M.Sc.

IN

MATHEMATICS

CURRICULUM AND SYLLABI

(Applicable from 2023 admission onwards)

Department of Mathematics
NATIONAL INSTITUTE OF TECHNOLOGY CALICUT
Kozhikode - 673601, KERALA, INDIA

The Program Educational Objectives (PEOs) of M.Sc. in Mathematics

PEO1	Provide a strong foundation in different areas of Mathematics, so that the students can compete with their contemporaries and excel in the various careers in Mathematics.
PEO2	Motivate and prepare the students to pursue higher studies and research, thus contributing to the ever increasing academic demands of the country.
PEO3	Enrich the students with strong communication and interpersonal skills, broad knowledge and an understanding of multicultural and global perspectives, to work effectively in multidisciplinary teams, both as leaders and team members.

Programme Outcomes (POs) & Programme Specific Outcomes (PSOs) of M.Sc. in Mathematics

PO1	Students will demonstrate in-depth knowledge of Mathematics, both in theory and application.
PO2	Students will attain the ability to identify, formulate and solve challenging problems in Mathematics.
PO3	Students will be able to analyse complex problems in Mathematics and propose solutions using research based knowledge.
PSO 1	Students will be able to work individually or as a team member or leader in uniform and multidisciplinary settings.
PSO 2	Students will develop confidence for self-education and ability for lifelong learning.

CURRICULUM

Total credits for completing M.Sc. in Mathematics is 75.

COURSE CATEGORIES AND CREDIT REQUIREMENTS:

The structure of M.Sc. programme shall have the following Course Categories:

Sl. No.	Course Category	Minimum Credits
1.	Program Core (PC)	47
2.	Program Electives (PE)	15
3.	Institute Elective (IE)	02
4.	Projects	11

The effort to be put in by the student is indicated in the tables below as follows:
L: Lecture (One unit is of 50 minute duration)
T: Tutorial (One unit is of 50 minute duration)
P: Practical (One unit is of one hour duration)
O: Outside the class effort / self-study (One unit is of one hour duration)

PROGRAMME STRUCTURE

Semester I

Sl. No.	Course Code	Course Title	L	T	P	O	Credits	Category
1.	MA6201E	Real Analysis	3	1	0	7	4	PC
2.	MA6202E	Abstract Algebra	3	1	0	7	4	PC
3.	MA6203E	Numerical Analysis	3	0	0	6	3	PC
4.	MA6204E	Linear Algebra	3	1	0	7	4	PC
5.	MA6205E	Ordinary Differential Equations	3	0	0	6	3	PC
6.	XXXXXE	Institute Elective	2	0	0	4	2	IE
		Total	**17**	**3**	**0**	**34**	**20**	**--**

Semester II

Sl. No.	Course Code	Course Title	L	T	P	O	Credits	Category
1.	MA6211E	Topology	3	1	0	7	4	PC
2.	MA6212E	Complex Analysis	3	1	0	7	4	PC
3.	MA6213E	Measure Theory	3	0	0	6	3	PC
4.	MA6214E	Graph Theory and Combinatorics	3	0	0	6	3	PC
5.	MA6215E	Probability and Statistics	3	1	0	7	4	PC
6.	MA6291E	Mini Project	0	0	0	4	2	Projects
		Total	**15**	**3**	**4**	**34**	**20**	**--**

Semester III

Sl. No.	Course Code	Course Title	L	T	P	O	Credits	Category
1.	MA7201E	Operations Research	3	1	0	7	4	PC
2.	MA7202E	Functional Analysis	3	1	0	7	4	PC
3.	MA7203E	Partial Differential Equations	3	0	0	6	3	PC
4.	MAXXXXE	Elective I	3	0	0	6	3	PE
5.	MAXXXXE	Elective II	3	0	0	6	3	PE
6.	MA7292E	Project Phase I	0	0	0	6	3	Projects
		Total	**15**	**2**	**6**	**36**	**20**	**--**

Semester IV

Sl. No.	Course Code	Course Title	L	T	P	O	Credits	Category
1.	MAXXXXE	Elective III	3	0	0	6	3	PE
2.	MAXXXXE	Elective IV	3	0	0	6	3	PE
3.	MAXXXXE	Elective V	3	0	0	6	3	PE
4.	MA7293E	Project Phase II	0	0	0	12	6	Projects
		Total	**9**	**0**	**12**	**30**	**15**	**--**

List of Electives

Sl. No.	Course Code	Course Title	L	T	P	O	Credits
1.	MA7221E	Reliability of Systems	3	0	0	6	3
2.	MA7222E	Multivariate Statistical Analysis	3	0	0	6	3
3.	MA7223E	Regression Analysis	3	0	0	6	3
4.	MA7224E	Software Reliability	3	0	0	6	3
5.	MA7225E	Stochastic Processes	3	0	0	6	3
6.	MA7226E	Applied Probability Models	3	0	0	6	3
7.	MA7227E	Advanced Topics in Graph Theory	3	0	0	6	3
8.	MA7228E	Fuzzy Set Theory and Applications	3	0	0	6	3
9.	MA7229E	Numerical Methods for Partial Differential Equations	3	0	0	6	3
10.	MA7230E	Fluid Dynamics	3	0	0	6	3
11.	MA7231E	Computational Methods for Ordinary Differential Equations	3	0	0	6	3
12.	MA7232E	Methods in Applied Mathematics	3	0	0	6	3
13.	MA7233E	Perturbation Methods	3	0	0	6	3
14.	MA7234E	Introduction to Fractional Calculus	3	0	0	6	3
15.	MA7235E	Fractal Geometry	3	0	0	6	3
16.	MA7236E	Chaotic Dynamical Systems	3	0	0	6	3
17.	MA7237E	Fuzzy Graph Theory	3	0	0	6	3
18.	MA7238E	Fourier Analysis	3	0	0	6	3
19.	MA7239E	Differential Geometry	3	0	0	6	3
20.	MA7240E	Distribution Theory	3	0	0	6	3
21.	MA7241E	Multivariable Calculus	3	0	0	6	3

22.	MA7242E	Statistical Digital Signal Processing	3	0	0	6	3
23.	MA7243E	Time Series Modelling	3	0	0	6	3
24.	MA7244E	Wavelets and Applications	3	0	0	6	3
25.	MA7245E	Numerical Linear Algebra	3	0	0	6	3
26.	MA7246E	Spectral Theory of Hilbert Space Operators	3	0	0	6	3
27.	MA7247E	Operator Theory	3	0	0	6	3
28.	MA7248E	Advanced Complex Analysis	3	0	0	6	3
29.	MA7249E	Special Functions	3	0	0	6	3
30.	MA7250E	Univalent Function Theory	3	0	0	6	3
31.	MA7251E	Number Theory & Cryptography	3	0	0	6	3
32.	MA7252E	Algebraic Number Theory	3	0	0	6	3
33.	MA7253E	Introduction to Algebraic Geometry	3	0	0	6	3
34.	MA7254E	Introduction to Dynamical Systems	3	0	0	6	3
35.	MA7255E	Galois Theory	3	0	0	6	3
36.	MA7256E	Sobolev Spaces with Applications	3	0	0	6	3
37.	MA7257E	Calculus of Variations and PDEs	3	0	0	6	3
38.	MA7258E	Fixed Point Theory and Application	3	0	0	6	3
39.	MA7259E	Approximation Theory	3	0	0	6	3
40.	MA7260E	Geometric Function Theory	3	0	0	6	3
41.	MA7261E	Introduction to Lie Algebras	3	0	0	6	3
42.	MA7262E	Introduction to Lie Superalgebras	3	0	0	6	3
43.	MA7263E	Kac-Moody Algebras	3	0	0	6	3
44.	MA7264E	Representation Theory	3	0	0	6	3
45.	MA7265E	Banach Algebras	3	0	0	6	3
46.	MA7266E	Finite Element Methods and Applications	3	0	0	6	3
47.	MA7267E	Transform Techniques	3	0	0	6	3
48.	MA7268E	Algebraic Topology	3	0	0	6	3
49.	MA7269E	Advanced Topology	3	0	0	6	3
50.	MA7270E	Generalized set Theory	3	0	0	6	3
51.	MA7294E	Computer Programming	1	0	3	6	3

Students may also credit PG level courses offered by other departments or Ph. D. level courses offered in the institute.

Detailed Syllabi

MA6201E REAL ANALYSIS

L	T	P	O	C
3	1	0	7	4

Pre-requisites: Nil

Total Lecture sessions: 39

Course Outcomes:

CO1: Appreciate the role of least upper bound property in real analysis which underlies all crucial results.
CO2: Learn the crucial concept of continuity of functions and uniform continuity and will be able to work on problems emphasizing these ideas of real analysis.
CO3: Study thoroughly the metric topology and discuss the ideas connecting compactness and continuity and connectedness and continuity.
CO4: Inculcate interest in analysis and understand how pictures and leading questions get into the strategy of proofs.

Real number system and its structure, infimum, supremum, LUB Axiom, Countable and uncountable sets. Sequences and series of real numbers, subsequences, monotone sequences, limit inferior, limit superior, convergence of sequences and series, Cauchy criterion.

Functions of a single real variable, limits of functions, continuity of functions, uniform continuity, Differentiation, properties of derivatives, chain rule, Rolle's theorem, mean value theorems, L'Hopital's rule. Riemann Integration- Darboux Integrability- Properties of the Integral- Fundamental theorem of calculus.

