Engineering Mechanics

Continued…(3)

Mohammed Ameen, Ph.D

Professor of Civil Engineering

National Institute of Technology Calicut

Equations of Equilibrium

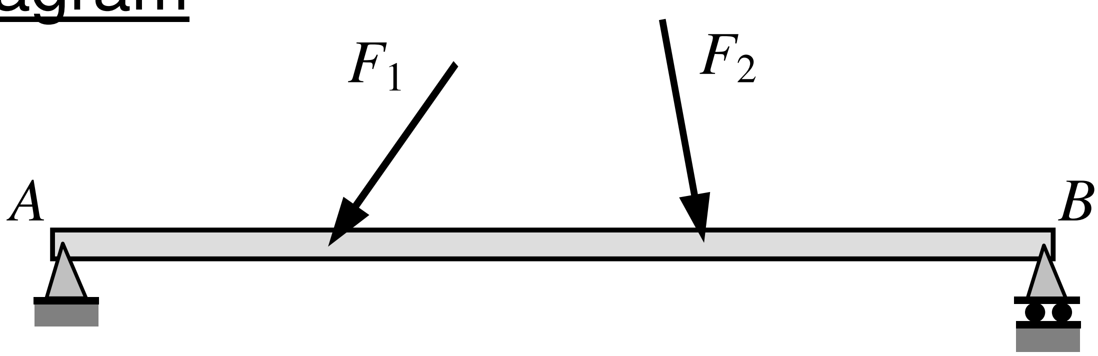
- A “particle” is in equilibrium if it is stationary or moves uniformly with respect to an inertial reference
- A “body” is in equilibrium if all the particles that constitute the body are in equilibrium
- A rigid body in equilibrium cannot, therefore, rotate with respect to an inertial reference
- For bodies in equilibrium, there are certain simple equations which are useful in determining certain unknowns

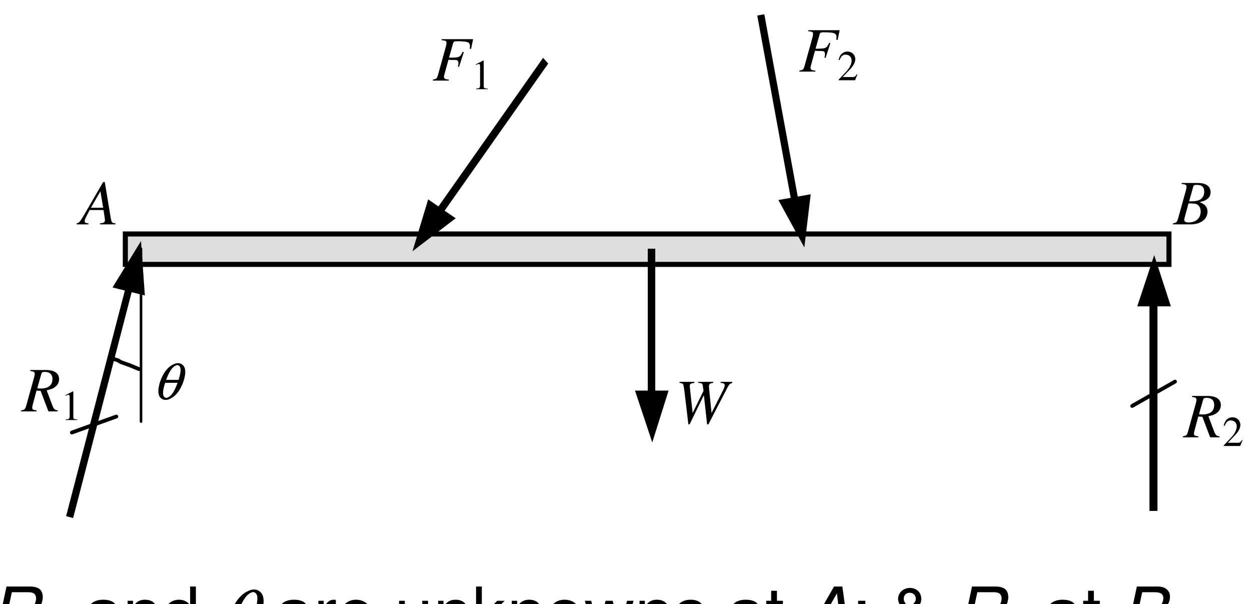
Free Body Diagram

For analysis purposes we

- isolate a body or part of a body fully from its surroundings
- draw a simple diagram of the isolated body
- show all the forces from the surroundings that act on the body

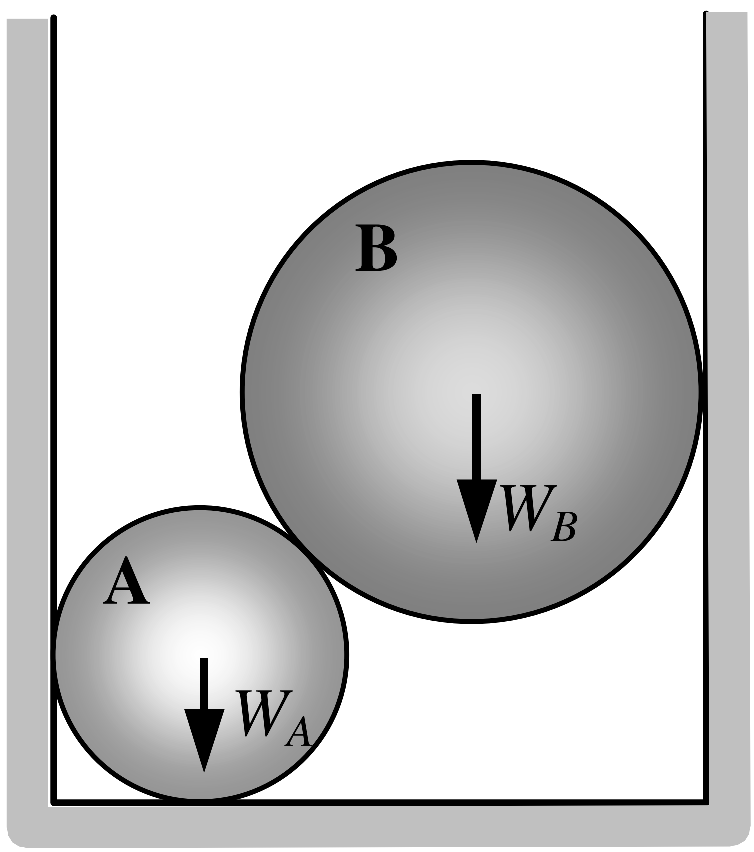
Such a diagram is called a "free body diagram"

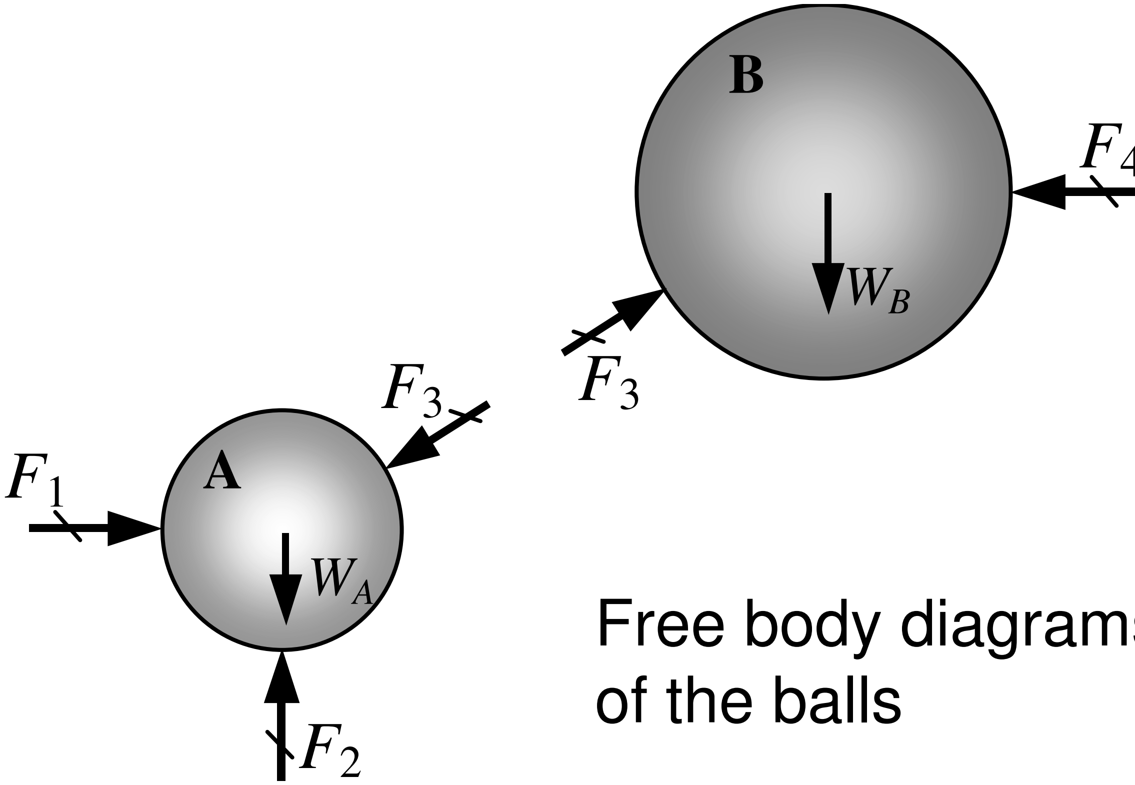




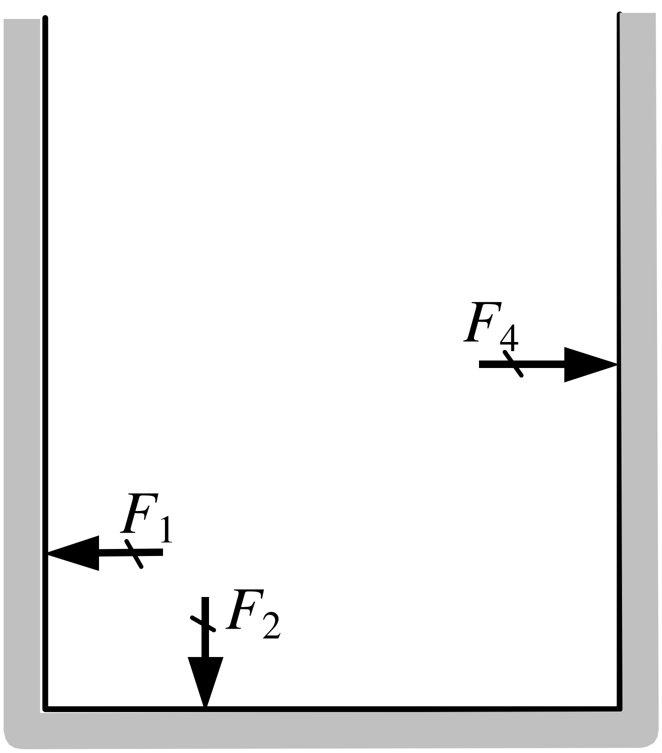
R_1 and θ are unknowns at A; & R_2 at B

- Mentally move the bodies relative to each other in each of the three orthogonal directions
- In those directions the movement is impeded, mark a force
- Next, mentally rotate about each of the orthogonal directions
- In those directions the movement is impeded, mark a couple



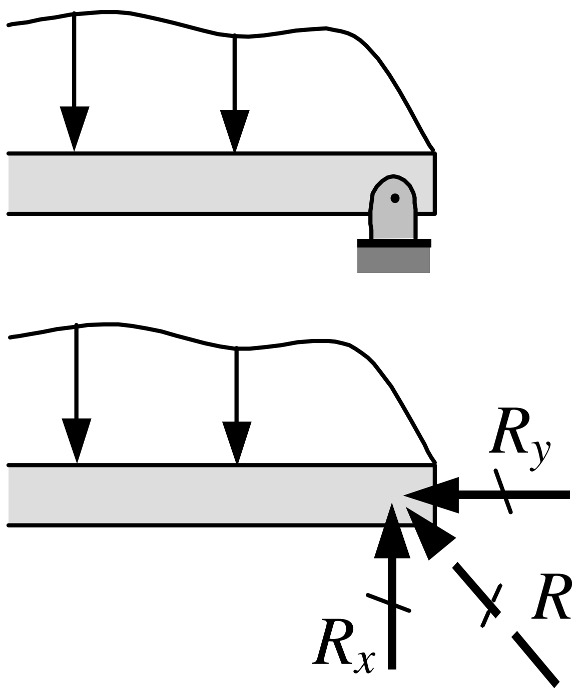


Free body diagrams of the balls

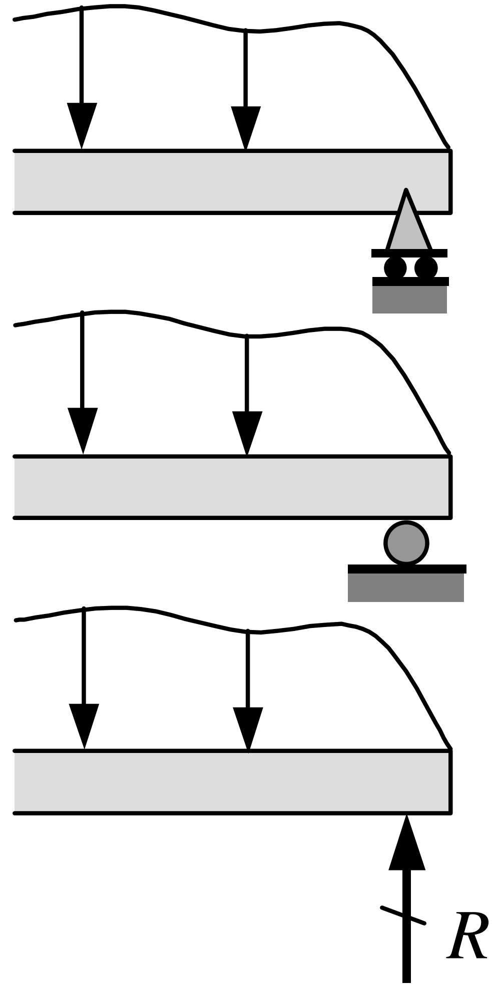


Not a free body diagram!

(Why not?)

