Module - IV: Numerical Analysis and Computational Methods for PDE

Preliminaries: Round-off error, Truncation error, Absolute error, Relative error, Percentage error; Solving system of linear equations: Direct methods: Gauss elimination, LU decomposition, Iterative methods: Gauss Seidel, Gauss Jacobi, Relaxation methods; Vector norms, matrix norms and condition number. Eigenvalues, eigenvectors, Gerschgorin circle theorem. Newton's method to solve nonlinear system of equations. Numerical methods to solve ODEs: Euler method, modified Euler method, Runge-Kutta methods, Stiff and nonstiff ODEs, finite difference methods, Backward difference formulas, Stability Theory, A-stability, L-stability, B-stability.

Numerical methods for PDEs; Heat equation, advection equation, elliptic equation; types of boundary conditions; finite difference approximations; FTCS, BTCS and Crank-Nicolson schemes; ADI schemes; Naïve, Upwind, Downwind, Lax Wendroff and Lax Friedrichs schemes, Beam-Warming Method; Numerical methods for Laplace Equations; Stability, Von-Neumann stability, CFL condition, consistency and convergence analysis, Lax theorem; Method of lines; Numerical solution of convection-diffusion, diffusion-reaction and convection-diffusion-reaction problems; Implementation of numerical schemes, Numerical error, Computational order.

Module - V : Fractal Theory and Applications

The completeness of space of fractals, Transformations on the real line, Affine transformations in the Euclidean plane, Mobius transformations on the Riemann sphere, Analytic transformations, The contraction mapping theorem, Condensation sets, Addresses of points on fractals, Continuous transformations from code space to fractals, Dynamical systems, Dynamics on fractals, Equivalent dynamical systems, Shadowing theorem, Chaotic dynamics on fractals, Fractal dimension, Theoretical and experimental determination of fractal dimension, Hausdorff-Besicovitch dimension. Fractal interpolation functions, Fractal dimension of fractal interpolation functions, Hidden variable fractal interpolation, Space filling curves, Escape time algorithm, Julia sets, IFS for Julia sets, Map of fractals, Mandelbrot's sets,

Module - VI : Sobolev Spaces and its Applications to PDEs

Sobolev Spaces and its properties: Fundamental solutions, non existence of classical solutionS, weak convergence, test functions, distributions, weak and distributional derivative, compact support, mollifiers, partition of unity. Sobolev spaces: definition and basic properties, approximation by smooth functions. extension theorem, Poincaré inequality, Sobolev inequality, embedding theorems, compactness theorems, traces, dual space, fractional Sobolev spaces, Hardy's inequality.

Variational Methods: First variation, second variation, Euler-Lagrange equation, existence of minimizers: classical method and direct method, Dirichlet principle, Dirichlet integral and p-Dirichlet Integral, existence and nonexistence of minimizers: examples and counterexamples, Wierstrass function, weak form of the Euler-Lagrange equations, convexity, coercivity, lower semicontinuity,

Applications to PDEs: Second-order elliptic equations: Laplace and Poisson equation, weak solutions, existence of weak solutions, Lax-Milgram lemma, weak formulations of elliptic boundary value problems, weak convergence method, Schauder's fixed point theorem, method of subsolutions and supersolutions, comparison principle, nonlinear eigenvalue problems, Pohozaev identity for nonexistence of solutions, critical points and saddle points of energy functional, Palais-Smale sequence, mountain pass lemma, interior and boundary regularity, Harnack inequality, Hopf lemma, weak and strong maximum principles.

Module - VII : Fuzzy Set Theory

Basic concepts of fuzzy sets, Membership functions, Methods of generating membership functions, Defuzzification methods, Extension principle, Operations on fuzzy sets, Fuzzy complement, Fuzzy union, Fuzzy intersection, combinations of operations, General aggregation operations. Fuzzy numbers, Arithmetic operations on intervals, Arithmetic operations on fuzzy numbers, Fuzzy equations, Fuzzy relations, Projections and Extensions, Binary Fuzzy relations, Similarity relations, Compatibility relations. Fuzzy measures, Evidence Theory, Belief and Plausibility measures, Joint Basic Assignment, Dempester's rule of Combination, Marginal bodies of Evidence, Possibility and Necessity measures, Possibility distribution, Basic distribution, Probability measures.