Sequences and series of functions, pointwise and uniform convergence, Consequences of Uniform convergence- Series of functions- Power series- equicontinuity, pointwise and uniform boundedness, Arzela-Ascoli's theorem.

Metric spaces - Definition and examples - open balls and open sets- Convergent sequences in metric spaces - limit and cluster points, Cauchy sequences - Bounded sets - Dense sets- Compact spaces and their properties -Continuous functions on Compact spaces- Characterization of Compact Metric spaces - Connected spaces- Complete metric spaces - Examples- Baire Category Theorem - Banach Contraction Principle.

References:

1. R.G. Bartle and D.R. Sherbert, Introduction to Real Analysis 4th Edn, Wiley India Edition, 2014.
2. S. Kumaresan, Topology of Metric Spaces, 2nd Edition, Narosa Publishing House, New Delhi, 2011.
3. Ajit Kumar and S. Kumaresan, A Basic Course in Real Analysis, CRC Press, Third Indian Reprint, 2015.
4. Edward D. Gaughan, Introduction to Analysis, AMS, Indian edition, 2010.
5. Kenneth A. Ross, Elementary Analysis: The Theory of Calculus, Springer Verlag, 2004.
6. Tom M. Apostol, Mathematical Analysis 2nd edn, Narosa, New Delhi, Reprint, 2002.
7. Walter Rudin, Principles of Mathematical Analysis, Third Edition, McGraw Hill, 2017.

MA6202E ABSTRACT ALGEBRA

L	T	P	O	C
3	1	0	7	4

Pre-requisites: Nil

Total Lecture Sessions: 39

Course Outcomes:

CO1: Define algebraic structures such as (groups, rings and fields) and construct substructures.
CO2: Analyse given algebraic structures and develop new structures based on given structures.
CO3: Understand the homomorphism and isomorphism by using the relationship between groups and rings.
CO4: Understand field extension and can work with mathematical problems that involve polynomial equations.

Group Theory

Binary operation, Group, Cyclic groups, Dihedral groups, Symmetric groups, Matrix groups, Subgroups, Cosets, Normal subgroups, Quotient groups, Homomorphisms, Finite Groups, Cayley's theorem, Group actions, Class equation, Sylow's theorems, Direct product of groups, Structure theorem for finite abelian groups.

Ring Theory

Ring, Formal power series rings, Matrix rings, Group rings, Characteristics of a ring, Integral domain, Field, Division ring, Ideals and Quotient ring, Homomorphism, Isomorphism theorems, Prime and Maximal ideals, Chinese Remainder Theorem, Euclidean algorithm, Euclidean domain, Principal Ideal Domain, Unique Factorization Domain(UFD), Gauss lemma, Polynomial Rings, Irreducibility of Polynomials.

Field Theory

Field extensions, Prime subfield, Algebraic and transcendental elements and extensions.

References:

1. D. S. Dummit and R. M. Foote, Abstract Algebra, 3rd Edition, Wiley, 2003.
2. S. Lang, Algebra, 3rd Edition, Springer, 2010.
3. S. Lang, Undergraduate Algebra, 3rd Edition, Springer, 2005.
4. M. Artin, Algebra, Prentice Hall of India, 1994.
5. I.N. Herstein, Topics in Algebra, Wiley, 2006.
6. T. W. Hungerford, Algebra, Graduate Texts in Mathematics, Vol. 73, Springer-Verlag, 2011.

MA6203E NUMERICAL ANALYSIS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Nil

Total Lecture sessions: 39

Course Outcomes:

CO1: Approximate the solution of scalar and system of equations using numerical methods.
CO2: Approximate eigenvalues using numerical methods.
CO3: Learn interpolation, numerical differentiation and integration.
CO4: Find the numerical solution of ordinary differential equations.

Solutions of nonlinear equations: Bisection method, Method of false position, Newton-Raphson method, Fixed-point iteration, Rates of convergence of these methods, Muller method, Chebyshev method, accelerating convergence. Solution of Linear system of equations, Direct methods: Gauss–Jordan methods, pivoting strategies, LU decomposition method, Condition number, Error estimates. Iterative methods: Gauss-Jacobi and Gauss-Seidel methods, Relaxation method, conjugate gradient method. Newton's method for nonlinear simultaneous equations. Eigenvalues and Eigenvectors: The power method, Inverse power method, Given's method for symmetric matrices.

Interpolation: Lagrange's interpolation, interpolation error, Neville's Algorithm, Newton's interpolation, Piecewise and Spline interpolation, Hermite interpolation. Curve fitting, least squares approximation. Differentiation formulas in the case of equally spaced points. Richardson's extrapolation. Numerical integration: Trapezoidal and Simpson rules, Gaussian integration, errors of integration formulas, Romberg integration.

Numerical solution of ordinary differential equations: Single step methods: Taylor series method, Picard's Method, Euler and Modified Euler methods, Runge-Kutta methods of 2^{nd} and 4^{th} order. Multi-step methods: Milne's Predictor-Corrector methods, Adam-Bashforth and Adam-Moulton 3^{rd} and 4^{th} order methods. Finite difference methods for linear and nonlinear boundary value problems, shooting method for boundary value problems.

References:

1. Jain M.K., Iyengar S.R.K., Jain R.K., Numerical methods for Scientific and Engineering Computation, 8th edition, New Age International (P) Ltd, 2022.

2. Brian Bradie, A friendly introduction to Numerical analysis, 1st edition, Pearson, 2007.

3. Gerald C. F., Wheatley P.O., Applied Numerical Analysis, 7th edition, Pearson, 2007.

MA6204E LINEAR ALGEBRA

L	T	P	O	C
3	1	0	7	4

Pre-requisites: Nil

Total Lecture Sessions: 39

Course Outcomes:

CO1: Solve systems of linear equations using Gauss elimination to reduce to echelon form and solve linear systems of equations using the language of matrices.
CO2: Understand an axiomatic description of an abstract vector space and be able to determine a basis and the dimension of a finite-dimensional space.
CO3: Find a matrix representation for the linear transformation and discuss how those matrices change when the bases are changed.
CO4: Understand and determine properties of eigenvalues and eigenvectors of special operators.

Vector Spaces

System of linear equations- Gauss elimination method, Matrices, Elementary Row operations, Row-Reduced Echelon matrix, Invertible matrices, Vector Spaces- Subspaces, Linear combination of vectors, Linear dependence and independence of vectors, Bases and dimension, Coordinates, Row-equivalence.

Linear Transformation

Linear transformation, The algebra of linear transformations, Linear operators, Isomorphism, Representation of linear transformations by matrices, Change of basis, Linear functional, Dual space, Double dual, Transpose of a transformation.

Invariant Subspaces

Characteristic polynomials- Eigenvalues and Eigenvectors, Diagonalization, annihilating polynomial-minimal polynomial, Cayley Hamilton theorem, Invariant subspaces, Direct sum decompositions, Invariant direct sums.

Inner Product Spaces

Inner product, Inner product space, Existence of orthonormal basis, Adjoint and Self- adjoint operator, Normal and Unitary operator, Real Spectral theorem, Complex Spectral theorem.

References:

1. K. Hoffman and R. Kunze, Linear Algebra, Second Edition, Pearson Education, 2015.
2. G. Strang, Introduction to Linear algebra, 3rd Edition, Wellesley- Cambridge Press, U.S., 2003.
3. S. H. Friedberg, A. J. Insel, and L. E. Spence, Linear Algebra, 4th Edition, Pearson Education, 2003.
4. S. Axler, Linear Algebra Done Right, 3rd Edition, Springer, 2015.
5. S. Lang, Linear Algebra, 3rd Edition, Springer, 2004.

MA6205E ORDINARY DIFFERENTIAL EQUATIONS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Nil

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand theory and solve first and second order differential equations.
CO2: Find the series solution of differential equations.
CO3: Understand Green's function and be able to solve boundary value problems.
CO4: Able to solve system of ordinary differential equations.
CO5: Learn phase-plane analysis and stability.

Ordinary differential equations, basic concepts, Existence and uniqueness theorems, Gronwall's inequality and applications, Cauchy-Peano and Picard-Lindelöf theorems. Second order linear differential equations, Linear dependence and Wronskian, Method of variation of parameters. The method of undetermined coefficients, Applications of linear second order equations, Linear differential equations of order n.

Power series solutions of second-order equations, Legendre's equation, Legendre polynomials and its properties, regular and singular points, Frobenius method, Bessel's equation, Bessel's function and its properties, Sturm-Liouville problem. Oscillation theory, qualitative properties of solutions, Sturm separation and comparison theorems, Green's functions, construction of Green's functions, nonhomogeneous boundary conditions.

System of ODEs, systems with constant coefficients, Matrix methods for first order linear systems, Matrix exponentials, Autonomous system of differential equations, Phase-space, Stability, Basic definitions, Conditions for asymptotic stability, Lyapunov stability.