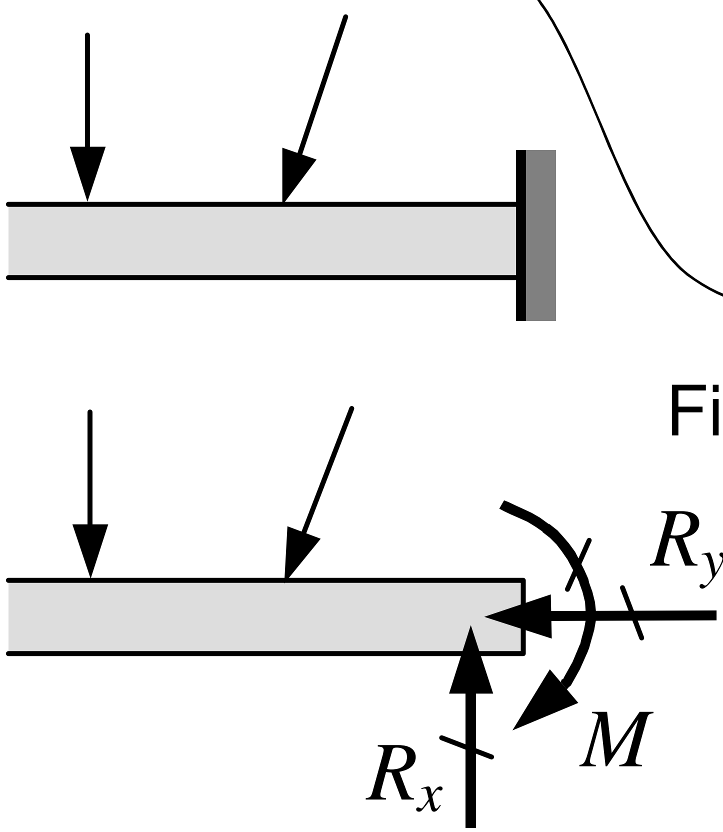


Hinge (pin)



Roller support

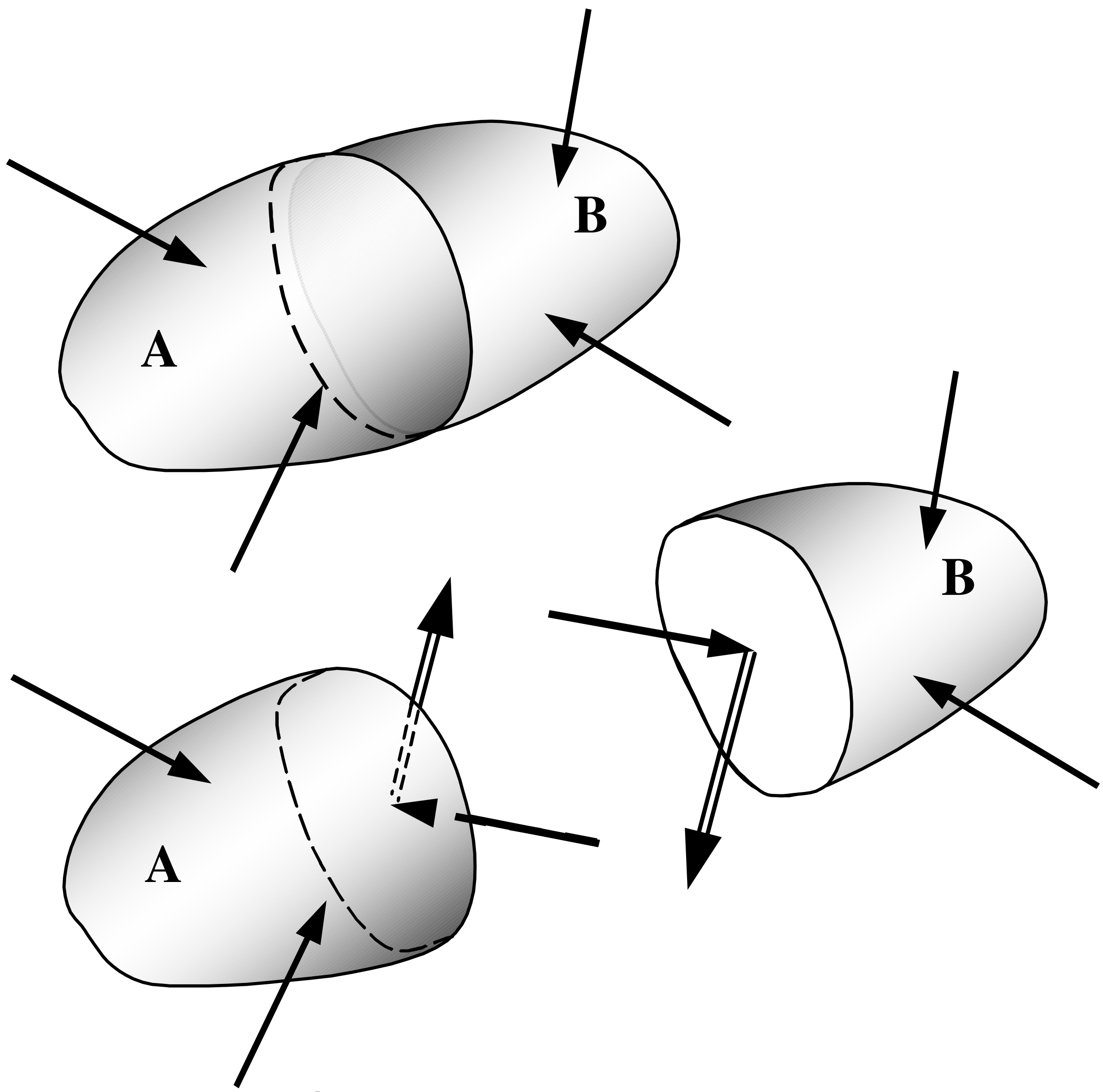
Fixed support



Free Bodies Involving Interior Sections

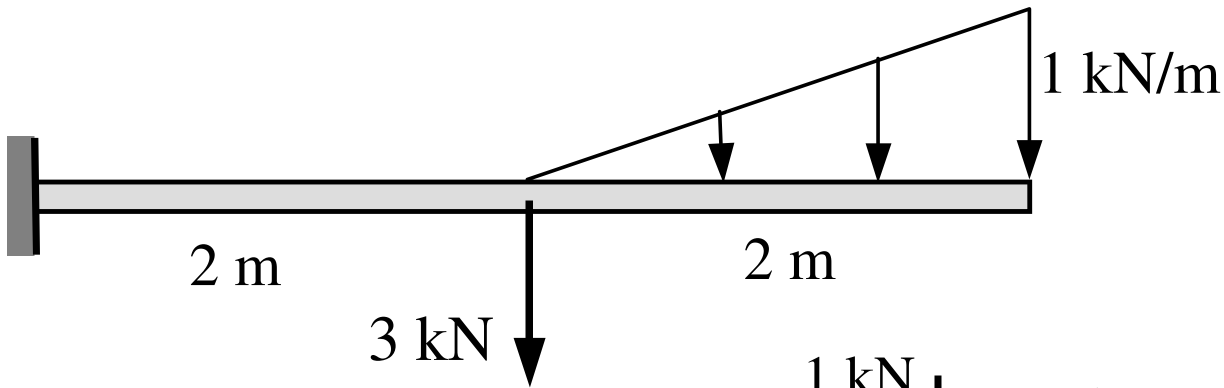
If a body is in equilibrium, every portion of it must also be in equilibrium

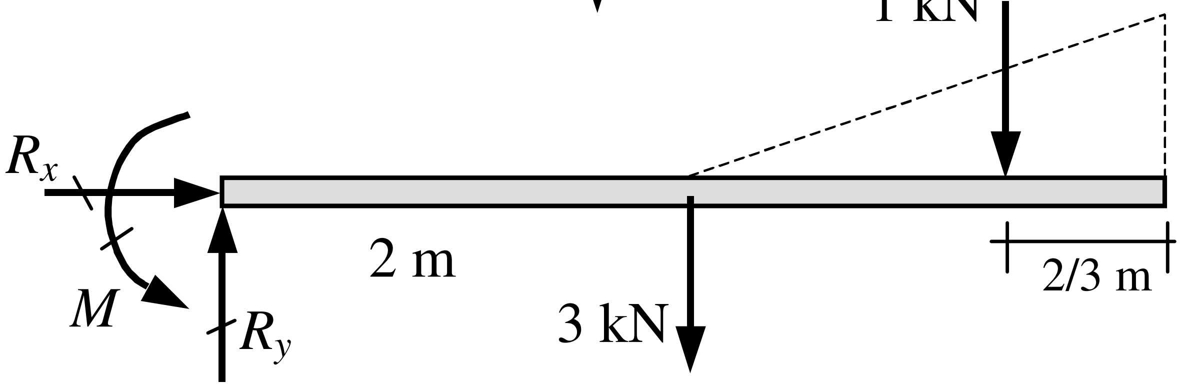
- over the cut surface, there will be a continuous force distribution
- such a force can be replaced by a single force and a single couple



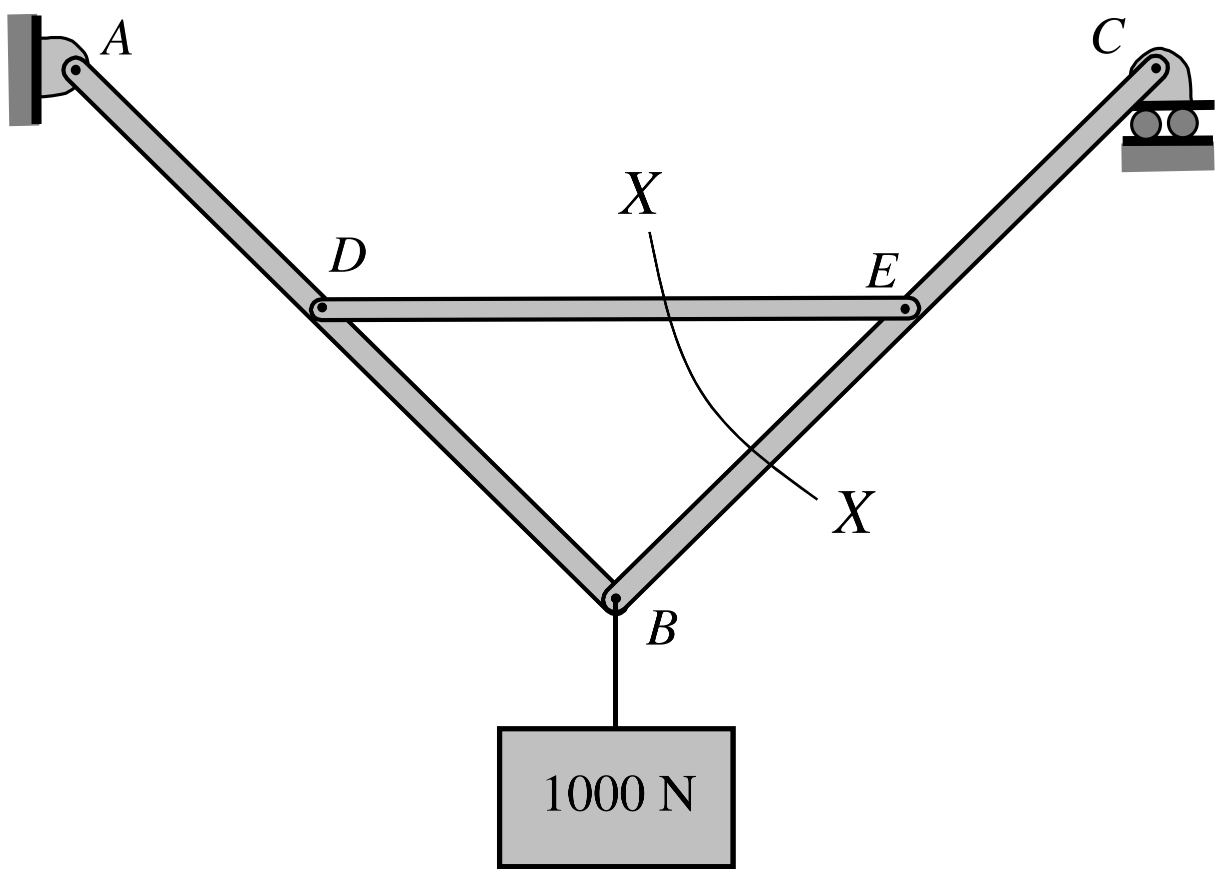
Example: Draw the free body diagrams of

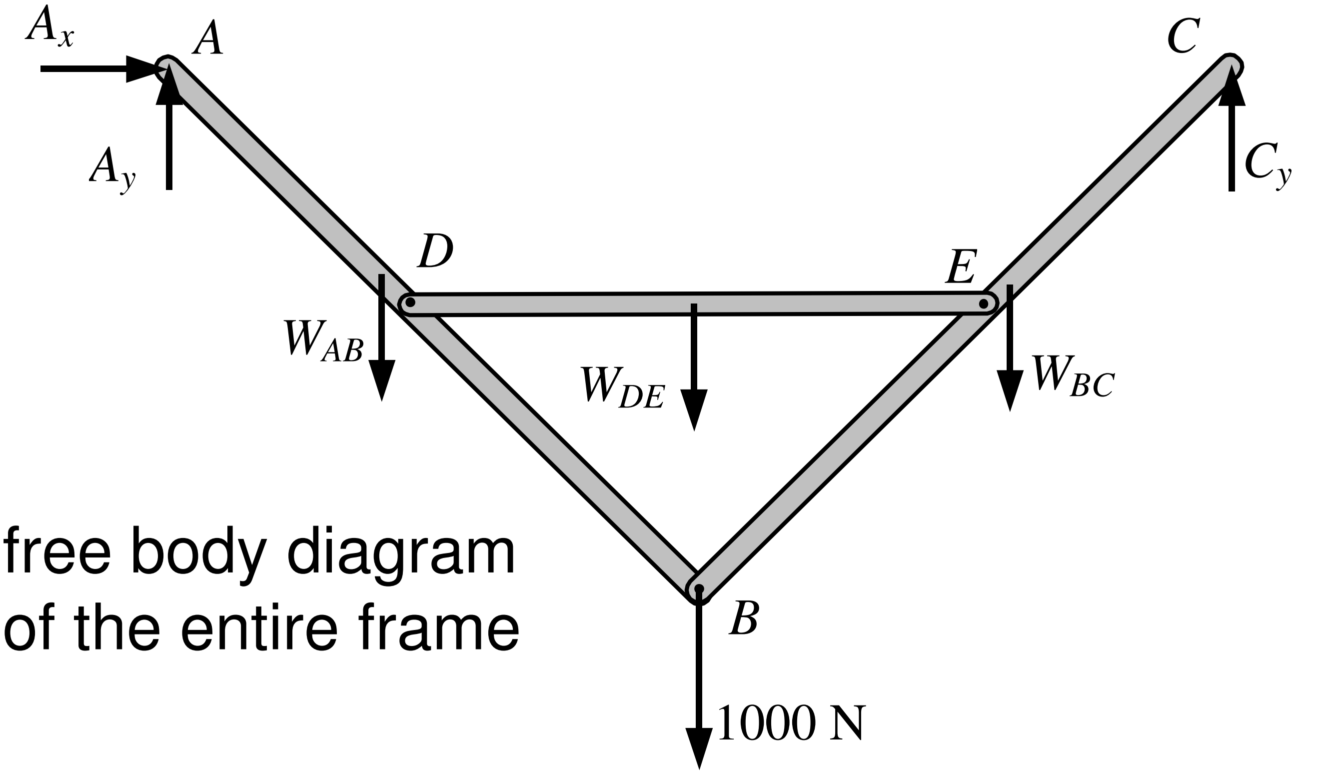
(i)