Module - VIII : Generalized Set Theory

An overview of basic operations on Fuzzy sets, Intuitionistic fuzzy sets, Hesitant fuzzy sets, Multisets, Multiset relations, Compositions, Equivalence multiset relations and partitions of multisets, Multiset functions, Fuzzy Multisets.Rough sets, Knowledge representation, Information systems, Exact sets, rough sets, approximations, Set-algebraic structures,Topological structures, Decision systems, Knowledge reduction,Reducts via boolean reasoning,discernibility approach . Reducts in decision systems, Rough membership functions Soft sets, Tabular representation of a soft set, Operations with Soft sets: soft subset, complement of a soft set, null and absolute soft sets, AND and OR operations, Union and intersection of soft sets, DeMorgan laws, Fuzzy soft sets and soft fuzzy sets, Intuitionistic Fuzzy Soft Sets and Soft Intuitionistic Fuzzy Sets, Hesitant Fuzzy Soft Sets, Soft Rough Sets and Rough Soft Sets, Fuzzy rough sets and rough fuzzy sets.

Module - IX : Algebraic Topology

Geometric Complexes and Polyhedra , Orientation of Geometric Complexes, Simplicial Homology Groups - Chains, cycles, Boundaries, Homology groups, examples of Homology Groups, The structure of Homology Groups, The Euler Poincare Theorem, Pseudo manifolds and the Homology Groups of Sn.Simplicial Approximation, induced Homomorphisms on the Homology groups, the Browder Fixed point theorem and related results. The Fundamental group - Introduction, Homotopic paths and the fundamental group, the covering homotopy property for S1, Examples of Fundamental group, the relation between H1(K) and π1([K]). Covering Spaces – Definition and examples, basic properties of Covering spaces, Classification of covering spaces, Universal covering spaces and applications.

Module X: Probability statistics and system reliability

Basics of probability statistics: Random variables, Discrete and continuous random variables, Moments, Moment generating function and Characteristic function, Random vectors, Jointly distributed random variables, Joint probability distributions, Conditional expectation.
Distributions of random variables and limit theorems: Bi-variate normal distribution, Transformations of random variables, Transformations of random vectors, Order statistics, Chebyshev's theorem, Limit theorems in probability, Modes of convergence, Weak law of large numbers, Strong law of large numbers, Limiting moment generating function, Central limit theorem.
Statistical inference: Introduction to population and samples, Sampling distribution of the mean and variance, Point estimation, Maximum Likelihood Estimation (MLE), Method of moments, Properties of estimators, Tests of hypothesis, Uniformly Most Powerful (UMP) Tests, Newman-Pearson lemma, Inference concerning single mean and two means, Inference concerning one variance and two variances, Inference concerning one proportion and several proportions, Chi -square test for goodness of fit.
System reliability: Basic concepts, Cut sets, Path sets, Minimal cut and path sets, Bounds for reliability, Reliability and Quality, Maintainability and Availability, Reliability analysis, Causes of failures, Catastrophic and Degradation failures, Useful life of components, Component reliability and hazard models, Mean time to failure, system reliability models, System with components in series, parallel, *k/n* systems, System with mixed mode failures.

Redundancy techniques: Basics of redundancy techniques, Component v/s unit redundancy, Weakest link techniques, Mixed redundancy, Stand by redundancy, Redundancy optimization, Double failure and redundancy, Maintainability and availability concepts, Two-unit parallel system with repair, Signal redundancy, Time redundancy, Software redundancy
Reliability evaluation and allocation: Hierarchical systems, Path determination method, Boolean Algebra method, Cut set approach, Logic diagram approach, Conditional probability approach, System cost and reliability approximations, Reliability allocation problems

Module XI: Reliability, stochastic process, acceptance sampling plans and optimization

Reliability estimation: Life testing: Introduction, hazard rate functions, Exponential distribution in life testing, Simultaneous testing-stopping at *r-th* failure, stopping by fixed time, sequential testing, Accelerated testing, Equipment acceptance testing.
Stochastic process: Elements of stochastic processes, Classification of general stochastic processes. Markov Chains: Definition, examples, transition probability matrix, classification of states, basic limit theorem, limiting distribution of Markov Chains, Continuous time Markov Chains: General pure birth processes and Poisson processes, more about Poisson processes, A counter model, Birth and Death processes with absorbing states, Finite state continuous time Markov Chains.
Acceptance sampling plans: Sampling plans by attributes and variables, Single, double, multiple, and sequential sampling plans-Acceptable quality level, LTPD-producer's risk, consumers risk.
Optimization techniques: Linear programming problems (LPP)-Formulation of LPP, Simplex method-Simplex algorithm-Charles M Method, Two phase method, Duality in LPP, Dual simplex method, Advanced linear LPP, Sensitivity analysis- Parametric programming, Bounded variable problem, Dynamic programming-Bellman's optimality principle.

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