References:

1. E. A. Coddington & N. Levinson, Theory of Ordinary Differential Equations, Tata-McGraw Hill, 2012
2. W. Walter, Ordinary Differential Equations, Springer, 6th edition, 1996.
3. P. Blanchard, R. L. Devaney, & G. R. Hall, Differential Equations, Brooks/Cole, 3rd edition, 2006.
4. Suman Kumar Tumuluri, A first course in ordinary differential equations, A Chapman and Hall book, 1st edition, 2021.
5. G. F. Simmons, Differential Equations with Applications and Historical Notes, McGraw Hill, 2nd edition, 2017.
6. Ross, Shepley L., Introduction to Ordinary Differential Equations, fourth edition, John Wiley & Sons, Inc., New York, 3rd edition, 2007.

MA6211E TOPOLOGY

L	T	P	O	C
3	1	0	7	4

Pre-requisites: Nil

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand the concepts of metric spaces and topological spaces and their role in mathematics.
CO2: Create new topological spaces by using subspace, product and quotient topologies.
CO3: Use continuous functions and homeomorphisms to understand structure of topological spaces.
CO4: Explore the foundations of mathematics (logic and set theory) at a level and depth appropriate for someone aspiring to study higher-level mathematics and/or to become a professional mathematician.

Topological spaces, Basis for a topology, The order topology, Product topology on X x Y, The subspace topology, quotient topology

Continuous functions and sets with imposed topologies. Closed sets, Limit points, Continuous functions, The Product topology, The metric topology, Properties

Connected sets, Connected sets in the real line, Components and path components Local

connectedness, Compact spaces, Compact sets in the real line

Countability axioms, Separation axioms, Urysohn Lemma, Urysohn Metrization Theorem,

Tychonoff Theorem.

References:

1. James R. Munkres: “Topology – A First Course”, Pearson Education, New Delhi, 2000.

2. G. F. Simmons; “Introduction to Topology and Modern Analysis”, McGraw Hill, 2017.

3. K. D. Joshi; “Introduction to General Topology, New Age Publications, New Delhi, 2022.

MA6212E COMPLEX ANALYSIS

L	T	P	O	C
3	1	0	7	4

Pre-requisites: Basic real analysis

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand Cauchy-Riemann equations, analytic functions, and various properties of analytic functions.
CO2: Find images of regions under the complex functions.
CO3: Apply the appropriate techniques of complex integration for establishing theoretical results and for solving related problems.
CO4: Use techniques of complex analysis to evaluate integrals of real-valued functions.
CO5: Represent functions as Taylor, power, and Laurent series, classify singularities and poles, find residues, and evaluate complex integrals using the residue theorem.

Geometry and topology of the complex plane, Riemann sphere, Limits, Continuity, Differentiability, Analytic functions, Cauchy-Riemann equation, Harmonic functions, Harmonic conjugate, Multi-valued functions, Mappings by elementary functions, Sequences and series, Uniform convergence, Radius of convergence of power series, power series as an analytic function, Complex exponential and Complex logarithm functions.

Elementary conformal mappings, Linear fractional transformations, Cross-ratio and its invariance property, Inverse Points, Mappings of disks and half-planes.

Complex integration, Arcs and closed curves, Line integral, Analytic functions in regions, Length and area, Cauchy's theorem, Index of a point with respect to a closed curve, Cauchy's integral formula, Morera's theorem, Weierstrass's convergence theorem.

Classification of singularities, Taylor's and Laurent's series (and theorems), Casorati-Weierstrass theorem, Cauchy's residue theorem, Evaluation of definite integrals. Zeros of analytic functions, Liouville's theorem, Fundamental theorem of algebra, Uniqueness theorem, Maximum modulus principle (and theorem), Schwarz lemma, Argument principle, Rouche's theorem, Open mapping theorem, Inverse function theorem.

References:

1. S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, Second Edition, New Delhi, 2005.
2. S. Ponnusamy, H. Silverman, Complex Variables with Applications, Birkhäuser Boston, 2006.
3. L. V. Ahlfors, Complex Analysis, McGraw-Hill International Editions, Third Edition, New Delhi, 1979.
4. Dennis G. Zill and Patrick Shanahan, A first course in complex analysis with applications, Jones & Bartlett Learning, 2003
5. Conway J.B, Complex Analysis, Narosa Publishing House.1995.

MA6213E MEASURE THEORY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Real Analysis

Total Lecture sessions: 39

Course Outcomes:

CO1: Identify measurable sets, non-measurable sets, measurable functions and evaluate the Lebesgue integral.
CO2: Determine various notions of convergence like L^p-convergence, convergence in measure and convergence almost everywhere, pointwise and uniform convergence.
CO3: Evaluate the iterated Lebesgue integral by Tonelli and Fubini's theorem on product measure spaces.
CO4: Identify the nature of a measure and be able to decompose a measure by using Hahn-Jordan decomposition and Radon-Nikodym theorem.

Algebra of sets, σ-algebra, Borel σ-algebra, outer measure, measurable sets, non-measurable sets, complete measure, Carathodory's extension, measure space, Lebesgue measure in R, measurable functions, Cantor set, Cantor function, simple functions, Lebesgue integration, almost everywhere convergence, monotone convergence theorem, Fatou's lemma, Lebesgue dominated convergence theorem, Egoroff's Theorem, Lusin's Theorem.

Definition of p-norm, convex functions, Jensen's inequality, L^p space, dense subsets of L^p space, L^p convergence, convergence in measure, Chebychev's inequality, support of a function, Lebesgue integral via limit of continuous functions with compact support, Hölder's and Minkowski's inequality, L^p space is complete: Riesz-Fischer Theorem.

Products of measurable spaces, elementary rectangles, product measure, monotone class lemma, Tonelli and Fubini's theorem, Lebesgue integration on R^N. Dini's derivative, Existence and regularity of convolutions for L^p functions, Young's theorem.

Functions of bounded variation, absolute continuity of functions and measures, Signed measure, Hahn decomposition, Jordan decomposition, Lebesgue Decomposition Theorem, Radon-Nikodym theorem, fundamental theorem of calculus.

References:

1. H. L. Royden & P.M. Fitzpatrick, Real Analysis, Pearson Education India, 4th edition, 2015.
2. S. Axler, Measure, Integration & Real Analysis, Springer, 2020.
3. G. B. Folland, Real analysis: Modern Techniques and Their Applications, Wiley-Interscience, 2nd edition, 1999.
4. E. M. Stein & R. Shakarchi, Real analysis: Measure Theory, Integration, and Hilbert Spaces, Princeton University Press, 2005.
5. M. T. Nair, Measure and Integration: A First Course, CRC Press, 2020.

MA6214E GRAPH THEORY AND COMBINATORICS

L	T	P	O	C
3	0	0	6	3

Prerequisites: Nil

Total lecture sessions: **39**

Course Outcomes:

CO1: Understand the main concepts of graph theory and to recognize graphs as an important modelling technique in several applications.
CO2: Learn the idea of colouring, matching and planarity in graphs and implement to solve real life problems.
CO3: Integrate core theoretical knowledge of graph theory to solve problems.
CO4: Learn generating functions and recurrence relations to solve various combinatorial problems.

Graph, subgraph, path and cycle, connected graph, component and complement of a graph, tree and partite graphs, graph operations, graph isomorphism, block and girth, eccentricity, radius and diameter, vertex and edge cut, Whitney's Theorem.

Separating set, *k*-connected graphs, Menger's Theorem, colouring, clique and chromatic number, five color theorem, Vizing's theorem, plane and planar graphs, Euler formula, Plane duality.

Eulerian graph, necessary and sufficient condition for Eulerian graph, Hamiltonian graph, Dirac's theorem. Independent sets, coverings, perfect and maximum matching. Hall's Theorem.

Equivalent sets, equivalence relation and partition of a set, countable and uncountable set, Cantor's diagonalization, pigeonhole principle, generating functions, closed form, linear and non-linear recurrence relation.

References

1. A. Bondy, U.S.R. Murty, Graph Theory, Springer, New York, 2008
2. R. Diestel, Graph Theory, Springer, New York, 2017.
3. J. Harris, J. L. Hirst, M. Mossinghoff, Combinatorics and Graph Theory, Springer, New York, 2010.
4. K. H. Rosen. K. Krithivasan, Discrete Mathematics and Its Applications, McGraw Hill Education Private Limited, New Delhi, 2021.
5. C. L. Lieu, Elements of Discrete Mathematics, McGraw Hill Publishing Company Limited, New Delhi, 2008.

MA6215E PROBABILITY AND STATISTICS

L	T	P	O	C
3	1	0	7	4

Pre-requisites: Nil

Total Lecture Sessions: 39

Course Outcomes:

CO1: Apply the basics of probability theory in solving real life problems.
CO2: Identify the distribution and transformation of random variables.
CO3: Use techniques of statistical inference and its applications.
CO4: Apply regression and correlation in solving real life problems.

Fundamentals of probability

Random variables, Discrete and continuous random variables, Moments, Moment generating function and Characteristic function, Random vectors, Jointly distributed random variables, Joint probability distributions, Conditional expectation.

Distributions of random variables and limit theorems

Bi-variate normal distribution, Transformations of random variables, Transformations of random vectors, Order statistics, Chebyshev's theorem, Limit theorems in probability, Modes of convergence, Weak law of large numbers, Strong law of large numbers, Limiting moment generating function, Central limit theorem.

Statistical inference

Introduction to population and samples, Sampling distribution of the mean and variance, Point estimation, Maximum Likelihood Estimation (MLE), Method of moments, Properties of estimators, Tests of hypothesis, Uniformly Most Powerful (UMP) Tests, Newman-Pearson lemma, Inference concerning single mean and two means, Inference concerning one variance and two variances, Inference concerning one proportion and several proportions, Chi -square test for goodness of fit.

