Quantum mechanics- notes

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1 Introduction

General method of Physics is as follows: we isolate the aspects of nature we want to study and call it a system. We then try to answer the following 4 questions about the system:

1. What describes the *state* of the system?
2. How does the state change with time?
3. How can I derive information about the system, once I know the state of the system?
4. Given a complex system (like a system containing a huge number of particles) with incomplete information, how do I approximately describe the overall behavior of the system?

Classical mechanics or quantum mechanics provides the basic framework to answer the first three questions and the fourth concerns Statistical Mechanics. Depending on the system under consideration, we get various branches of physics, like classical mechanics of particles, classical electrodynamics (classical theory of a system of electromagnetic fields), General relativity (classical theory of gravitational (metric) fields), Quantum mechanics of particles, Quantum electrodynamics (Quantum theory of a system of electromagnetic fields) etc.

State of the system is the minimum information to be given for a complete physical description [1] of the system at an instant of time. In classical mechanics, the state of the system is specified by the position and momentum (or velocity) of all the particles the system is made of. All physical observables are functions of these and can be computed given values of these. A definition of the system involves how the particles under consideration interacts with the external world (and also a specification of certain parameters like the masses of the particles etc.). This is specified by the forces acting on the particles or equivalently the potential in which the particles are in. State of the system at an instant of time, along with the equation of motion (given the system is defined by specifying the potential and the parameters as mentioned earlier) determines the state at any other instant of time [2]. Usually, in classical mechanics one is concerned with the motion of an idealized particle (or a small number of particles) acted upon by a force (or potential) field This is because when the number of particles becomes large, the complexity of description become so high that there is no practical way of describing the state of the system, let alone keeping track of the changes in the state. For eg, a description of the state of a system of

[1] Complete description means that given the state of the system at an instant of time, one can answer all the *answerable* questions about the physical observables of the system. As we will see later, in quantum mechanics there are physical observables that cannot be simultaneously measured accurately. So, a complete description doesn't mean a simultaneous knowledge of all the physical observables as in classical mechanics.

[2] This is true in quantum mechanics also.

1 gram of Hydrogen gas at an instant of time, would require $10^{14} GB$ of storage space. Therefore, we shift gear and try to describe only the overall behavior of the system. To be fair, this overall behavior is all we need (Why would we want to know the position of each hydrogen atom in the Hydrogen gas. In contrast, macroscopic questions like how much heat is needed to increase the temperature of the gas by 1 K are of interest). This is the domain of statistical mechanics. Clearly, this is of immense practical interest. However, we won't be interested in these questions in this course. We will restrict ourselves to the study of a single particle[3] (or a small number of particles) in an external potential. In particular, we will focus on the first 3 questions mentioned in the beginning.

By the end of the 19th century, physicists knew how to answer these four questions. Their confidence in their answers is reflected in the following quote often attributed to Lord Kelvin [4] :

"There is nothing new to be discovered in physics now. All that remains is more and more precise measurement."

Their answers are what we usually call classical physics. Then came experimental facts that couldn't be explained by classical physics, some of which were as follows:

1. **Stability of atom**

   By classical electrodynamics a negatively charged electron going around a positively charged nucleus will emit radiation, lose energy and spiral into the nucleus. Then why is atom stable?

2. **Seemingly infinite energy radiated by a black body**

   A classical calculation of energy of a black body gives infinity as answer.

3. **Frequency cutoff in photo electric effect**

   Contrary to the classical description, emission of electrons from a metal when it is exposed to light only happens when the frequency of light is above certain frequency.

4. **Compton effect**

   Light scattering from charges also doesn't agree with the classical description of light.

An explanation of these experimental facts required a major revision in our understanding of nature and led to Quantum mechanics. The above 4 questions remain the same. Answers are different. It's been 100 years since Quantum mechanics was developed. The quantum mechanical viewpoint is well established and it will not benefit us to dive into all the confusions of the first quarter of 20th century when it was developed. Therefore, we will not look at the above experiments in detail. Once we understand the basic framework of Quantum mechanics, we will be in a position to think about these experiments.

[3]By particle, we don't necessarily mean an elementary particle. A cricket ball may be a treated as a particle depending on the context

[4]It seems Lord Kelvin never really uttered those words.

Any physical theory consists of a set of axioms or postulates, which originates from experimental input. Sometimes these axioms are so obvious and ingrained in our experience that we don't think of them as axioms, but those are still there. Nobody really thought of Galilean invariance as axioms of classical mechanics, until these were revised by special relativity. Once we agree on the postulates, we can derive other physical conclusions or predictions by a process of logical deduction from these axioms. These predictions are then looked for in experiments. As long as, nothing contrary is found, the theory is fine. We gradually expand the domain of applicability of the theory until we hit a region were the theory fails and requires revision. In the end of nineteenth century we reached the boundaries of applicability of classical mechanics [5] and it had to be revised into Quantum mechanics, which has a larger domain of applicability. In fact, we have not come across any experiment which violates quantum mechanics, even though we do not know how to apply quantum mechanics in certain domains.

The dust has settled on quantum mechanics. In this course, instead of reliving the confusions of our forefathers, we will start with the postulates or axioms of Quantum mechanics and learn how to derive physical conclusions from it.

1.1 Postulates of Quantum mechanics

1. State of the system is a vector $|\psi\rangle$ in a complex Hilbert space.
2. Associated with every physical observable is a Hermitian operator. A measurement of an observable O yields an eigenvalue o_i of the corresponding operator $\hat{O}$ with probability $|\langle o_i|\psi\rangle|^2$ where $|o_i\rangle$ is an eigenstate of $\hat{O}$ with eigenvalue o_i and $|\psi\rangle$ is the state of the system. If a measurement yields an eigenvalue o_i, the state of the system becomes $|o_i\rangle$ and any subsequent measurement of O immediately after will yield the same value o_i with certainty.
3. The state of the system evolves unitarily in time.
4. The Hilbert space of a composite system consisting of various parts is the tensor product of the Hilbert space of the parts.

In this course, we will answer the 4 questions raised in the beginning within the framework of the above postulates which defines quantum mechanics. The outline of the course is as follows:

- In the next few lectures, we will define all the underlined stuff above and make sense of the above postulates. With a proper understanding of these postulates and an exceptional mathematical ability, one can in principle make most of the rest of the course redundant.
- In the rest of the course we will consider various example systems and apply the above postulates to derive physics out of these systems. We will first start with one dimensional systems. Once we have some proficiency dealing with 1D systems, we will look at 3D systems.

[5] basically we gained experimental access to atomic scales where Quantum mechanics is essential for any physical understanding.

- We will then briefly look at how to answer the fourth question raised above. A detailed understanding would require a course in quantum statistical mechanics.