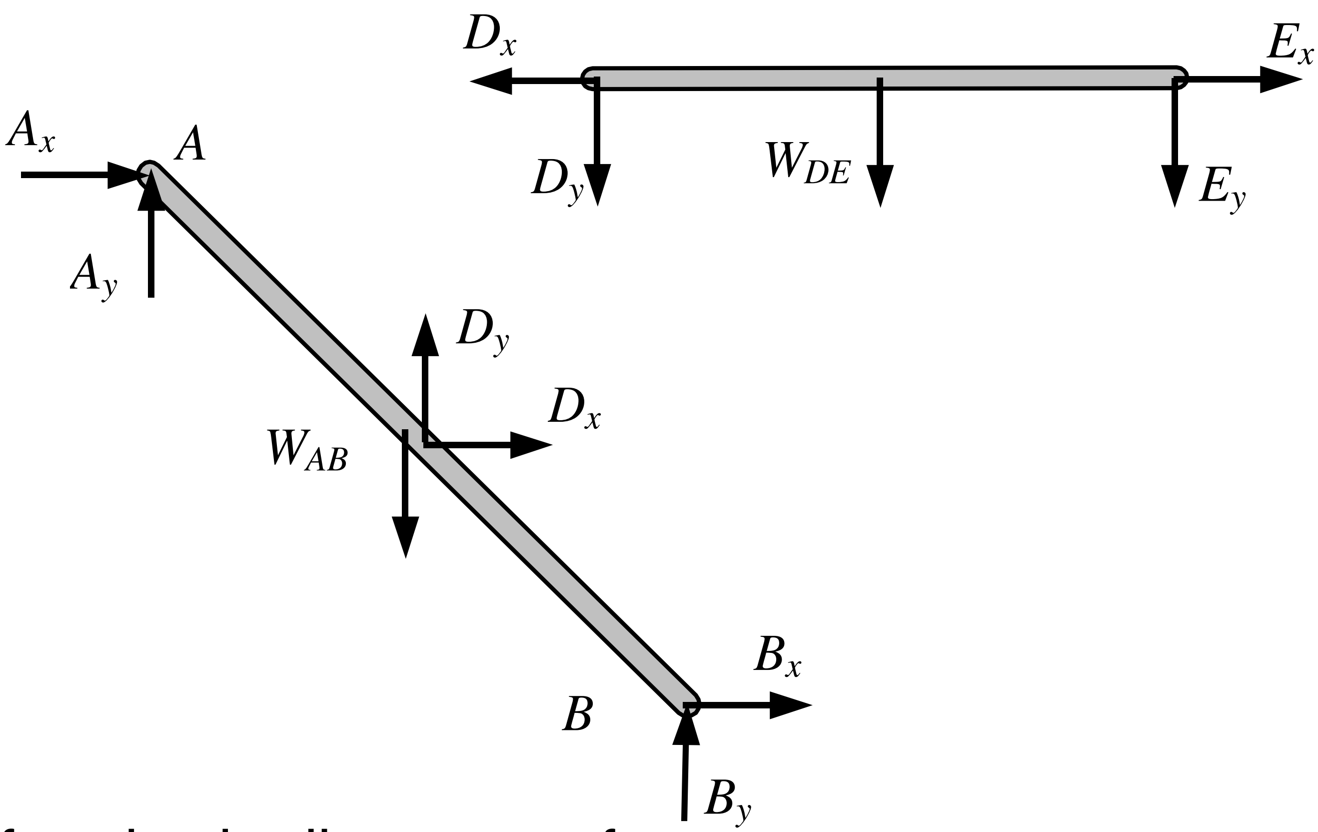


(ii)



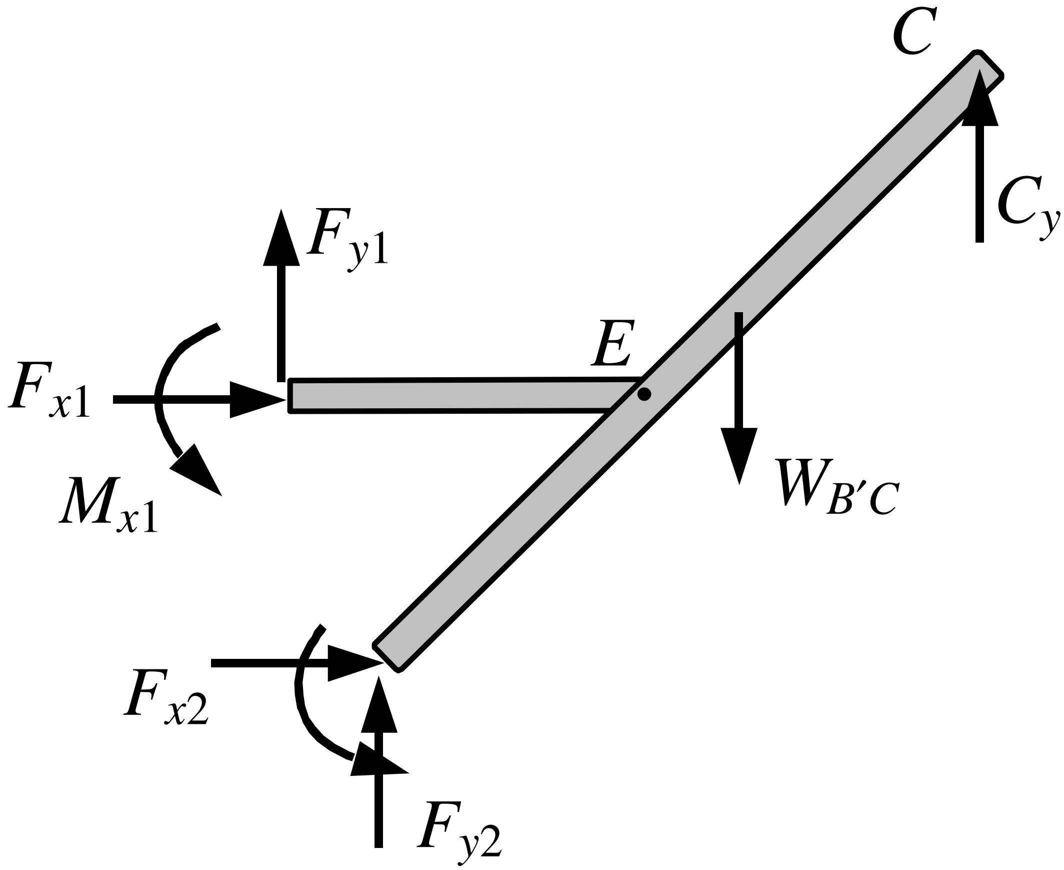


free body diagram
of the entire frame

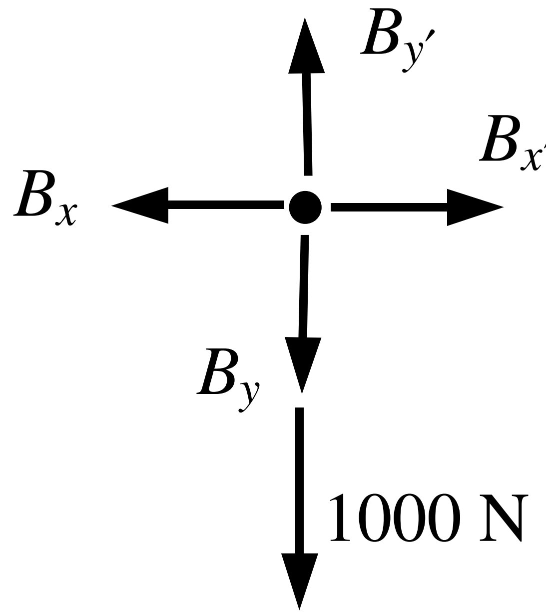


free body diagrams of
members *AB* and *DE*

Free body diagram to the right of section XX



Free body diagram of the pin at *B*



General Equations of Equilibrium

For every free body diagram we can replace the system of forces and couples acting on the body by

- a single force $\mathbf{F_R}$ and
- a single couple $\mathbf{C_R}$ through some point "a" (as "a" is moved to a different point, only $\mathbf{C_R}$ will change)

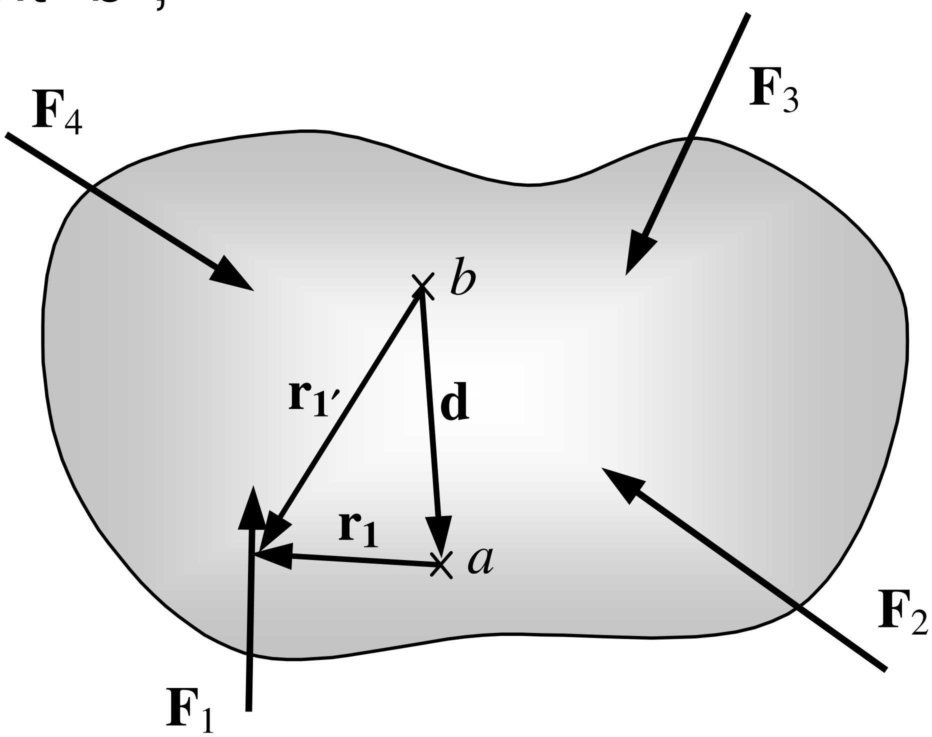
The necessary conditions for a rigid body to be in equilibrium are that

- the resultant force $\mathbf{F_R}$ and the resultant couple $\mathbf{C_R}$ for any point "a" be null vectors.

That is $\mathbf{F_R} = \mathbf{0}$ and $\mathbf{C_R} = \mathbf{0}$. That is

$$\sum_{i=1}^{n} \mathbf{F}_i = \mathbf{0} \quad \text{and} \quad \sum_{i=1}^{n} \mathbf{r}_i \times \mathbf{F}_i + \sum_{j=1}^{m} \mathbf{C}_j = \mathbf{0}$$

If, instead of taking moments about “a”, we consider moments about another point “b”,



we get $$\sum_{i=1}^{n} (\mathbf{r}_i + \mathbf{d}) \times \mathbf{F}_i + \sum_{j=1}^{m} \mathbf{C}_j = \mathbf{0}$$

which amounts to be the same as before as

$$\sum_{i=1}^{n} \mathbf{d} \times \mathbf{F}_i = \mathbf{d} \times \sum_{i=1}^{n} \mathbf{F}_i = \mathbf{d} \times \mathbf{0} = \mathbf{0}$$

Hence, moment about any point in space must be zero; that is, there are only two independent vector equations

$$\sum_{i=1}^{n} \mathbf{F}_i = \mathbf{0} \quad \text{and} \quad \sum_{i=1}^{n} \mathbf{r}_i \times \mathbf{F}_i + \sum_{j=1}^{m} \mathbf{C}_j = \mathbf{0}$$

The above is equivalent to six scalar equations:

$$\sum_{i=1}^{n} (F_x)_i = \sum_i F_x = 0 \qquad \sum_i M_x = 0$$

$$\sum_i F_y = 0 \qquad \sum_i M_y = 0$$

$$\sum_i F_z = 0 \qquad \sum_i M_z = 0$$

Thus, no more than 6 unknown scalar quantities can be solved by the methods of statics for a single free body diagram.

Special Cases of Equilibrium

A Concurrent System of Forces

Since, the resultant in this case is a single force at the point of concurrency, this force $\mathbf{F_R}$ must be zero for equilibrium.

This amounts to the following <u>three scalar equations of equilibrium</u>:

$$\sum_i F_x = 0 \quad \sum_i F_y = 0 \quad \sum_i F_z = 0 \qquad \boxed{\mathbf{A}}$$

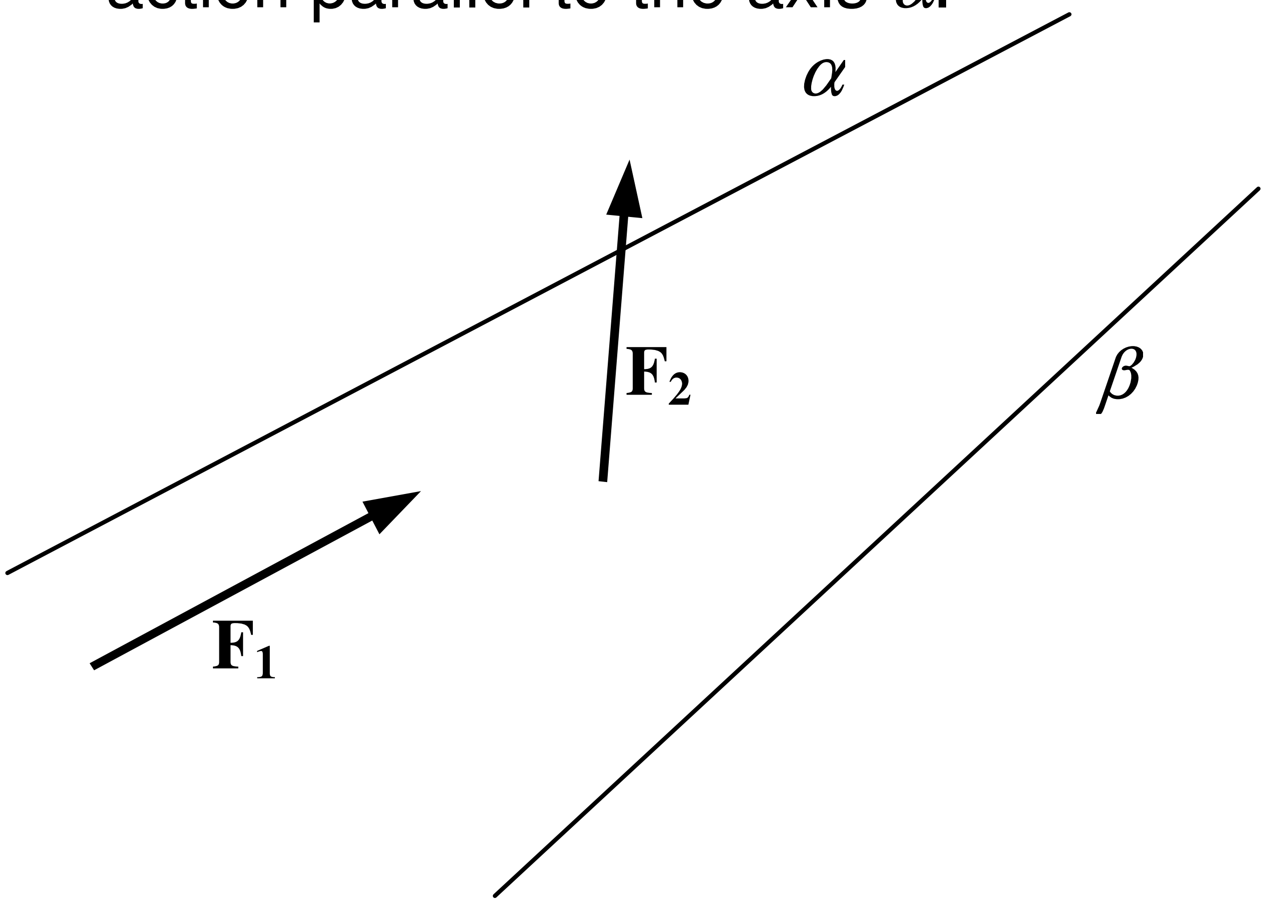
There are, however, other ways of ensuring a zero resultant force.