Regression and correlation

Introduction to regression and correlation- Bi-variate relationships, Correlation coefficient, Two variable linear regression, Least square estimation, Inference in two-variable linear regression model.

References:

1. S. Ross, A First Course in Probability, 9th Edition, Pearson, 2014.

2. R. V. Hogg, J. McKean, and A. T. Craig, Introduction to Mathematical Statistics, 7th Edition, Pearson Education, 2012.

3. W. H. Hines, Montgomery, et. al., Probability and Statistics for Engineering, John Wiley & Sons, Inc.,2003.

4. R. V. Hogg and E. A. Tanis, Probability and Statistical Inference, 6th Edition Pearson, 2001.

5. J. Johnston and J. DiNardo, Econometric Methods, 4th Edition, The Mc-Graw- Hill, 1997.

6. V. K. Rohatgi and A K Md. Ehsanes Saleh, An Introduction to Probability and Statistics, 3rd Edition, John Wiley and Sons, 2015.

MA6291E MINI PROJECT

L	T	P	O	C
0	0	0	6	2

Pre-requisites: Nil

Course Outcomes:

CO1: Carry out literature review and gather information from multiple sources.
CO2: Identify the key concepts and steps in the mathematical results.
CO3: Explain their findings in written/oral modes.

The 'Mini Project' aims at introducing the students to various areas in Pure and Applied Mathematics. The student shall choose a topic in consultation with their guide(s), carry out extensive literature survey from books, journals and other sources and identify research gaps. For the course evaluation, the student shall submit a written report as well as an oral presentation to a duly constituted committee.

MA7201E OPERATIONS RESEARCH

L	T	P	O	C
3	1	0	7	4

Pre-requisites: Basic Linear Algebra and Calculus

Total Lecture Sessions: 39

Course Outcomes:

CO1: Model managerial problems in industries or civilian sectors into linear programming problems.
CO2: Find feasible solutions to linear programming problems using various techniques and algorithms.
CO3: Formulate and solve travelling salesman, transportation and assignment problems.
CO4: Manage large-scale projects using PERT/CPM.

Introduction to linear programming: Formulation of a linear programming problem through examples, Preliminary theory and geometry of linear programs, Lines and hyperplanes, Convex sets, Convex hull, Convex functions, Theorems dealing with vertices of feasible regions and optimality, Graphical solution, Basic feasible solution, Simplex method, Variants of simplex method (two-phase method and revised simplex method), Charne's method of penalty (Big-M method).

Computational complexity of simplex method, Interior point algorithms, Karmarkar's algorithm; Dual problem, Duality theory, Interpretation of dual variables, Dual simplex method, Primal-dual method, Sensitivity analysis, Bounded variable problem.

Integer linear programming: applications in real decision-making problems, Branch and bound algorithm, Cutting plane algorithm, Travelling salesman problem, Transportation problems, Integrity property, MODI method, Degeneracy, Unbalanced problem, Assignment problem, Development of Hungarian method, Routing problems.

Project management with PERT (program evaluation and review technique) and CPM (critical path method): Critical path analysis, Probability consideration in PERT, Distinction between PERT and CPM, Resources analysis in networking scheduling, Time cost optimization algorithm, Linear programming formulation, Introduction to optimization softwares.

References:

1. M. S. Bazaraa, J.J. Jarvis, and H.D. Sherali. Linear Programming and Network Flows, John Wiley & Sons. 4th Edition. 2011.

2. H. A. Taha. Operations Research: An Introduction. Prentice Hall, 10th Edition, 2017.

3. G. Hadley. Linear Programming. Narosa Publishing House, 1961.

4. F. S. Hillier, and G. J. Lieberman. Introduction to Operations Research. McGraw Hill Companies, Inc., 11th Edition, 2021.

MA7202E FUNCTIONAL ANALYSIS

L	T	P	O	C
3	1	0	7	4

Pre-requisites: Linear Algebra

Total Lecture sessions: 39

Course Outcomes:

CO1: Identify the classical normed spaces and analyse their properties
CO2: Analyse operators for their continuity properties and to apply results on operators to problems of practical interest.
CO3: Analyse the spectrum and spectral properties of bounded linear operators.

Normed linear spaces and bounded linear operators

Review of vector spaces and metric spaces, normed linear spaces, Banach spaces, examples including Lp spaces and sequence spaces, Schauder basis, separable spaces, Riesz Lemma, finite dimensional spaces, bounded linear maps, bounded linear functionals, dual spaces and transpose operators, reflexive spaces.

Inner product spaces and operators between Hilbert spaces

Inner product spaces, Hilbert spaces, orthonormal sets and orthonormal basis, Bessel's inequality, Fourier expansion and Parseval's identity, Projection theorem, Riesz-Fischer Theorem, Riesz representation theorem, Adjoint of bounded operators.

Fundamental theorems on operators

Hahn-Banach extension Theorem and applications, Hahn-Banach separation theorem, Uniform boundedness principle, Banach-Steinhaus theorem, Arzela-Ascoli theorem, Open mapping theorem, Closed graph theorem.

Spectral theory of bounded linear operators

Spectrum of bounded linear operators and its variants, spectral radius, spectral mapping theorem, Gelfand-Mazur theorem, spectral radius formula, normal, unitary and self-adjoint operators and their spectral properties.

References:

1. B. V. Limaye, Functional Analysis, New Age International Publishers, Revised Third Edn, 2014.
2. M. T. Nair, Functional Analysis: A First Course, PHI, second edition, 2021
3. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 2017
4. E. Kreyszig, Introductory Functional Analysis with Applications, Wiley, 2007
5. S. Kumaresan and D. Sukumar, Functional Analysis: A First Course, Narosa, 2020

MA7203E PARTIAL DIFFERENTIAL EQUATIONS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Ordinary differential equations

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand the derivation of methods to solve first order linear and nonlinear PDEs and solve them.
CO2: Learn classification and solving linear second order PDEs with constant coefficients.
CO3: Model and solve Parabolic PDEs using analytic methods.
CO4: Solve wave and Laplace equations analytically.
CO5: Learn Green's identity, Mean value theorem, Maximum principle, Green's function.

Partial differential equations of first order, Pfaffian differential equations, Hadamard's definition of well-posedness, Compatible systems, Method of characteristics, Lagrange method, Charpit's method, integral surfaces through a given curve, Jacobi's method.

Partial differential equation of second order, Classification of second order PDEs, Canonical forms, Hyperbolic, Parabolic and Elliptic equations, Linear second order partial differential equations with constant coefficients.

Parabolic differential equations, Modelling of heat equation, Boundary conditions, Dirichlet, Neumann and Robin type boundary conditions, Method of separation of variables, Solutions in cylindrical and spherical equation, The maximum principle for the heat equation, Duhamel's principle for heat equation.

Modelling of wave equation, Solution of the wave equation by separation of variables, D'Alembert's solution, Vibrations of a string of infinite, semi-infinite and finite lengths, Boundary and initial value problem of two dimensional wave equation, Laplace equation, Harmonic function, Green's identity, Mean value theorem, Maximum and minimum principles, Harnack inequality, Fundamental solution, Green's function, Poisson equation, Existence and uniqueness of solutions.

References:

1. I. Sneddon, Elements of Partial Differential Equations, Dover Publications, Inc.,2006.
2. D. Greenspan, Introduction to Partial Differential Equations, Dover Publications; 1st edition, 2000.
3. E. Kreyszig, Advanced Engineering Mathematics, John Wiley and Sons, 1995.
4. K. S. Rao, Introduction to partial differential equations, Phi Learning Pvt. Ltd., 2011.
5. Jacob Rubinstein and Yehuda Pinchover, An Introduction to Partial Differential Equations, Cambridge, 2005.

MA7292E PROJECT PHASE I

L	T	P	O	C
0	0	0	9	3

Pre-requisites: Nil

Course Outcomes:

CO1: Able to locate information regarding a chosen mathematical topic.
CO2: Learn skills to read and research a chosen mathematical topic.
CO3: Write an expository account of the chosen topic.
CO4: Document research findings and explain them in presentations.

The student, in consultation with their guide(s), shall identify a mathematical topic and carry out investigation on it. The evaluation will be based on the internal evaluation by the guide, a written report documenting the work they have carried out and an oral presentation to a duly constituted committee.

MA7293E PROJECT PHASE II

L	T	P	O	C
0	0	0	18	6

Pre-requisites: Nil

Course Outcomes:

CO1: Able to locate information regarding a chosen mathematical topic.
CO2: learn skills to read and research a chosen mathematical topic.
CO3: Write an expository account of the chosen topic.
CO4: Document research findings and explain them in presentations.

The student, in consultation with their guide(s), shall identify a mathematical topic and carry out investigation on it. The evaluation will be based on the internal evaluation by the guide, a written report documenting the work they have carried out and an oral presentation to a duly constituted committee.

