Name:_____
Roll No:_____

Department of Civil Engineering

NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

Winter Semester 2023-24

Mid-semester exam, 26 Feb 2024

CE6111E FINITE ELEMENT METHOD

Duration: 2 hours | Maximum Marks: [30]

Note: *Answer* ***all*** *the questions after reading it carefully; Assume any missing data after stating*

1. Write the equations of equilibrium, stress-strain relations and strain-displacement relations in matrix form for a two-dimensional idealisation (either plane stress problem or plane strain problem). Combine them into a single equation in terms of displacements. [4]

2. Consider the bar subjected to uniformly distributed and concentrated axial loads as shown in Fig. 1. Analyse it using 2-noded bar elements. Use the element and node numbering shown in figure and obtain the nodal displacements and element stresses. Given: areas of members: $A_1 = 80$ mm^2 and $A_2 = 100$ mm^2; lengths: $L_1 = 1200$ mm and $L_2 = 800$ mm; modulus of elasticity: $E = 160$ GPa (same for both elements); the loads are: $p = 2$ kN/m; $P_1 = 4$ kN and $P_2 = 6$ kN. [6]

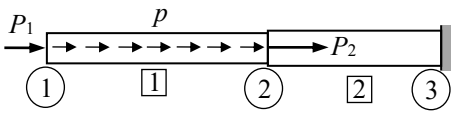


Figure 1

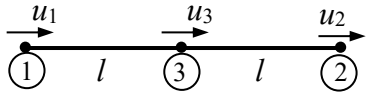
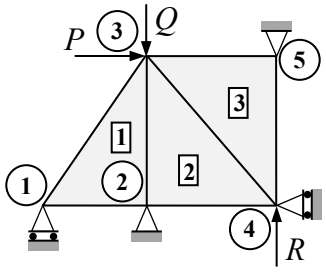


Figure 2

3. For the 3-noded bar element of length $2l$ in Fig. 2 with constant axial stiffness AE and one axial degree of freedom at each node: (a) Obtain the (Lagrangian) interpolation function at node 1 and sketch it. (b) Obtain k_{11} of element stiffness matrix $[\mathrm{k}^e]$. (c) If the element is subjected to a uniformly distributed axial load p_0, obtain the first element r_1 of the element load vector $\{\mathrm{r}^e\}$. [4]

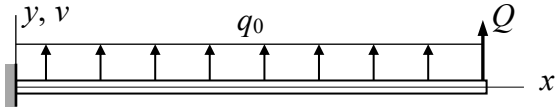
4. Consider the finite element mesh for plane stress problem shown in the figure below. The element numbers and node numbers (encircled) are also shown. The element connectivity matrix (the node numbers of each element given as columns) is as:

$$\begin{bmatrix} 3 & 3 & 3 \\ 1 & 2 & 4 \\ 2 & 4 & 5 \end{bmatrix}$$



(a) Number the free degrees of freedom and determine the sizes of the reduced stiffness matrix **K** and the global load vector **R**. (b) *Assemble* the element matrices to obtain the *global stiffness matrix* and *global load vector* corresponding to the free degrees of freedom. Assume the element stiffness coefficients for elements [1], [2] and [3] respectively as a_{ij}, b_{ij} and c_{ij}, $(i, j = 1$ to 6) and the element load vectors due to a body force as p_i, q_i and r_i, $i = 1$ to 6. In addition to the body force field, three concentrated loads P, Q and R act at nodes as shown. [6]

5. (a) Write the total potential energy of the cantilever beam of span l and flexural rigidity EI subjected to uniformly distributed load q_0 and end load Q as shown below. Use the principle of stationary potential energy and obtain the Euler equation and natural boundary condition(s) of the problem. [6]



(b) Using Rayleigh-Ritz procedure, obtain approximate values for the slope and deflection at the free end of the above cantilever assuming the deflection as: $v(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$. Given: $q_0 = 2$ kN/m, Q = 5 kN and the span of the beam is 3 m. [6]

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