If the moment of this force about an axis α is zero, this would imply

(a) the resultant force is zero,

(b) the resultant force has a line of action through the axis, or

(a) the resultant force has a line of action parallel to the axis α.



If another nonparallel axis β is chosen, and the moment about this axis is also zero, we have:

(a) the resultant force is zero, or

(b) the resultant force has a line of action through both α and β axes

Now, let a third axis γ be chosen such that the line of action of the resultant neither intersects all three axes nor intersects any two axes while being parallel to the third.

Then, if the moment of the resultant about γ axis is also zero, then

(a) the resultant is zero.

Hence, another way is

$$\sum_i M_\alpha = 0 \quad \sum_i M_\beta = 0 \quad \sum_i M_\gamma = 0 \qquad \boxed{\textbf{B}}$$

where α, β and γ have the restrictions as noted earlier.

Combinations of $\boxed{\textbf{A}}$ and $\boxed{\textbf{B}}$ is also possible.

However, only <u>three equations of equilibrium</u> are available for a concurrent system of forces.

B Coplanar Force System

For a coplanar force system, the simplest resultant is a single force, a single couple or the null vector.

Hence, $\sum_i F_x = 0$ and $\sum_i F_y = 0$

(to ensure that the resultant force is zero), and $\sum_i M_\alpha = 0$

(to ensure that the couple is also zero).

Thus, there are three equations of equilibrium in the case of coplanar force systems too.

C Parallel Forces in Space

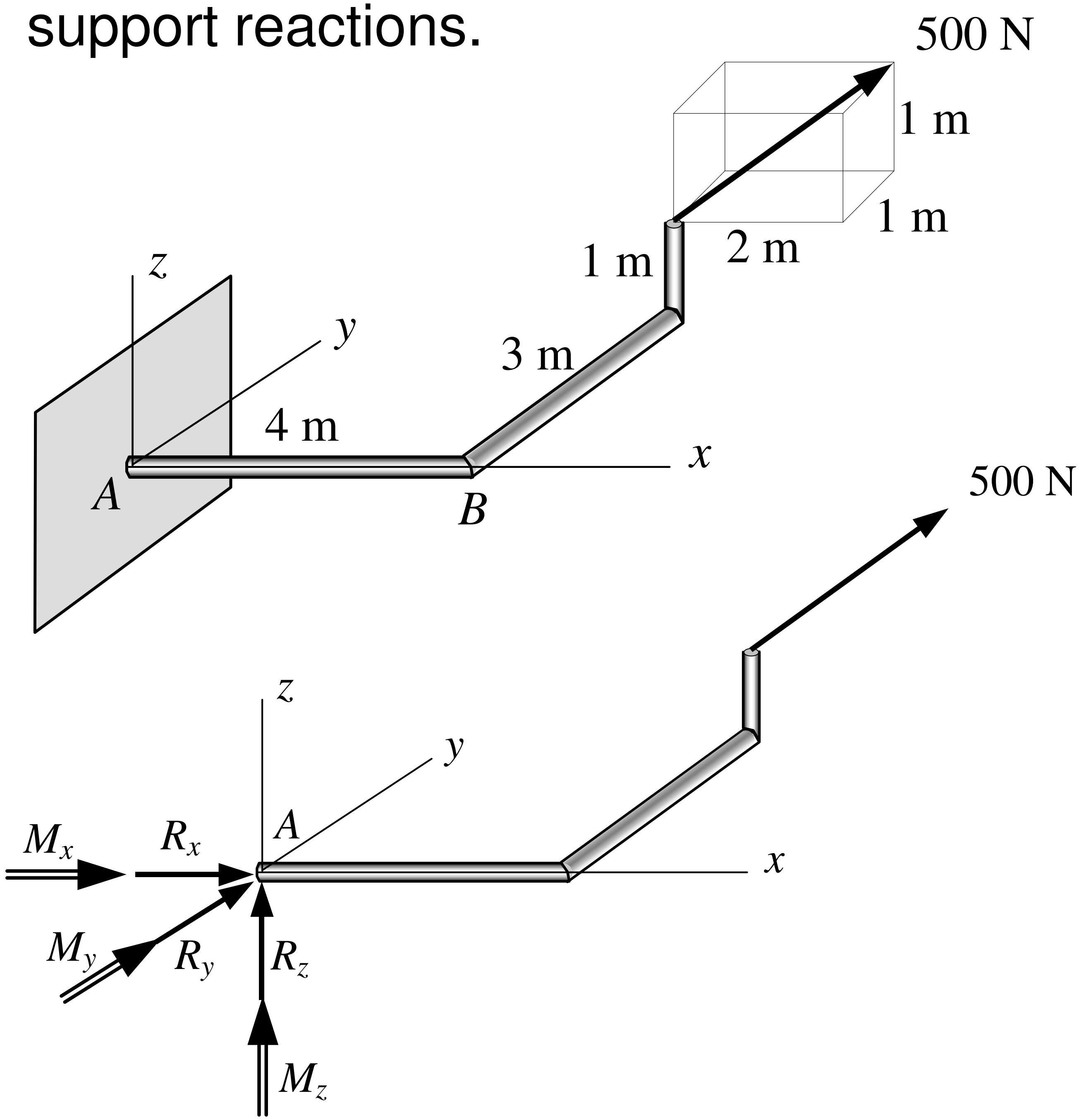
The resultant could be a single force or a single couple.

Hence, $\sum_i F_z = 0$ (to ensure that the resultant force is zero), and

$$\sum_i M_x = 0 \text{ and } \sum_i M_y = 0$$

(to make the couple zero).

<u>Example 1</u>: Draw the free body diagram of the bracket shown, and determine the support reactions.



The support reactions are found by invoking the equations of equilibrium. Thus, we get

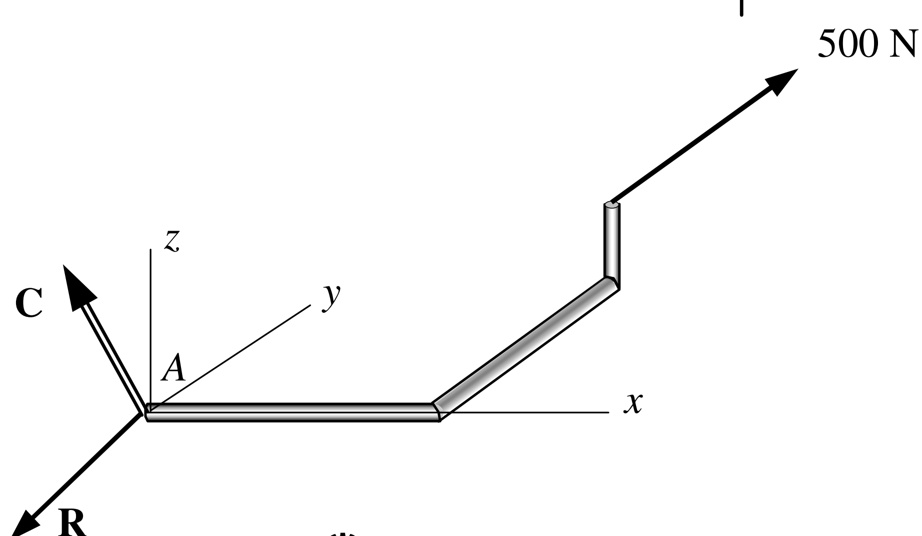
$$R_x = -500\left(\frac{2}{\sqrt{6}}\right) \qquad R_y = -500\left(\frac{1}{\sqrt{6}}\right) \text{ and}$$

$$R_z = -500\left(\frac{1}{\sqrt{6}}\right)$$

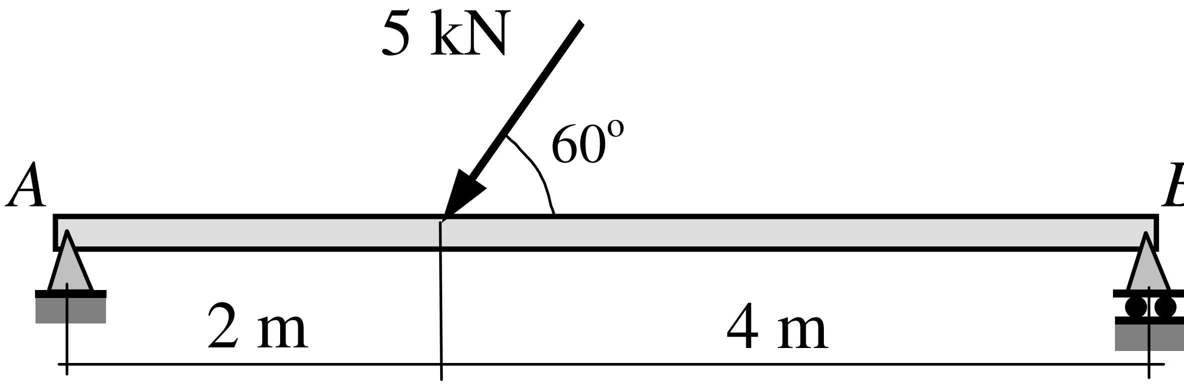
which can also be obtained by solving the simple vector equation **F + R = 0**.

In order to get the moment reactions, we can take moments about *A*. Thus,

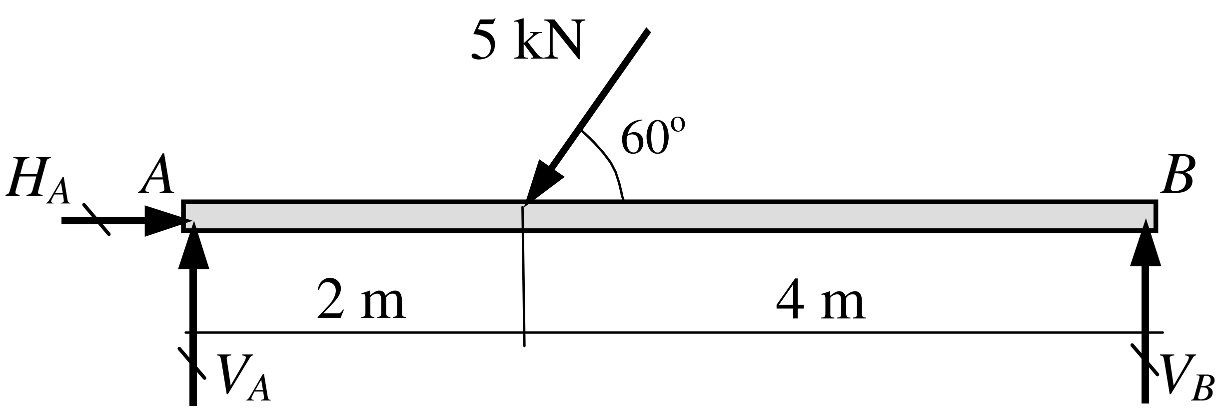
$$\mathbf{C}+\mathbf{M}=\mathbf{C}+\mathbf{r}\times\mathbf{F}=\mathbf{C}+\frac{500}{\sqrt{6}}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 4 & 3 & 1\\ 2 & 1 & 1\end{vmatrix}=\mathbf{0}$$



Example: Determine the support reactions of the simply supported beam shown in figure below.



Draw the free body diagram; identify the support reactions; write equations of equilibrium; and solve for the unknowns.

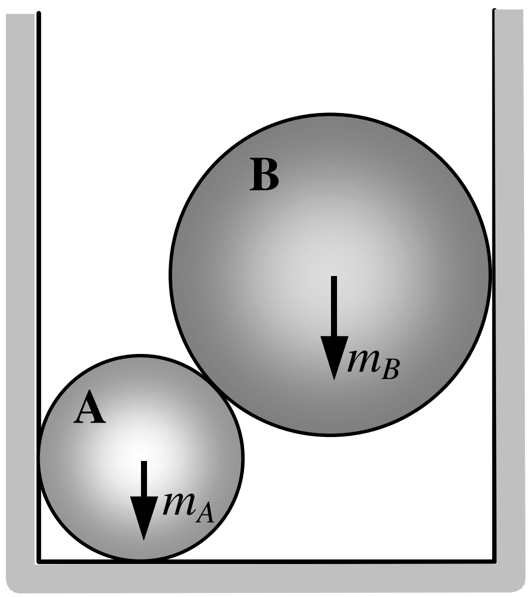


$$\sum F_x = 0; \quad H_A - 5\cos 60 = 0; \quad \therefore H_A = 2.5 \text{ kN}$$

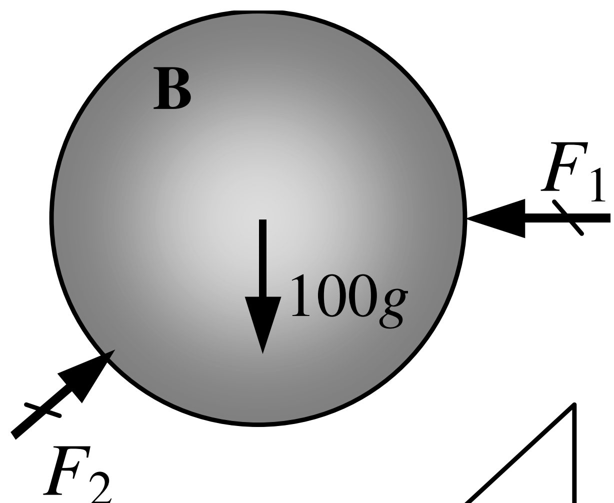
$$\sum M_A = 0; \quad 6V_B - 5\sin 60 \times 2 = 0; \quad \therefore V_B = 1.443 \text{ kN}$$

$$\sum F_y = 0; \quad V_A + V_B = 5\sin 60; \quad \therefore V_A = 2.887 \text{ kN}$$

<u>Example</u>: Cylinder A has a mass of 30 kg and B has a mass of 100 kg. They are of diameters 300 mm and 1000 mm respectively. The width of the container is 1 m. Compute all contact forces.