Electives

MA7221E RELIABILITY OF SYSTEMS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Probability theory

Total Lecture sessions: 39

Course Outcomes:

CO1: Able to apply fundamentals of system reliability in reliability evaluation
CO2: Able to solve reliability redundancies problems using various techniques of redundancy.
CO3: Able to solve reliability allocation problems to improve reliability.
CO4: Able to apply reliability estimation procedure by using data from life testing

Fundamentals of system reliability

Introduction to reliability, Basic concepts, Cut sets, Path sets, Minimal cut and path sets, Bounds for reliability, Reliability and Quality, Maintainability and Availability, Reliability analysis, Causes of failures, Catastrophic and Degradation failures, Useful life of components, Component reliability and hazard models, Mean time to failure, system reliability models, System with components in series, parallel, *k/n* systems, System with mixed mode failures.

Redundancy techniques

Basics of redundancy techniques, Component v/s unit redundancy, Weakest link techniques, Mixed redundancy, Stand by redundancy, Redundancy optimization, Double failure and redundancy, Maintainability and availability concepts, Two-unit parallel system with repair, Signal redundancy, Time redundancy, Software redundancy

Reliability evaluation and allocation

Hierarchical systems, Path determination method, Boolean Algebra method, Cut set approach, Logic diagram approach, Conditional probability approach, System cost and reliability approximations, Reliability allocation problems.

Reliability estimation

Life testing: Introduction, hazard rate functions, Exponential distribution in life testing, Simultaneous testing-stopping at r^{th} failure, stopping by fixed time, sequential testing, Accelerated testing, Equipment acceptance testing.

References:

1. Balagurusamy, E., Reliability Engineering, Tata McGraw-Hill, 2011.
2. Shooman, M.L, Probability Reliability, An Engineering Approach, McGraw-Hill. New York, 1968.
3. Barlow, R.E. and Proschen, F., Mathematical Theory of Reliability, John Wiley, New York, 1965
4. Aggarwal, K.K., Reliability Engineering, Springer, 2007.
5. Ross, S.M. Introduction to Probability and Statistics for Engineers and Scientists, 4/e, Elsevier, 2009.

MA7222E MULTIVARIATE STATISTICAL ANALYSIS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Probability and Statistics

Total Lecture sessions: 39

Course Outcomes:

CO1: Able to apply the basics of multi-variate normal distribution in real life problems.
CO2: Able to apply multi-variate distributions and its applications in problem solving
CO3: Able to apply techniques of classification of multi-variate populations in real life problems.
CO4: Able to apply testing of hypothesis and classification of data in data analysis related problems.

Multi-variate normal distribution

Introduction and notions of multi-variate normal distributions, Distribution of linear combinations of normally distributed variates, Independence of variates, Marginal distributions, Conditional distributions, Multiple correlation coefficient, Characteristics function, Moments.

Multi-variate distributions

Maximum likelihood estimation of mean vector and covariance matrix, The distribution of sample mean vector and the covariance matrix is known, Properties of estimators of mean vectors, Correlation coefficients of a bi-variate sample, Partial correlation coefficients, Multiple correlation coefficients, Conditional distributions, Wishart distribution and properties.

Classification of multi-variate populations

Generalized T^2 statistics and its distribution, Applications of T^2 statistics, The two sample problem with unequal covariance matrices, Classification of observations, Standards of good classification, Classification of one of the two known multi-variate normal populations, Classification of one of the two known multi-variate normal populations when the parameters are estimated, Classification of one of the several multi-variate normal populations, Probabilities of misclassification.

Testing of hypothesis and classification of data

Estimation of parameters in multi-variate linear regression, Likelihood ratio tests, Testing hypothesis of equality of means of several normal populations with common covariance matrix, Testing equality of several covariance matrices, Principal components, MLE of principal components and their variances, Canonical correlations and their estimation, Factor analysis and MLE of random orthogonal factors.

References:

1. T.W.Anderson, An Introduction to Multivariate Statistical Analysis, 3rd Ed.,Wiley,July2003.
2. R. Gnanadesikan, Methods for Statistical Data Analysis of Multivariate Observations, John Wiley, NewYork,1997.
3. R.A. Johnson and D.W. Wicheran, Applied Multivariate Statistical Analysis, 6th Edition, Wiley, 2007.
4. M.S. Srivastava and E.M. Carter, An Introduction to Multivariate Statistics, NorthHolland,1983.

MA7223E REGRESSION ANALYSIS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Nil

Total Lecture sessions: 39

Course Outcomes:

CO1: Apply the basics of regression for solving real life problems
CO2: Apply multiple regression and inferences in regression models in solving practical problems
CO3: Apply techniques of non-linear regression and its applications in real life problems.
CO4: Apply logistic regression and its applications in real life problems.

Regression fundamentals

Simple regression with one independent variable *(X)*, assumptions, estimation of parameters, standard error of estimator, testing of hypothesis about regression parameters, standard error of prediction, Testing of hypotheses about parallelism, equality of intercepts, congruence. Extrapolation, optimal choice of *X*. Diagnostic checks and correction: graphical techniques, tests for normality, uncorrelatedness, homoscedasticity, lack of fit, modifications like polynomial regression, transformations on *Y* or *X*, inverse regression *X(Y).*

Multiple regression and inferences

Introduction to multiple regression: Standard Gauss Markov setup, Least square(LS) estimation, Error and estimation spaces, Variance-Covariance of LS estimators, estimation of error variance, case with correlated observations, LS estimation with restriction on parameters, Simultaneous estimation of linear parametric functions, Test of Hypotheses for one and more than one linear parametric functions, confidence intervals and regions.

Nonlinear regression

Introduction to nonlinear regression (NLS) : Linearization transforms, their use and limitations, examination of non-linearity, initial estimates, iterative procedures for NLS, grid search, Newton- Raphson, steepest descent, and Marquardt's methods.

Logistic regression

Introduction to logistic regression: Logit transform, ML estimation. Tests of hypotheses, Wald test, LR test, score test, test for overall regression, multiple logistic regression, forward and backward method, interpretation of parameters relation with categorical data analysis, generalized linear model: link functions such as Poisson, binomial, inverse binomial, inverse Gaussian, and gamma.

References:

1. Draper, N. R. and Smith, H., Applied Regression Analysis, 3rd Ed., John Wiley, 1998.
2. McCullagh, P and Nelder, J. A., Generalized, Linear Models, Chapman & Hall, 1998.
3. Ratkowsky, D. A., Nonlinear Regression Modelling, Marcel Dekker, 1983.
4. Hosmer, D.W. and Lemeshow, S., Applied Logistic Regression, John Wiley, 1989.
5. Seber, G.E.F. and Wild, C.J., Nonlinear Regression, Wiley, 1989.

MA7224E SOFTWARE RELIABILITY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Probability and Statistics

Total Lecture sessions: 39

Course Outcomes:

CO1: Apply the basic concepts of software reliability in industrial problems.
CO2: Apply the models for software reliability in software reliability testing
CO3: Apply techniques of prediction analysis for software reliability analysis.
CO4: Apply the methods of software reliability testing and estimation in software development.

Software and system reliability

Introduction and basic concepts, Fault prevention, removal and tolerance, Dependability concepts, Failure behaviour on X-ware system, Reliability modelling and estimation, Availability modelling.

Software reliability models

Exponential failure time class of models, Weibull and Gamma failure time class of models, Infinite failure category models, Bayesian models, Software reliability prediction in the early phase of life, Software reliability growth models, Black box software reliability models, White box software reliability models, Soft computing technique

Prediction analysis

Simple short-term predictions, Long-term predictions, Methods of analysing predictive accuracy-The Prequential Likelihood Ratio (PLR)-U-plot, V-plot, Measurement based analysis of software reliability-Measurement techniques-On-line machine logging, Manual reporting, Preliminary analysis of data, Data processing-Fault and error classification, Error propagation, Error and recovery Distributions, Detailed analysis of data-Dependency analysis, Hardware-related software errors, Evaluation of software fault tolerance, Trend analysis-Reliability growth characterization, Definitions of reliability growth, Graphical interpretation of the sub-additive property, Sub-additive property analysis, Sub-additive property and trend change.

Reliability and testing

Overview of software testing, Kinds of software testing, Concepts from White-Box and Black -Box Testing, Operational profiles-Difficulties in estimating the operational profile, Estimating reliability with Inaccurate operational profiles, Time/structure- based software reliability estimation-Definitions and terminology-Basic assumptions, Testing methods and saturation effect-Testing effort, Limits of testing methods, Empirical basis of the saturation effect, Reliability overestimation due to saturation, Incorporating coverage in reliability estimation, Filtering failure data using coverage information, Selecting the compression ratio.

References:

1. M. R. Lyu, Handbook of Software Reliability Engineering, McGraw-Hill publishing, 1995.
2. J. D. Musa, A. Iannino, and K. Okumoto, Software Reliability: Measurement, Prediction, Application, McGraw-Hill Book Company, 1987.
3. A. K. Verma, S. Ajit, and D. R. Karanki, Reliability and Safety Engineering, 2nd, Springer. 2016.
4. H. Pham, System Software Reliability, Springer, 2006.

MA7225E STOCHASTIC PROCESSES

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Knowledge of elementary probability theory

Total Lecture Sessions: 39

Course Outcomes:

CO1: Analyse some real world problems as a DTMC and make inferences.
CO2: Find limiting distributions of finite state Markov Chains.
CO3: Identify the renewal nature of some stochastic models and write the renewal equations.
CO4: Check for the stationarity or covariance stationarity of stochastic processes.

Elements of stochastic processes, Classification of general stochastic processes. Markov Chains: Definition, examples, transition probability matrix, classification of states, basic limit theorem, limiting distribution of Markov Chains.

Continuous time Markov Chains: General pure birth processes and Poisson processes, more about Poisson processes, A counter model, Birth and Death processes with absorbing states, Finite state continuous time Markov Chains.

Renewal Processes: Definition of a renewal process and related concepts, examples of renewal processes, special renewal processes, renewal equation and elementary renewal theorem, the renewal theorem, generalizations and variations on renewal processes, applications of renewal theory.

Martingales: Preliminary definitions and examples. Brownian motion: Introduction and preliminaries. Stationary Processes: Definition and examples, Mean square distance, spectral analysis of covariance stationary processes.

References:

1. S. Karlin & H M Taylor; "A First Course in Stochastic Processes", 2nd Edn., Academic Press, New York, 1975.
2. S. M. Ross; "Stochastic Processes", 2nd Edn, John Wiley and Sons, New York, 2008.
3. J. Medhi; "Stochastic Processes", New Age International, 4th Edn, 2019.
4. E. Cinlar, Introduction to Stochastic Processes, Dover Edition, 2013.
5. H. C. Tijms, A first course in Stochastic Models, 1st Edn, John Wiley, 2003.

MA7226E APPLIED PROBABILITY MODELS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Knowledge of elementary probability theory

Total Lecture Sessions: 39

Course Outcomes:

CO1: Analyse some real world problems as a finite state MC and make inferences.
CO2: Analyse various waiting lines to find necessary performance measures.
CO3: Obtain the reliability of series/parallel/k out of n systems.
CO4: Find optimal solutions to stochastic inventory models.

Stochastic Processes

Stochastic Processes, Discrete time Markov Chains, Transition probabilities, Classification of states, Steady state distribution, Continuous time Markov chains, Rate matrix and Kolmogorov equations, Finite state continuous time Markov chains, Poisson processes, Birth and Death processes and its steady state solution.

Queueing Models

Introduction to Queueing theory, Markovian Queueing models – Single server queues, Finite capacity queues, Queues with parallel channels, Erlang's Loss formula, Queues with unlimited service, Finite source queues, Network of queues.

Reliability Models

Basic concepts of Reliability analysis, Modelling reliability problems, Series, parallel and series/parallel structures, repairable structures, k out of n redundant systems, life testing.

Inventory Models

Basic concepts of inventory, Inventory control, Deterministic and probabilistic inventory problems, EOQ models, Models with lead time, Safety stocks, Static inventory problems, Newspaper boy's problem.

References:

1. S M Ross, Introduction to Probability Models, 11th edn, Academic Press,2014.
2. D Gross and C M Harris, Fundamentals of Queueing Theory, 4th edition, John Wiley and Sons, 2008.
3. H A Taha, Operations Research: An Introduction,10th edn, Pearson, 2016.
4. C O Smith, Introduction to Reliability in Design, McGraw Hill, 1983.
5. F S Hillier and G J Lieberman, Operations Research, 11th edn, McGraw Hill, 2021.

MA7227E ADVANCED TOPICS IN GRAPH THEORY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Nil

Total Lecture sessions: 39

Course Outcomes:

CO1: Enhance ability for critical thinking on problem solutions in the area.
CO2: Analyse and synthesize proofs of mathematical statements.
CO3: Solve problems in the area by reinforcing concepts of problem abstraction, formulation, specification, complexity analysis and problem reductions or mappings:
CO4: Develop ability for breaking down a mathematical problem into simpler statements and synthesize proofs.
CO5: Develop research skills, and identify gaps in the knowledge base or open problems in the area.

Graphs, Connectivity and

Graphs**:** review of basics in graphs - Trees- Blocks- Matrices-Operations on graphs. Connectivity**:** Vertex Connectivity and edge connectivity – n- connected graphs-Menger's Theorem, Traversability: Euler Graphs-Hamiltonian Graphs-Planar and Nonplanar graphs.

Metric, Distance Sequences, Convexity and Symmetry

Metric in graph: Centre, Median, eccentric vertex, Eccentric graph, boundary vertex, complete vertex, interior vertex, Convexity**:** Closure Invariants-gin(G) –gn(G)-Hull number- Geodetic Graphs- Distance Hereditary Graphs, Symmetry**:** Graphs and groups-- Symmetric Graphs - Distance Symmetry-Distance transitive graphs-distance regular graphs, Distance Sequences**: Degree** sequence, Eccentric Sequence - Distance Sequences - The Distance Distribution, Mean distance.

Matchings, Factorization and Domination

Matchings: Maximum matching -Perfect matching -Matching in bipartite graphs, Factorization: Coverings and independence, 1-factorization, 2-factorization, Arboricity, Domination: Dominating set-Domination number-total dominating set –total domination number.

Digraphs, Networks and Algorithms

Digraphs**:** Digraphs and connectedness- Tournaments- directed trees-binary trees- weighted trees and prefix codes, Networks**:** Flows-cuts- The Max- Flow Min-Cut Theorem, Graph Algorithms**:** Polynomial Algorithms and NP completeness, Complexity, Search algorithms**,** Shortest path algorithms.

References:

1. Gary Chartrand, Ping Zhang, ‘Introduction to Graph Theory, McGraw Hill International Edition, 2005.
2. J.A. Bondy, U.S.R.Murty, Graph Theory, Springer, 2011.
3. F. Buckley and F. Harary, Distance in Graphs, Addison - Wesley ,1990.
4. F. Harary, Graph Theory, Narosa Publishing House, 2001.
5. R. Diestel, Graph Theory, 5th Edn, Springer, 2017

MA7228E FUZZY SET THEORY AND APPLICATIONS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Nil

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand basic concepts in fuzzy sets and Operation on fuzzy sets
CO2: learn techniques for converting a crisp data to fuzzy data and vice versa
CO3: Enhance ability to solve problems based on fuzzy arithmetic
CO4: Handle uncertain data involving fuzzy relations and its properties
CO5: Handle fuzzy data using different types of fuzzy measures
CO6: Apply concepts of fuzzy logic and fuzzy approximate reasoning for solving problems in various Engineering fields

Crisp sets and Fuzzy sets

Introduction – Crisp sets: An overview – Notion of fuzzy sets –Basic concepts of fuzzy sets – Membership functions – Methods of generating membership functions – Defuzzification methods- Extension principle- Operations on fuzzy sets - Fuzzy complement, Fuzzy union, Fuzzy intersection – combinations of operations – General aggregation operations.

Fuzzy arithmetic and Fuzzy relations

Fuzzy numbers- Arithmetic operations on intervals- Arithmetic operations on fuzzy numbers- Fuzzy equations- Fuzzy relations – Projections and Extensions - Binary Fuzzy relations –Similarity relations – Compatibility relations.

Fuzzy measures

Fuzzy measures – Evidence Theory - Belief and Plausibility measures – Joint Basic Assignment – Dempester's rule of Combination - Marginal bodies of Evidence-– Possibility and Necessity measures – Possibility distribution – Basic distribution - Probability measures.

Fuzzy Logic and Applications

Classical logic: an overview – Multi valued logic - Fuzzy logic – Approximate reasoning – Other forms of implication operations – Other forms of the composition operations – Fuzzy control systems - Fuzzy decision making– Fuzzy clustering - Fuzzy pattern recognition – Fuzzy Image processing - Fuzzy logic in database and information systems- Fuzzy linear programming.

References:

1. George J Klir and Tina A Folger, Fuzzy sets, Uncertainty and Information, 1st Edn., Pearson India Education, 2015.

2. H.J. Zimmerman, Fuzzy Set theory and its Applications, 4th Edition, Springer, 2015.

3. Goerge J Klir and Bo Yuan, Fuzzy sets and Fuzzy logic: Theory and Applications,

   Pearson India Education, 2015.

4. Kwang H Lee, First Course on Fuzzy Theory and Applications, Springer, 2009.

5. James J Buckley, Esfandiar Eslami , An Introduction to Fuzzy Logic and Fuzzy Sets, Springer, 2007.

6. Timothy J Ross, Fuzzy Logic with Engineering Applications, 3rd Edn., Wiley India Edition, 2011.

MA7229E NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS

L	T	P	O	C
3	0	0	6	3

Prerequisites: Numerical Analysis, Partial Differential Equations

Total Lecture sessions: 39

Course Outcomes:

CO1: Familiar with the fundamentals of developing numerical PDE schemes, especially finite difference and finite volume methods.
CO2: Analyse the consistency, stability and convergence of a numerical scheme.
CO3: Develop the ability to relate mathematical equations to its underlying physical meanings.
C04: Implement and test the numerical systems using the programming languages.