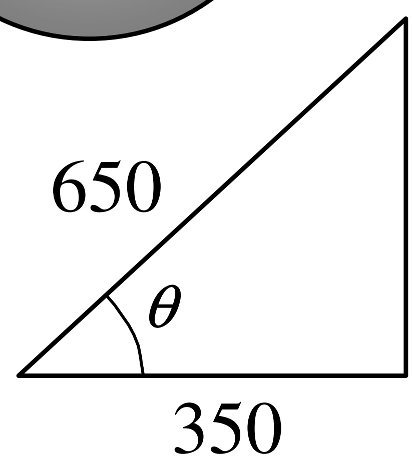
Free body diagram of ball B



$$\sum F_y = 0 \Rightarrow$$

$$F_2 \sin\theta - 100 \times 9.81 = 0;$$

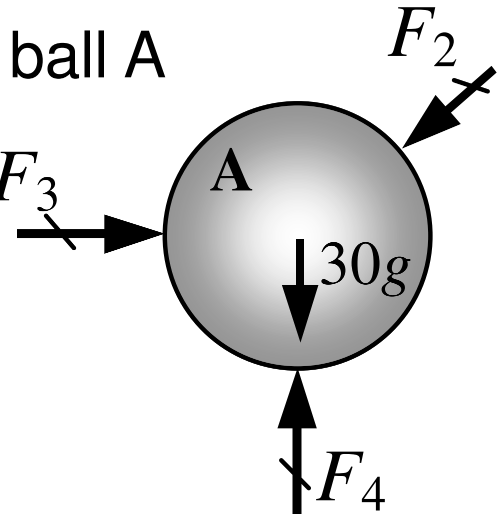
$$\therefore F_2 = \underline{\underline{1164.184 \text{ N}}}$$



$$\sum F_x = 0 \Rightarrow$$

$$F_1 = F_2 \cos\theta = \underline{\underline{626.868 \text{ N}}}$$

Free body diagram of ball A



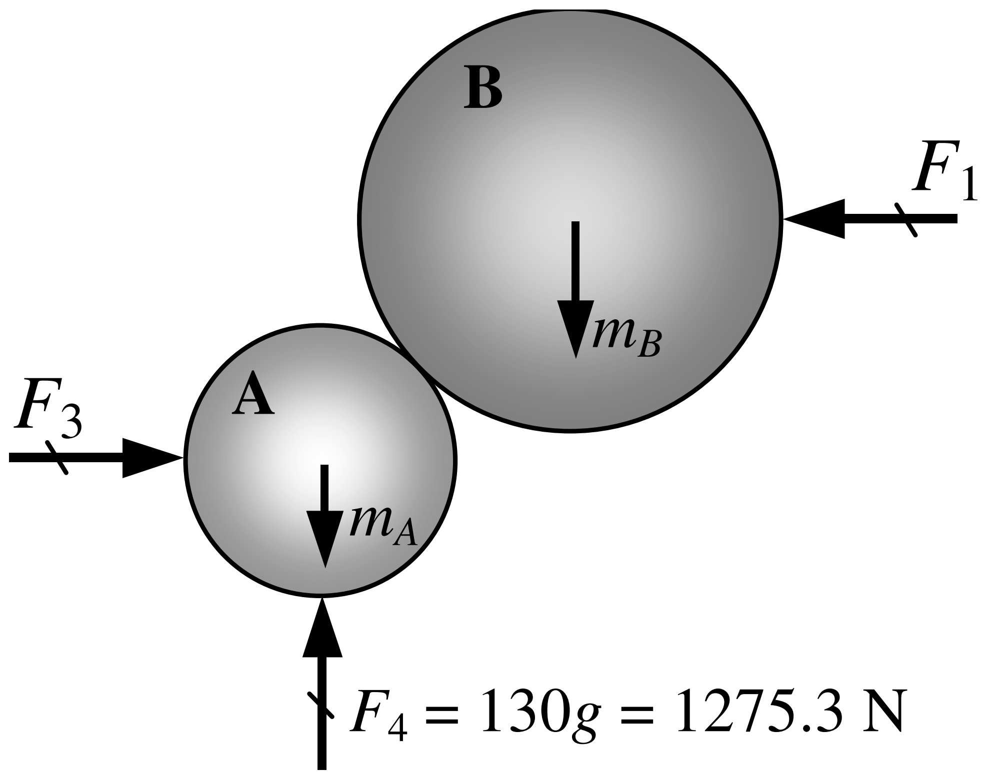
$$\sum F_x = 0 \Rightarrow$$

$$F_3 = F_2 \cos\theta$$

$$= \underline{\underline{626.868}} \text{ N}$$

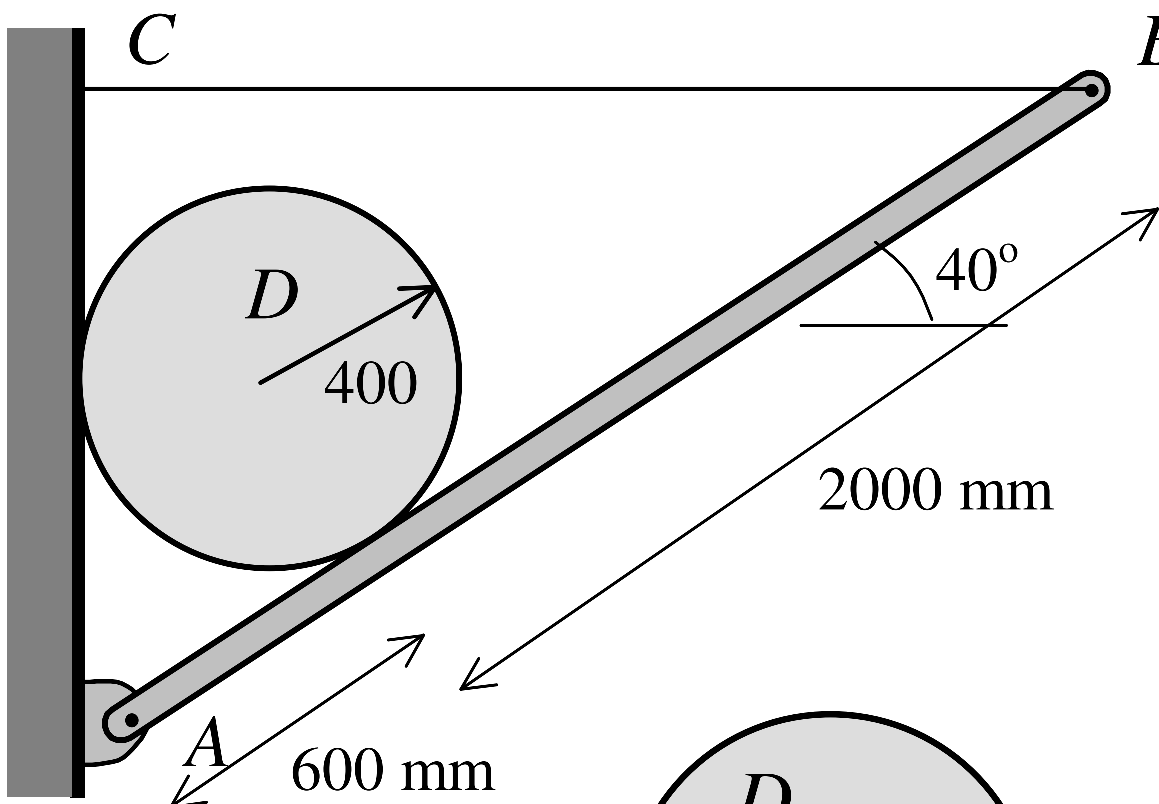
$$\sum F_y = 0 \Rightarrow$$

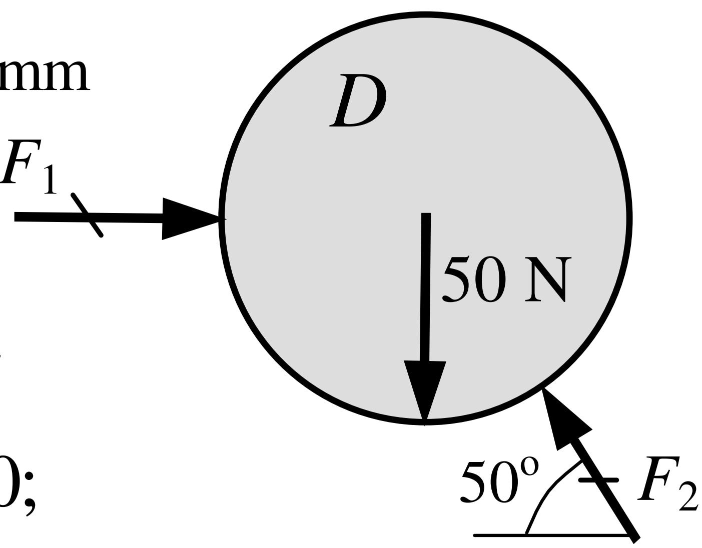
$$F_4 = 30 \times 9.81 + F_2 \sin\theta = \underline{\underline{1275.3}} \text{ N}$$



Joint free body diagrams of the two balls A and B

Example: Draw the free body diagram and find the tension in the string if the cylinder *D* has a weight of 50 N and the bar *AB* has a weight of 10 N.

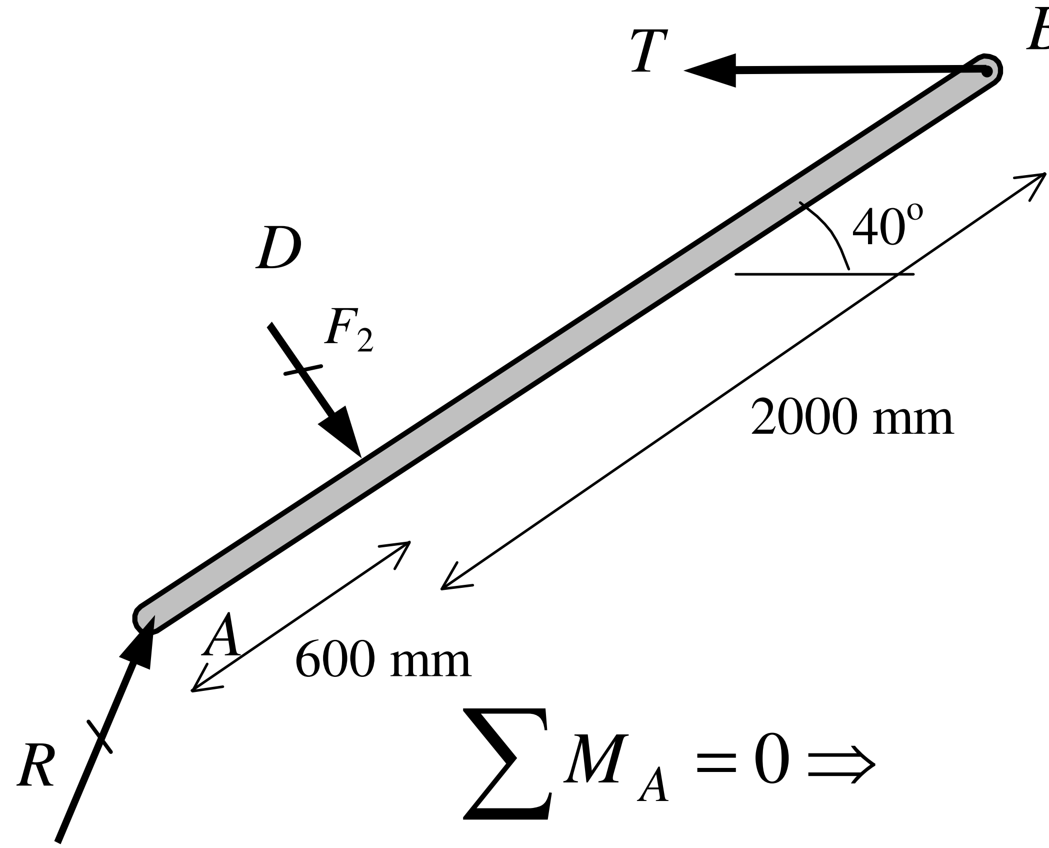




$$\sum F_y = 0 \Rightarrow$$

$$F_2 \sin 50 = 50;$$

$$\therefore F_2 = \underline{\underline{65.270 \text{ N}}}$$

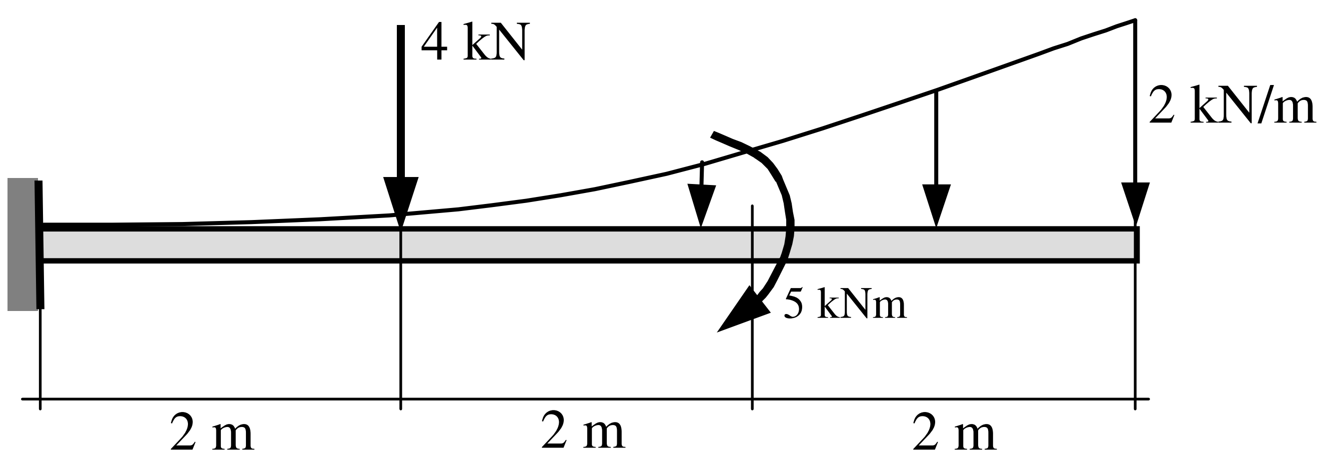


$$\sum M_A = 0 \Rightarrow$$

$$0.6 \times F_2 = T \times 2.6 \times \sin 40$$

$$\therefore T = \underline{\underline{23.433 \text{ N}}}$$

<u>Example</u>: Compute the support reactions of the cantilever shown.



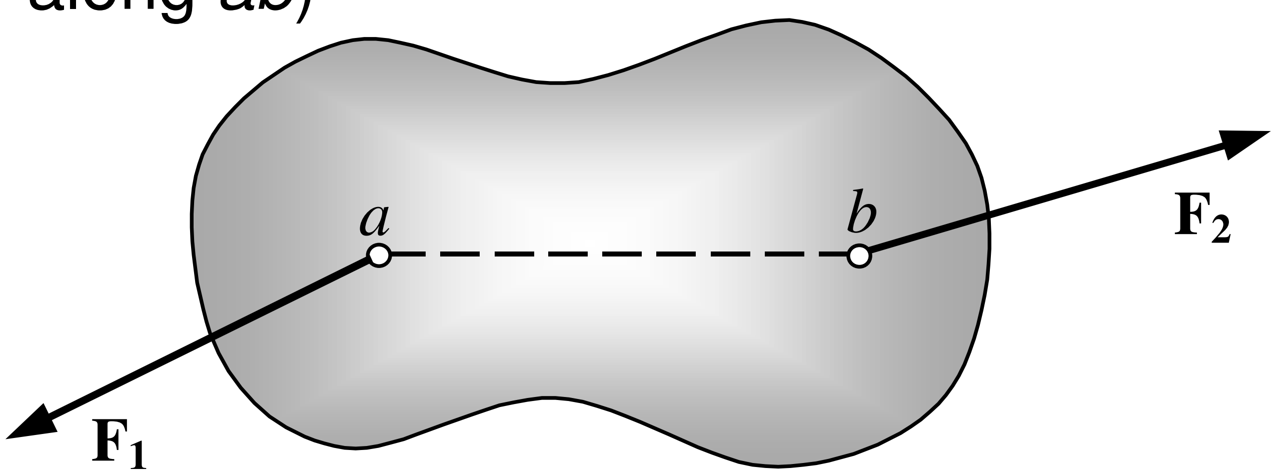
Simple Conclusions from Equilibrium

A Body in Equilibrium with Two Forces

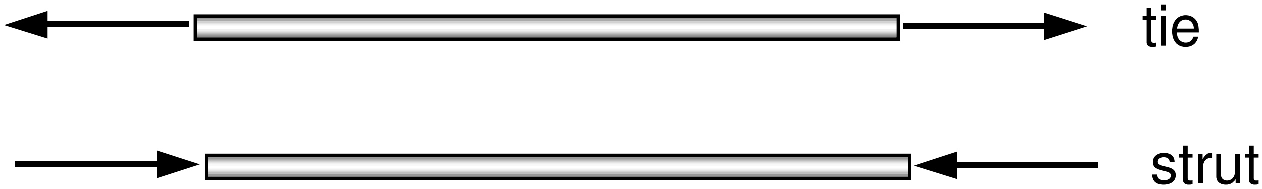
If a body is in equilibrium under the action of just two forces $\mathbf{F_1}$ and $\mathbf{F_2}$:

(i) $\mathbf{F_2 = - F_1}$

(ii) $\mathbf{F_1}$ and $\mathbf{F_2}$ should be collinear (i.e., along *ab*)

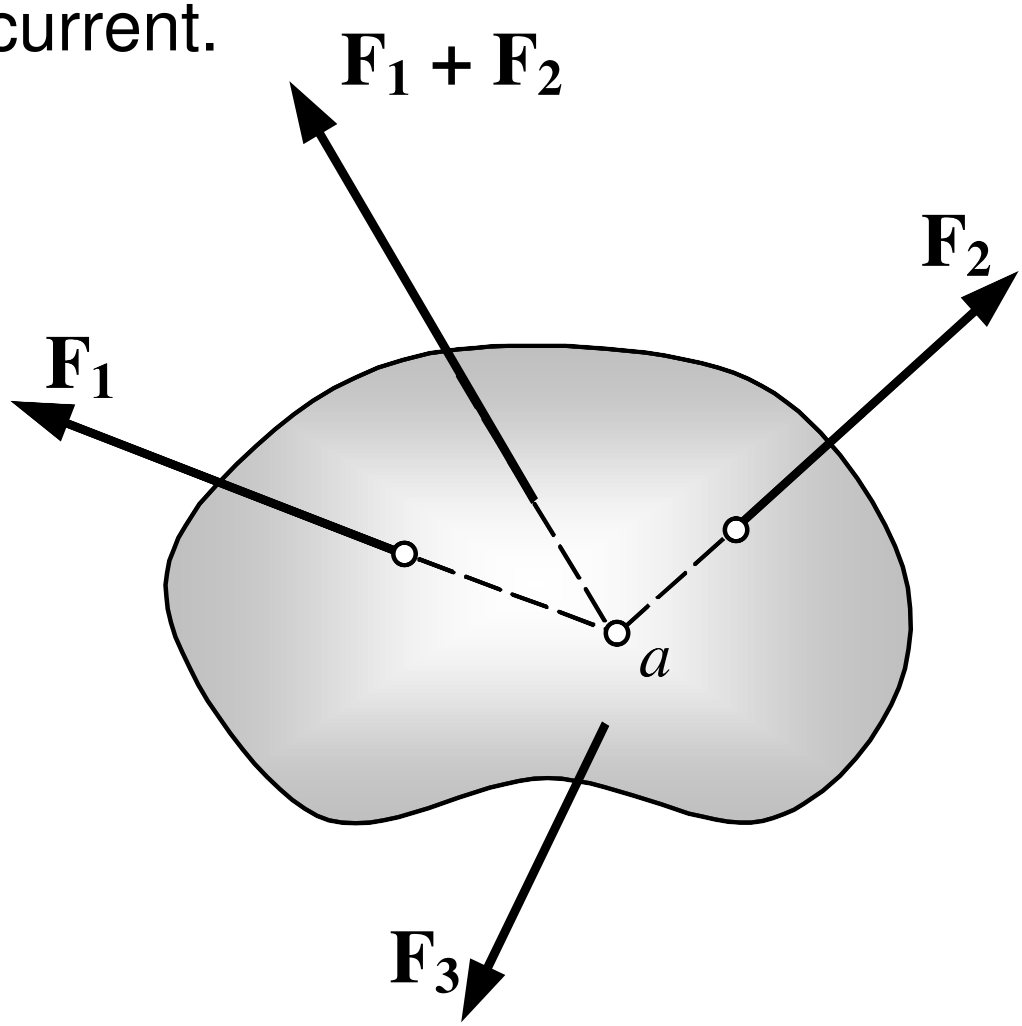


In the case of *trusses*—pin-connected frames if friction at the pin is ignored and the self-weight is also ignored, the member will be subjected to either compression or tension (“strut” and “tie”)

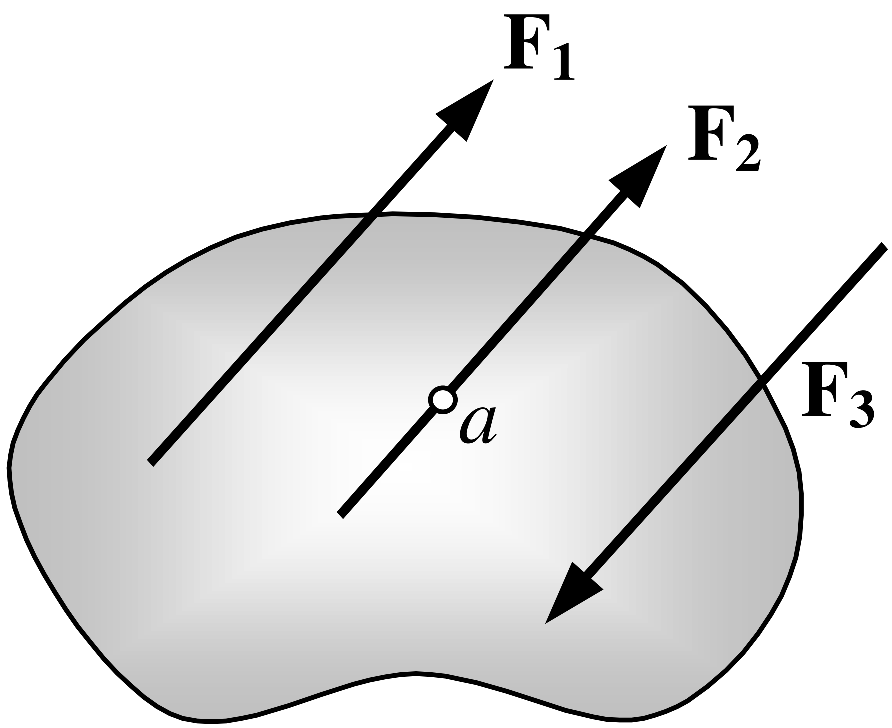


B Equilibrium of Three Forces

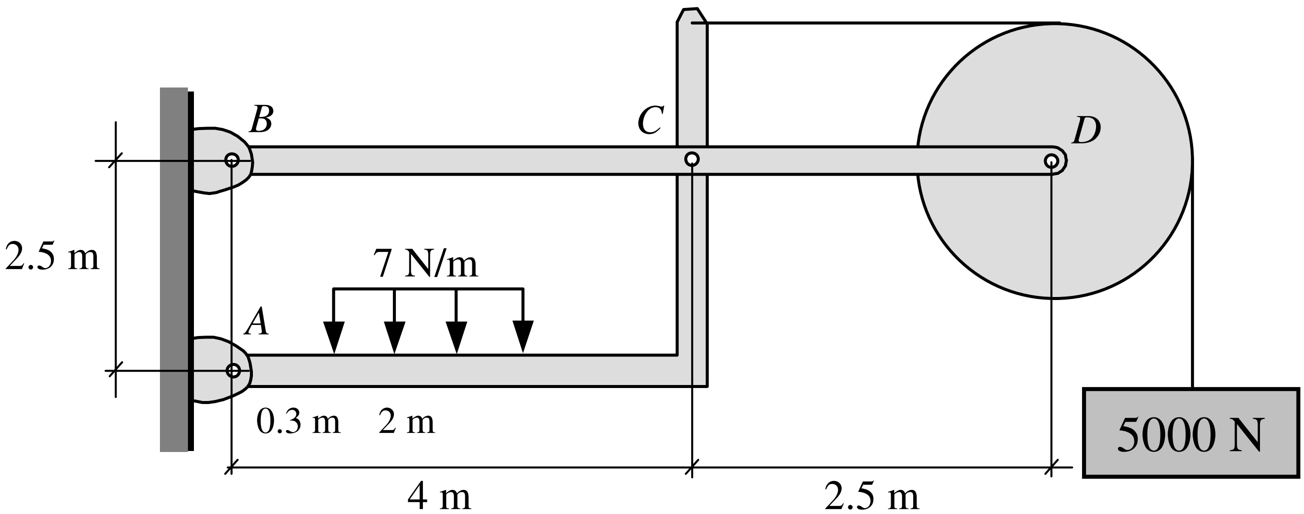
The forces must be coplanar, and the lines of action must be either parallel or concurrent.



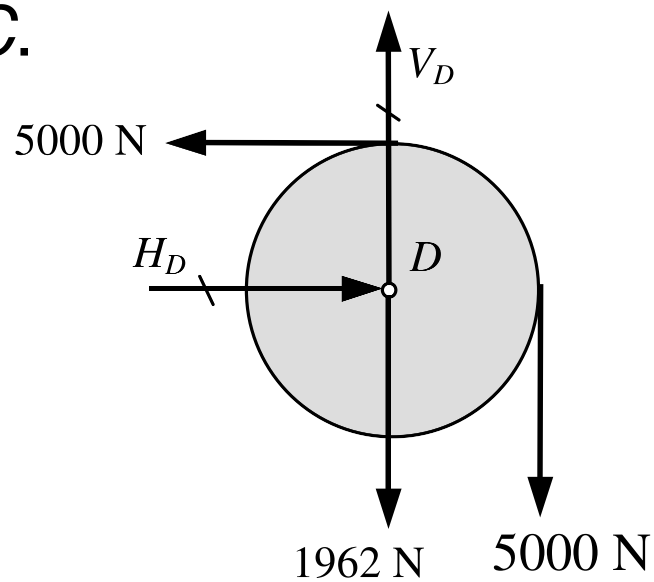
If $\mathbf{F_1}$ and $\mathbf{F_2}$ meet at "*a*", $\mathbf{F_3}$ must also pass through "*a*"

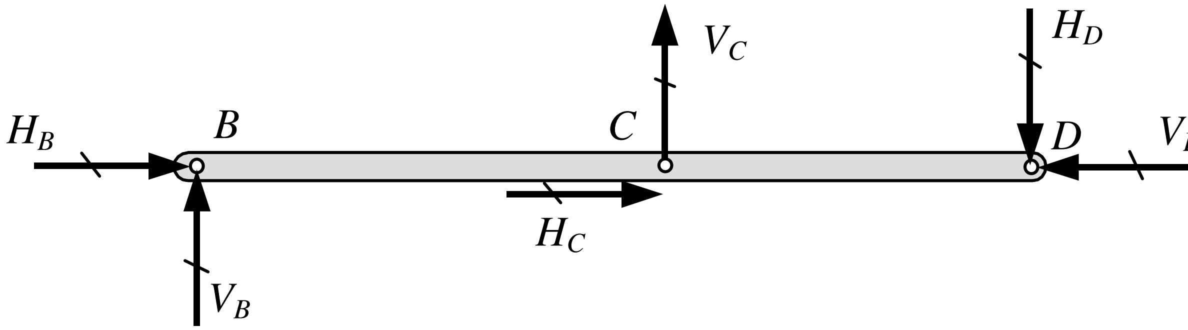


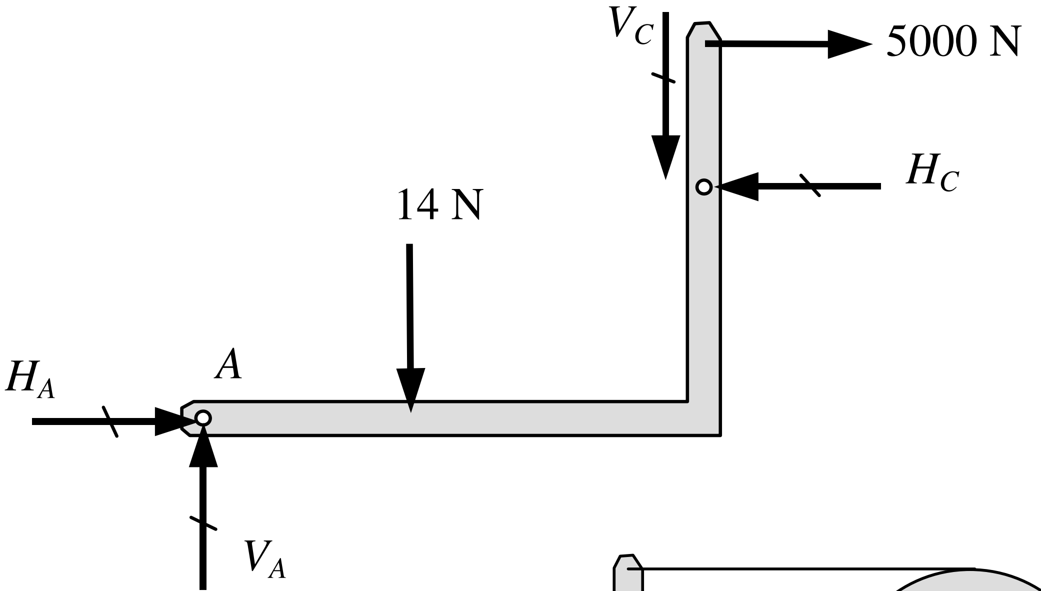
- If the three forces do not intersect, taking moments of $\mathbf{F_1}$ and $\mathbf{F_2}$ about "*a*", we conclude that $\mathbf{F_1}$, $\mathbf{F_3}$ and "*a*" must be coplanar.
- Now, choosing "*a*" elsewhere on $\mathbf{F_2}$, we conclude that $\mathbf{F_2}$ is also in the same plane.
- Since they do not intersect, $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ must be parallel.