Sources of Partial Differential Equations (PDEs), Classification of PDEs, Boundary Conditions: Dirichlet, Neumann and Robin type, Periodic boundary condition, Finite difference approximations to derivatives, Parabolic PDEs: One-dimensional heat equation, Finite difference methods for 1D heat equation: Explicit, Implicit and Crank-Nicolson schemes, Error analysis, Truncation errors and Consistency, Stability, Convergence, Richardson method, Du-Fort Frankel scheme, Method of lines, Two-dimensional heat equation: Explicit and Implicit finite difference scheme, ADI method,

Hyperbolic PDEs, Derivation of one-dimensional advection equation, Method of characteristics, Numerical methods for 1D advection equation: Naïve, Upwind, Downwind, Lax-Friedrichs, Consistency, Stability, Well-posedness and Convergence, CFL condition, Fourier Analysis, Von-Neumann theory, Lax-Wendorff method, Beam-Warming Method, Dissipation and Dispersion, Modified Equations, Finite difference method for the wave equation

Elliptic equation, Steady-state heat equation, Finite difference method for Laplace equation and its solutions through iterative methods, Jacobi, Gauss-Seidel, and SOR, Discretization of Laplacian operator in polar coordinates, 2D Poisson equation in Cartesian and polar coordinates

Finite Volume Method: General formulation of Conservation laws, a basis of the finite volume method, Mesh and notations, discretization, Numerical flux function, Consistent discretization, Solving one-dimensional heat equation with Neumann boundary condition. An advection-diffusion equation, Staggered grid, System of PDEs

References:

1. K W Morton and D F Mayers, Numerical solution of partial differential equations, Cambridge, 2nd Edn., 2011.
2. G D Smith, Numerical solution of partial differential equations, finite difference methods, Oxford, 3rd Edn., 2010.
3. R J Leveque, Finite difference methods for ordinary and partial differential equations, SIAM, 2007.
4. J W Thomas, Numerical partial differential equations: Finite difference methods, Springer, 2010.
5. R J Leveque, Finite Volume Methods for Hyperbolic Problems, Cambridge University Press, 2002.

MA7230E FLUID DYNAMICS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Vector Calculus, Differential Equations

Total Lecture sessions: 39

Course Outcomes:

CO1: Derive the governing equations of the fluid mechanics
CO2: Identify the non-dimensional parameters for given systems and use such numbers for the characterizations
CO3: Understand the use of the different approximation methods to solve the fluid and thermal problems
CO4: Exhibit knowledge of significant practical outcomes in typical fluid flows and their physical implications.

Motivation, Reviews of integral theorems of vector calculus, Fundamental conceptions about fluids, Kinematics of fluids, Visualization of a moving fluid; Eulerian and Lagrangian description; The Transport Theorem, Continuity equation, Equation of motion, Stress tensor, Euler's equation, Navier-Stokes equations.

The energy equation, Boundary and initial conditions, Dimensional analysis, Buckingham's Pi theorem, Transformation of Cartesian Coordinates, Curvilinear Coordinates, Boussinesq approximation

Basic properties of irrotational flow, Incompressible and irrotational flow, Stream lines, Path lines, Streak lines, Plane potential flow, Laplace equation, Bernoulli equation, Application of Bernoulli's equation, Vorticity dynamics, Kelvin's circulation theorem.

Laminar flow, Steady flow in a pipe, Steady flow between concentric cylinders, Flow due to an oscillating plate, High and low Reynold's number flows, Boundary layer theory, Similarity solutions, Blasius solution, gravity waves, Shallow water waves.

References:

1. M. Feistauer, Mathematical Methods in Fluid Dynamics, Longman Scientific and Technicals, 1993.
2. A. J. Chorin & J. E. Marsden, A mathematical introduction to fluid mechanics, Springer, 3rd Edn., 4th printing 2000.
3. L. D. Landau & E. M. Lifshitz, Fluid Mechanics, Butterworth-Heinemann Ltd; 2nd edition, 1987.
4. J. H. Spurk & N. Aksel, Fluid Mechanics, Springer-Verlag, 2rd Edn., 2007.
5. G. K. Batchelor, An Introduction to Fluid Dynamics, Cambridge Univ. Press, Cambridge, 2000.
6. P.K. Kundu & I.M. Cohen, Fluid Mechanics, Academic Press, 2008.

MA7231E COMPUTATIONAL METHODS FOR ORDINARY DIFFERENTALA EQUATIONS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Differential Equations

Total Lecture sessions: 39

Course Outcomes:

CO1: Solve a system of ODEs and difference equations.
CO2: Estimate error and stability of single step methods.
CO3: Construct general multistep methods with convergence analysis.
CO4: Implement predictor and corrector methods for initial and boundary value problems.

Introduction of ODE and Difference Equations

Introduction; Initial Value problem for first order ODEs, Initial Value problem for system of first order ODEs', Reduction of higher order ODEs to first order systems; First order system with constant coefficients, linear difference equation with constant coefficients

Numerical Methods for ODEs: Single Step Methods

Numerical Methods for ODEs: Single Step Methods: Order and convergence of the general explicit one-step methods, derivation of classical RungeKutta methods, error bounds and error estimate of RungeKutta methods,RungeKutta methods of order greater than four, numerical errors ; weak stability theory for Runge-Kutta methods, implicit Rungekutta methods.

General Linear Multi-Step Methods

General Linear Multi-Step Methods, derivation through Taylor expansions, derivation through numerical integration, derivation through interpolation, convergence, order and error constants, Local and global truncation error, consistency and zero stability, Error bounds and local and global truncation error, weak stability theory, interval of absolute and relative stability, comparison of implicit and explicit Linear Multistep methods,

Predictor-Corrector methods

Predictor-Corrector methods, Local truncation error of predictor-corrector methods: Milne's device, weak stability of predictor corrector methods. Stiff ODEs*;* Implicit Stability Theory: A-stability, L-stability, B-stability, Backward Difference Formulas Methods: formulas and stability regions, Two-Point Boundary Value Problems*;* Finite-Difference Methods, Shooting Methods, Collocation Methods.

References:

1. Atkinson, Han and Stewart, ‘Numerical Solution of Ordinary Differential Equations’, John Wiley & Sons, 2009.
2. Shampine, Gladwell and Thompson, ‘Solving ODEs with MATLAB’, Cambridge University Press, 2003.
3. Hairer, Nørsett and Wanner, ‘Solving Ordinary Differential Equations I -- Nonstiff Problems’, Springer, 2010,
4. Hairer and Wanner, ‘Solving Ordinary Differential Equations II -- Stiff and Differential-Algebraic Problems’, Springer, 2010.
5. Hairer, Lubich and Wanner, ‘Geometric Numerical Integration’, Springer, 2010.
6. Lambert, J. D., ‘Numerical Methods for Ordinary Differential Systems: The Initial Value Problems’, John-Wiley, 1991.

MA7232E METHODS IN APPLIED MATHEMATICS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Nil

Total Lecture sessions: 39

Course Outcomes:

CO1: Acquire the basic concepts of Fourier series, integrals, and transforms with applications.
CO2: Identify the relationship between differential and integral equations along with solution techniques.
CO3: Understand the notation and concepts of tensors algebra and calculus.
CO4: Implement the optimization techniques through calculus of variations.

Fourier Analysis, if any

Fourier Series, Dirichlet's conditions, convergence theorem, other forms of Fourier series. Integral transforms, Fourier integral, Gibbs phenomenon, properties and applications of Fourier transforms, Fourier integral to the Laplace transformation, finite Fourier transforms, finite Fourier sine and cosine transforms, convolution theorem, multiple _nite Fourier transforms and applications

Integral Equation

Classification, relation between differential and integral equations, Neumann's iterative method for Fredholm's equation of second kind, Volterra type integral equations, integral equations of first kind, solution of integral equations, Fredholm equations with separable Kernels, iterative methods for the solution of integral equations of the second kind, resolvent kernels, eigenvalues and eigen functions.

Tensor Analysis and Tensor Calculus

Coordinate transformations, contravariant, covariant and metric tensors, fundamental operation with tensors, quotient law, line element and metric tensors, conjugate Tensor, Christoffel symbols, covariant differentiation of tensors.

Calculus of Variation

Method of variations in problems with fixed boundaries, variation of a functional, Euler's equation, functionals involving derivatives of higher order, functionals depending on functions of several independent variables, variational, problems of constrained extrema, Rayleigh-Ritz method.

References:

1. Wylie C. R., & Barrett, L. C., Advanced Engineering Mathematics, 6th edition, McGraw-Hill,1995.
2. Sneddon, I. N., The Use of Integral Transforms, McGraw Hill, 1972.

3. Hilderbrand F. B., Methods of Applied Mathematics, Prentice Hall of India, 1961.
4. Spain B., Tensor Calculus, Oliver and Boyd, London, 1988.
5. Riley, K.F., Hofson M.P. & Bence, S.J., Mathematical methods for Physics and Engineering,Cambridge University Press, 1998.
6. Elsgolc, L. E., Calculus of Variations, Pergamon Press Ltd., 1961.

MA7233E PERTURBATION METHODS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Differential Equations

Total Lecture sessions: 39

Course Outcomes:

CO1: Acquire analytical experience in dealing with a class of problems where a small or large parameter can be identified for suitable asymptotic expansion.
CO2: Recognize the advantages and limitations of asymptotic approximations to challenging mathematical problems.
CO3: Compute asymptotic approximation of integrals.
CO4: Compute perturbation solutions to differential equations.
CO5: Combine numerical and asymptotic solutions to differential equations.