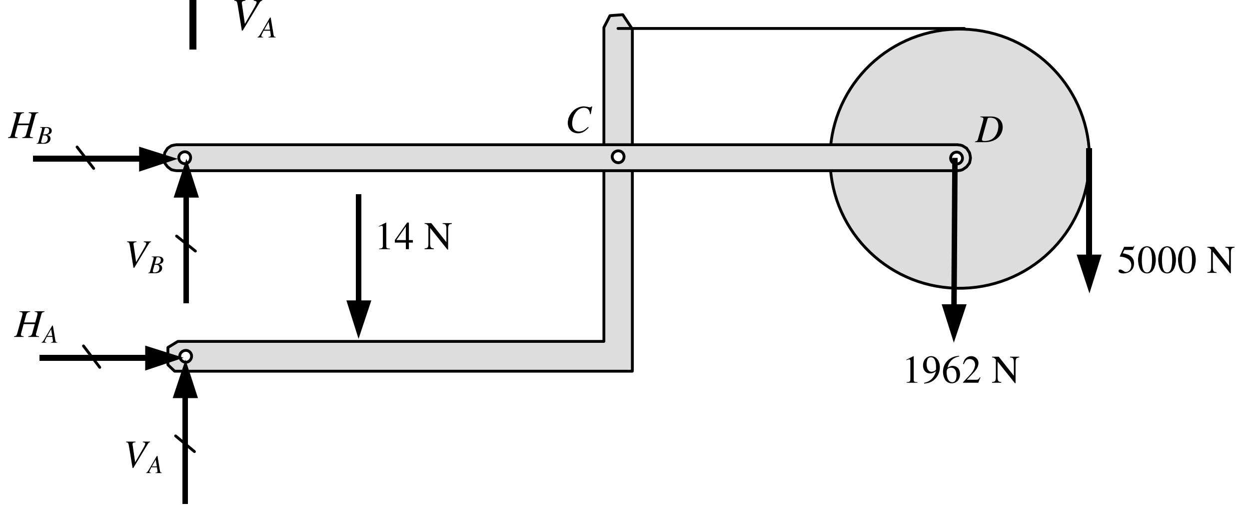


<u>Example</u>: Neglect the weight of the bars. The pulley has a mass of 200 kg. Find the force transmitted from one bar to the other at *C*.

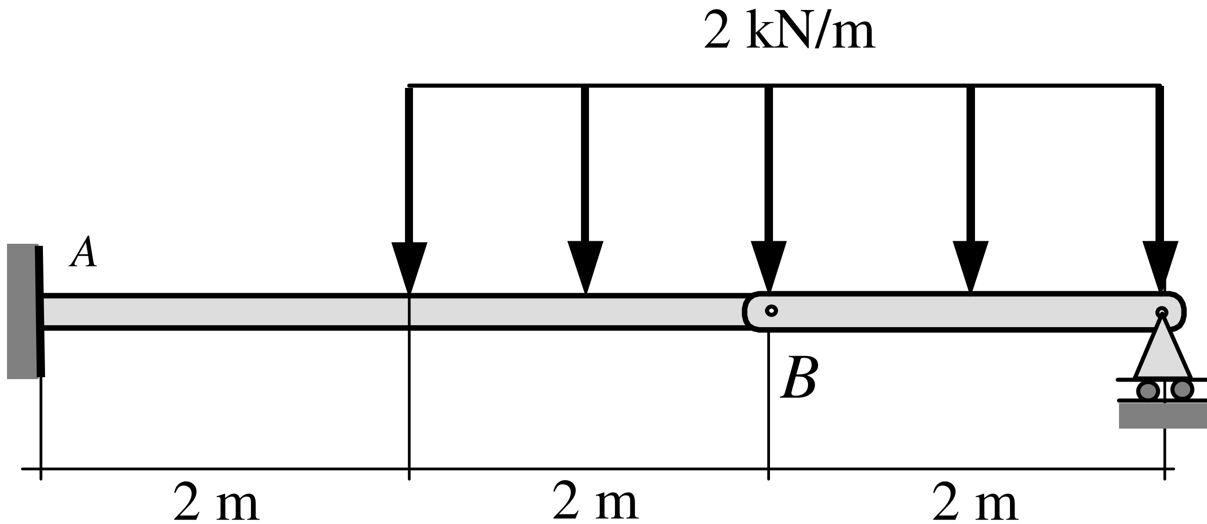




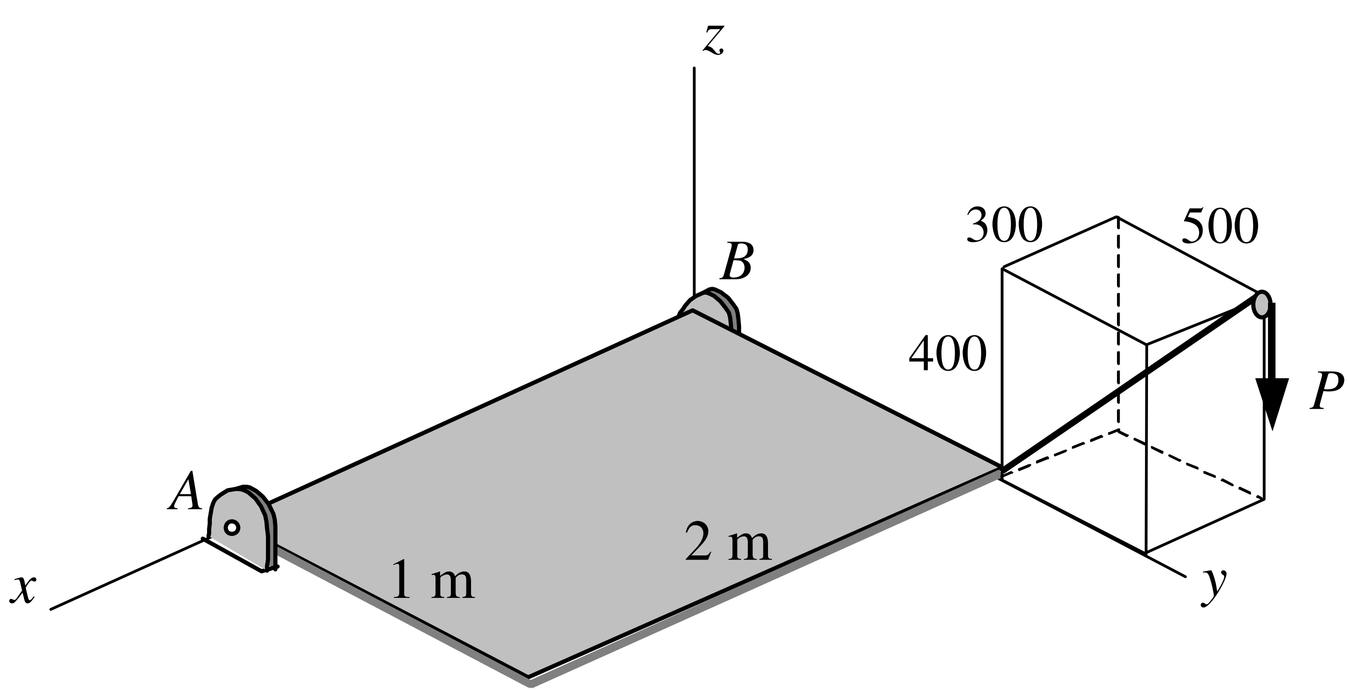




<u>Exercise</u>: Determine the support reactions of the beam. There is an internal hinge at *B*.

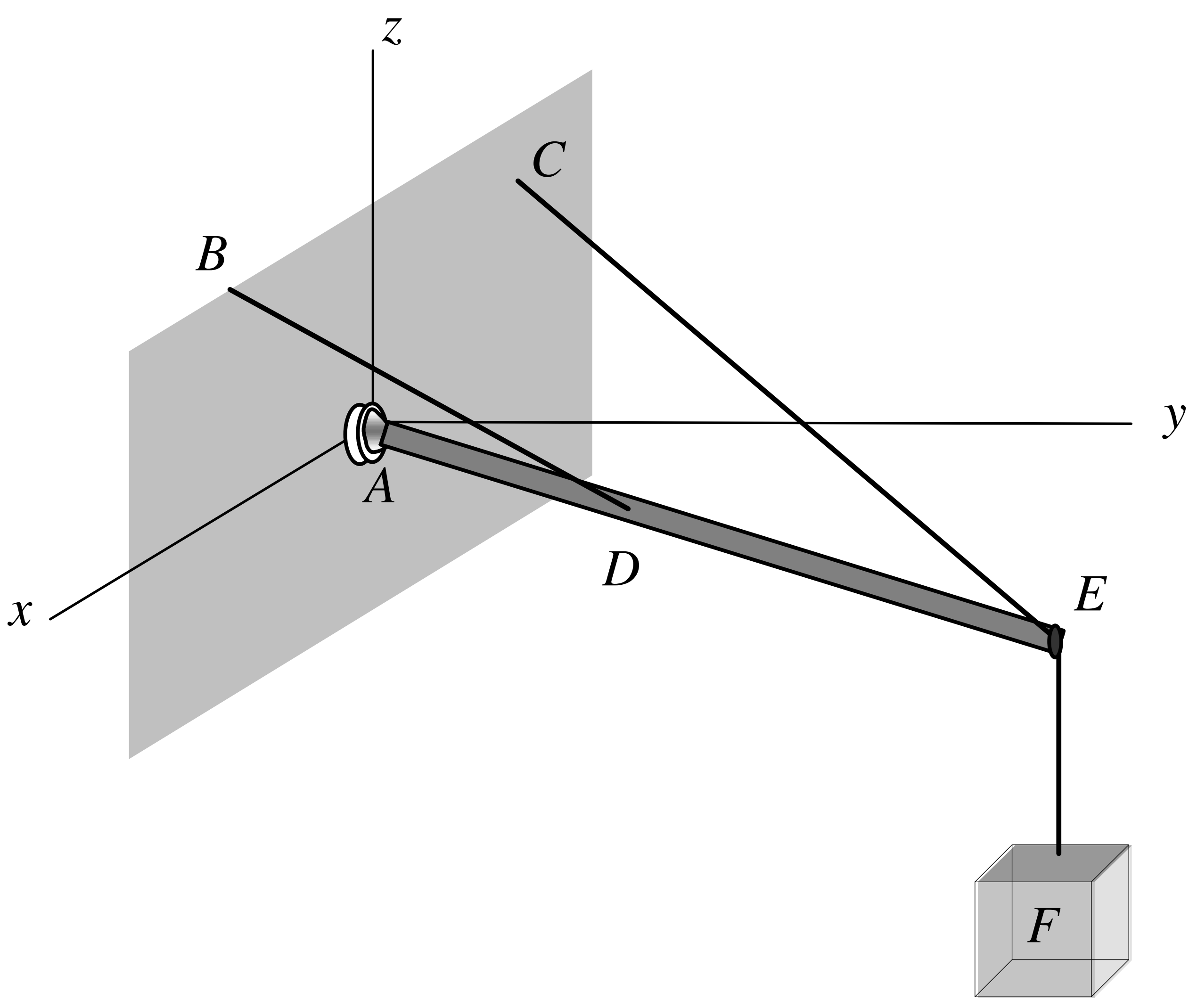


Exercise: Compute the force *P* needed to hold the door in a horizontal position. The door weighs 200 N. Determine the supporting forces at joints *A* and *B*. Joint *A* is a pin joint and *B* is a ball-and-socket joint.



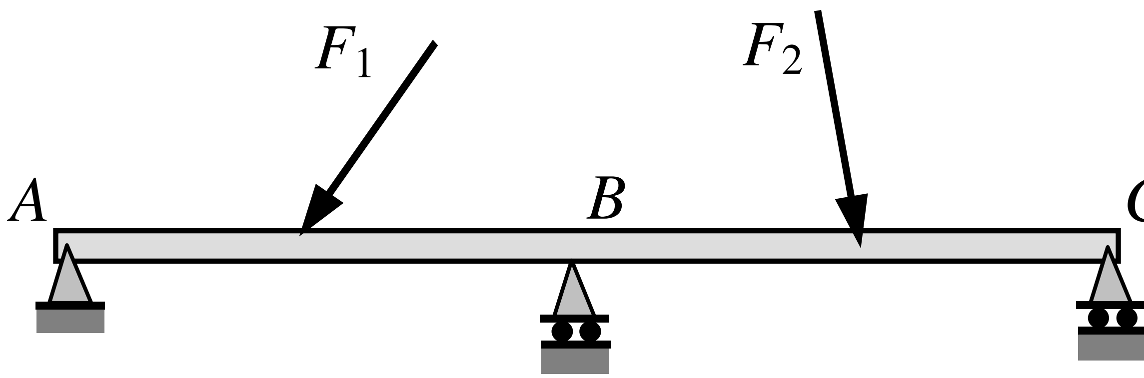
<u>Exercise</u>: Rod *AE* is held by ball-and-socket joint at *A* and supported by cables *BD* and *CE*. It carries a load *F* of 2000 N. Find the forces in the cables if

A is (0,0,0); *B* is (1,0,2) m and *C* is (–2,1.7,0) m. Rod *AE* is 5 m long and *D* is located at 2 m from end *A*. The direction cosines of line *AE* are *l* = 0, *m* = 0.9659.

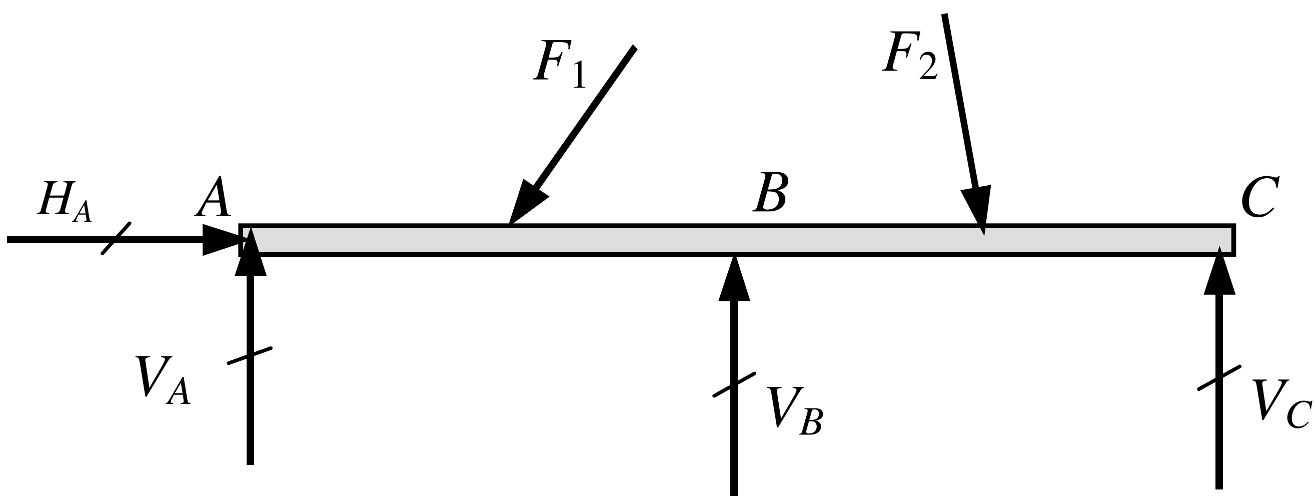


Static Indeterminacy

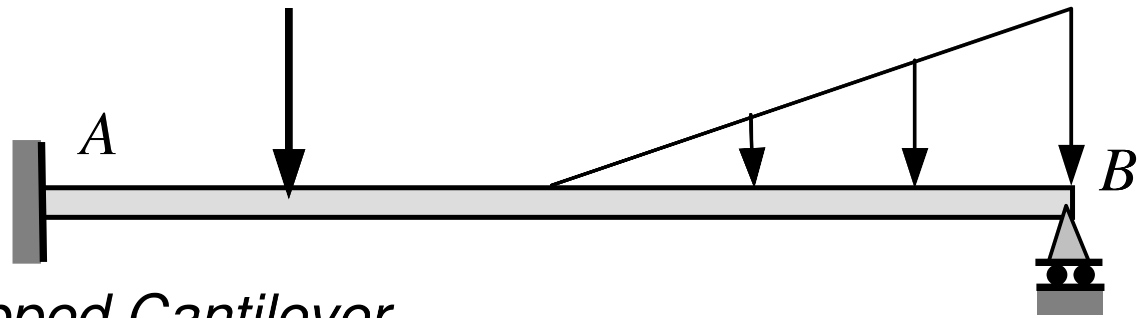
If the number of unknown support reactions exceed the number of equations of equilibrium available, the structure is said to be *statically indeterminate*.



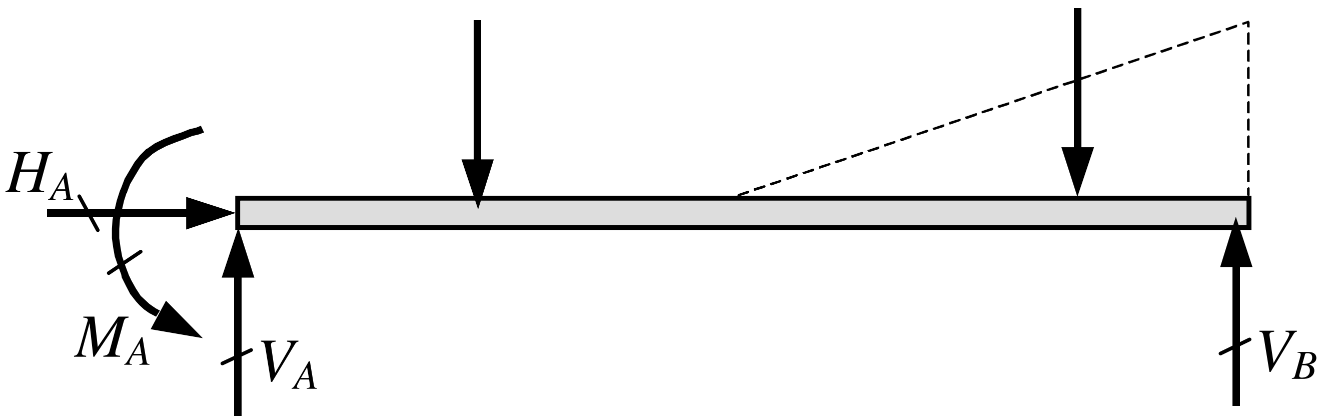
Continuous beam



Here there are 4 unknown reactions; but only 3 equations of equilibrium.

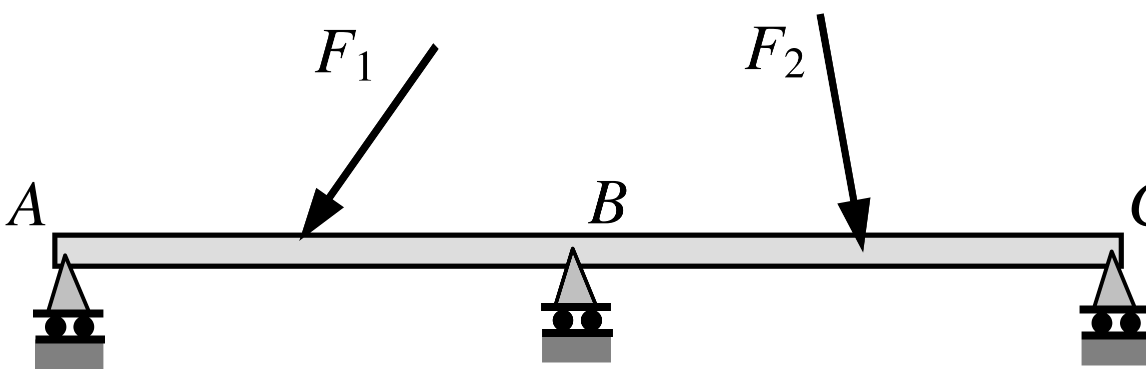


Propped Cantilever

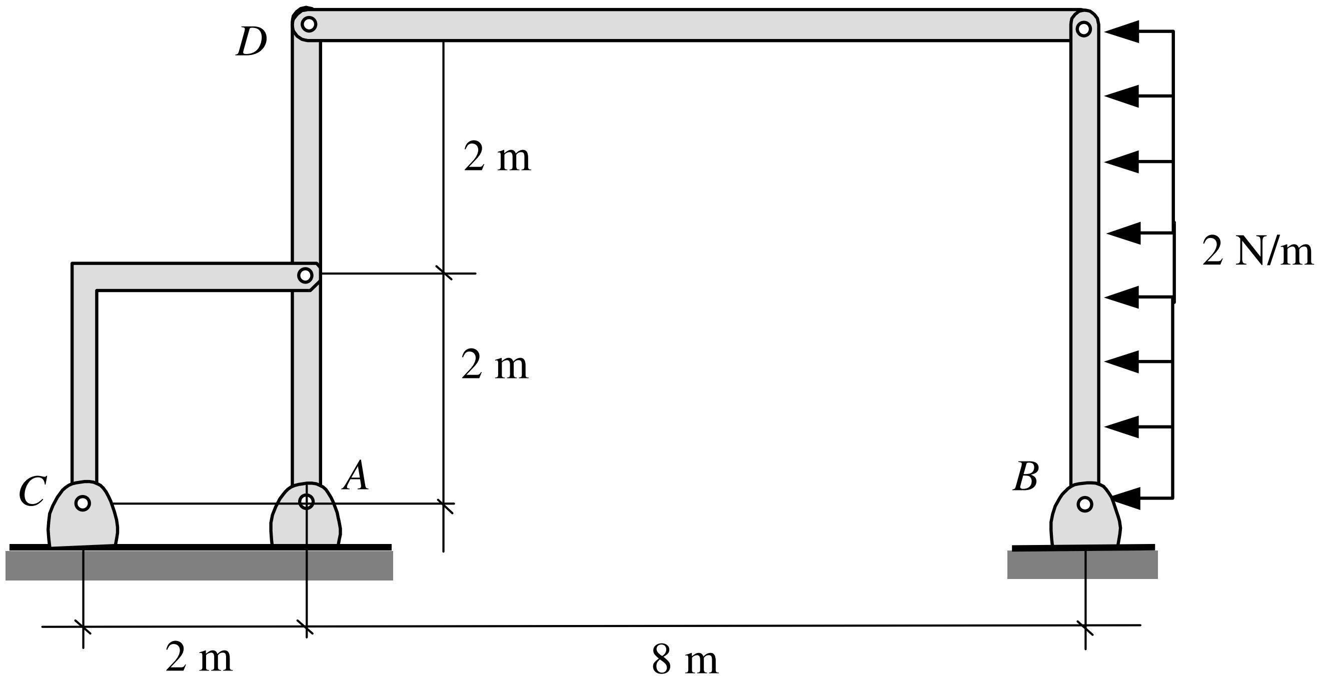


The number of reactions = 4

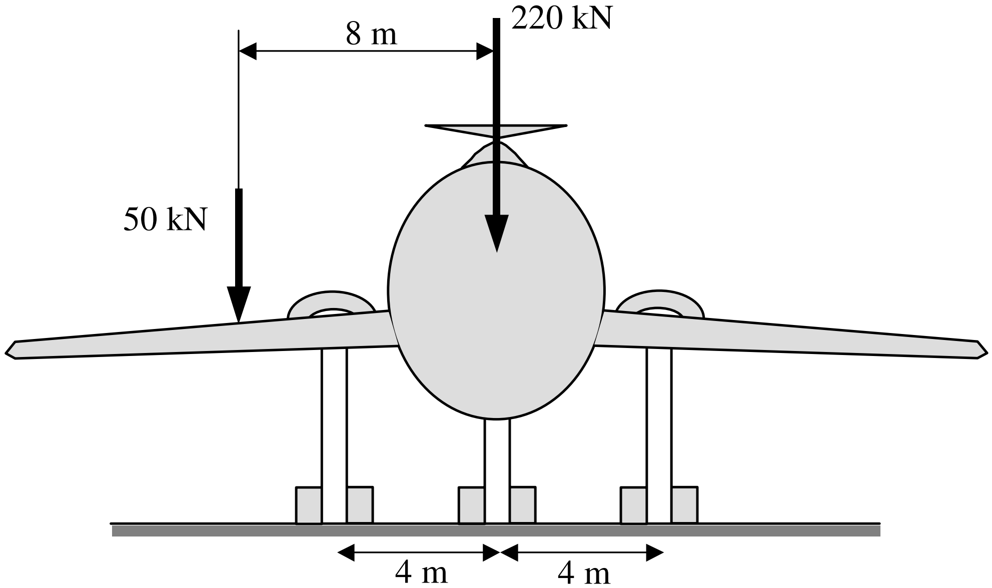
number of equations of equilibrium = 3

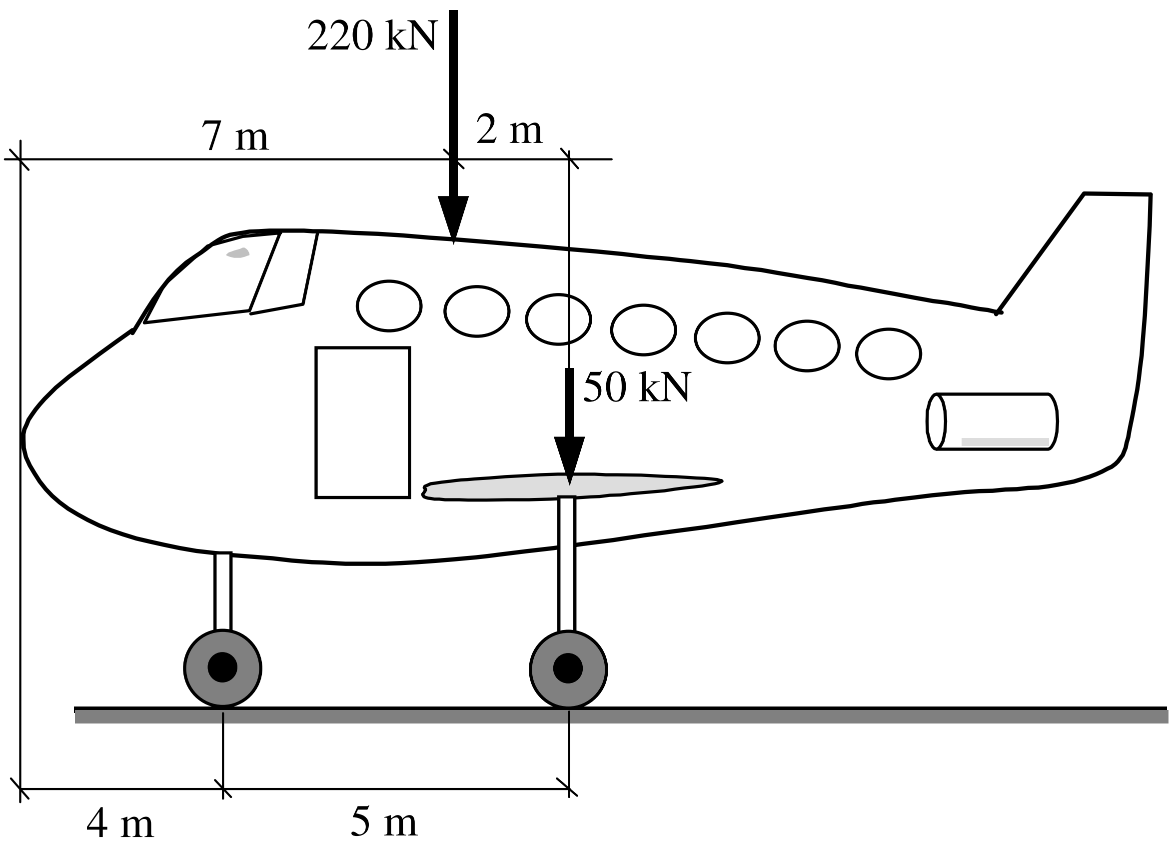


Unstable structure



Determine the reactions at *A* and *B* of the frame shown. Neglect the weight of the members.





A transport jet plane has a weight without fuel of 220 kN. If one wing is loaded with 50 kN of fuel, what are the forces in each of the three landing gear?