Asymptotic Approximations

Introduction to asymptotic approximations, Dimensional analysis, Order symbols, Asymptotic expansions, Accuracy versus convergence of an asymptotic series, Asymptotic solution of algebraic and transcendental equations

Matched Asymptotic Expansion

Regular perturbation of ordinary and partial differential equations, Error estimates, Singular perturbations of ordinary differential equations, Introductory examples, Outer solution, Inner solution, Matching, Composite expansion, A formal definition of boundary layer, Multiple boundary layers, A singular perturbed partial differential equation

Multiscale Expansions

Asymptotic expansion of integrals, Watson's lemma, Laplace approximations, Stationary phase approximation, Multiple scales, Slow varying coefficients, Multiple scales and boundary layers, Some general remark concerning multiple scales

WKB and Related Theory

Multiple turning points; Wave propagation and energy methods; Wave propagation and slender-body approximations; Ray methods – WKB expansion, Discrete WKB method. The method of homogenization, Example, Weak limit and rapidly oscillating functions, New concept of convergence, Flow in porous media.

References:

1. M. H. Holmes, "Introduction to Perturbation Methods", Springer 1995.
2. E. J. Hinch, "Perturbation Methods", Cambridge University Press, 1992.
3. M. Van Dyke, "Perturbation Methods in Fluid Mechanics", Parabolic Press, 1975.
4. R. E. O'Malley, "Singular Perturbation Methods for Ordinary Differential Equations", Springer 1991.

MA7234E INTRODUCTION TO FRACTIONAL CALCULUS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Real Analysis

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand the basic concept of fractional calculus and appreciate the differences and similarities between the classical derivatives and fractional derivatives.
CO2: Analyse the existence and uniqueness of the solution of fractional differential equations.
CO3: Solve linear fractional differential equations analytically.

Gamma function and its properties, Beta function, Contour integral representation. Mittag-Leffler function and its relation with other functions, Laplace transform and derivative of Mittag-Leffler function, Review of basic definitions of integer-order derivatives and integrals and their geometric and physical interpretations. Motivation and history of fractional calculus.

Fractional derivatives: Grunwald-Letnikov, Riemann-Liouville and Caputo's fractional derivative (Left and right fractional derivatives), Riesz fractional integro-differentiation, Properties of fractional derivatives, Leibniz rule for fractional derivatives, Geometric and physical interpretation of fractional integration and fractional differentiation. Computation of these FDs for some basic functions like constant, ramp, exponential, sine, cosine, etc.

Laplace transforms of fractional derivatives. Fourier transforms and Mellin transforms of fractional derivatives. Linear fractional differential equations: Equation of a general form, existence and uniqueness theorem as a method of solution, dependence of a solution on initial conditions, Laplace transform method, standard fractional differential equations, some methods for solving fractional order equations: Mellin transform, power series, numerical evaluation of fractional derivatives.

Fractional Differential Equations: Introduction, Linearly independent solutions, Solutions of the homogeneous and non-homogeneous fractional differential equations, Reduction of fractional partial differential equations to ordinary differential equations, applications of fractional calculus: Abel's integral equation, fractional diffusion equation.

References:

1. K. B. Oldham, J. Spanier, The Fractional Calculus, Academic Press, San Diego, 1974.
2. K. S. Miller, B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, Wiley, New York, 1993.
3. I. Podlubny, Fractional Differential Equations, Academic Press, San Diego, 1999.

MA7235E FRACTAL GEOMETRY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Real Analysis, Linear Algebra, Functional Analysis, Topology, Complex Analysis

Total Lecture sessions: 39

Course Outcomes:

CO1: Analyse the fundamental geometric patterns in an analytical point of view.
CO2: Present different methods for the construction of fractals.
CO3: Provide some efficient algorithms for the production of fractals.
CO4: Comprehend advanced iteration and interpolation techniques.

The space of fractals and Iterative function systems (IFS)

Housdorff metric, the completeness of space of fractals, transformations on the real line, affine transformations in the Euclidean plane, Mobius transformations on the Riemann sphere, Analytic transformations, the contraction mapping theorem, the deterministic algorithm, random iteration algorithm, condensation sets, the continuous dependence of fractals on parameters.

Chaotic dynamics on fractals and fractal dimension

The addresses of points on fractals, continuous transformations from code space to fractals, dynamical systems, dynamics on fractals, equivalent dynamical systems, shadow of deterministic dynamics, shadowing theorem, Chaotic dynamics on fractals, fractal dimension, theoretical and experimental determination of fractal dimension, Hausdorff-Besicovitch dimension.

Fractal interpolation

Applications of fractal functions, fractal interpolation functions, the fractal dimension of fractal interpolation functions, hidden variable fractal interpolation, space filling curves.

Julia sets and Mandelbrot's sets

Escape time algorithm, Julia sets, IFS for Julia sets, Application of Julia sets to Newton's method Invariant sets of continuous open mappings, map of fractals, Mandelbrot's sets, Mandelbrot's sets for Julia sets.

References:

1. M. Barnsley, Fractals Everywhere, Academic Press, Second Edition, 1993.

2. B. B. Mandelbrot, The fractal geometry of Nature, W.H. Freeman and Company, New York, 1982.

3. Peitgen, Jurgens, saupe, Chaos and Fractals, Springer- Verlag, 1992.

MA7236E CHAOTIC DYNAMICAL SYSTEMS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Topology, Real Analysis, Functional Analysis, Linear Algebra

Total Lecture sessions: 39

Course Outcomes:

CO1: Analyse the fundamentals of one dimensional linear dynamical systems
CO2: Analyse higher dimensional dynamical systems
CO3: Learn some of the essential features of dynamical systems
CO4: Present different characterization of dynamical systems

One dimensional dynamical Systems

Elementary definitions, Hyperbolicity, The quadratic family, Symbolic dynamics, Topological Conjugacy, Chaos, Structural Stability, Sarkovskii's theorem, The Schwarzian derivative, Bifurcation theory, Maps of the circle, Morse-Smale diffeomorphisms, Homoclinic points and bifurcations, The period-doubling route to chaos, The kneading theory, Geneology of periodic points.

Higher Dimensional Dynamical Systems

Preliminaries, The dynamics of linear maps: two and three dimensions, The horseshoe map, Hyperbolic automorphism, Attractors, The stable and unstable manifold theorem, Global results and hyperbolic sets, The Hopf bifurcation, The Henon map.

Chaotic Oscillations

A simple nonlinear mechanical oscillator: The Duffing oscillator, Chaos in the weather: The Lorenz model, The Rossler systems, Phase space, dimension and attractor form, spatially extended systems: coupled oscillators, Taylor-Couette flow, Mathematical Routes to chaos and turbulence.

Characterising Chaos

Preliminary Characterization: visual inspection & frequency spectra, Characterising chaos: Lyapunov exponents & dimension estimates, Attractor reconstruction, Embedding dimension for attractor reconstruction.

References:

1. Robert L Devaney, An introduction to Chaotic Dynamical Systems, CRC Press, Second Edition, 2003.

2. Paul S Addison, Fractals and Chaos: An illustrated Course, IOP Publishing ltd, 1997.

3. K. T. Alligood, T. D. Sauer, J. A. Yorke, Chaos, An Introduction to Dynamical Systems, Springer, 2006.

MA7237E FUZZY GRAPH THEORY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Fuzzy Set Theory, Graph Theory

Total Lecture sessions: 39

Course Outcomes:

CO1: Present basics of fuzzy set theory and fuzzy graph theory.
CO2: Analyse different connectivity concepts in fuzzy graph theory.
CO3: Familiar and equipped with different connectivity structures in fuzzy graphs.
CO4: Analyse different types of modern graph structures related to fuzzy graphs.

Fuzzy Sets, Fuzzy Relations and Fuzzy Graphs

Fuzzy sets, Fuzzy relations, Fuzzy graphs, connectedness, fuzzy cut vertices, cut sets, fuzzy bridges, fuzzy trees, blocks, Theta fuzzy graphs, Operations in fuzzy graphs, Cycle connectivity, Metrics in fuzzy graphs.

Connectivity in Fuzzy Graphs

Strong edges, vertex connectivity, edge connectivity, separating sets, Menger's theorem, connectedness level, acyclic level, bonds and cutbonds,

Fuzzy Trees and Blocks

Characterizations of fuzzy trees, fuzzy blocks, blocks of a fuzzy graph, critical blocks, block graphs, K2-block graphs, connectivity transitive and cyclically transitive fuzzy graphs.

Interval Valued and Bipolar Fuzzy graphs

Interval valued fuzzy graphs, operations, isomorphisms, strong interval valued fuzzy graphs, Bipolar fuzzy graphs, Strong bipolar fuzzy graphs, regular bipolar fuzzy graphs, connectivity in bipolar fuzzy graphs.

References:

1. S. Mathew, J. N. Mordeson, D. S. Malik, Fuzzy Graph Theory, Springer, 2018.
2. J. N Mordeson, P. S. Nair, Fuzzy Graphs and Fuzzy Hypergraphs, Physica Verlag, 2000.
3. J. N Mordeson, S. Mathew, Advanced Topics in Fuzzy Graph Theory, Springer, 2019.

MA7238E FOURIER ANALYSIS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Functional Analysis

Total Lecture Sessions: 39

Course Outcomes:

CO1: Solve problems related to summability and convergence of the Fourier series.
CO2: Understand the properties of the convolution of functions, approximate identities and use them to prove the existence of Fourier transform on various function spaces and derive its properties.
CO3: Acquire understanding of the uncertainty principles for Fourier transform.
CO4: Familiar with common invariant subspaces for Fourier transform.

Fourier Series

Trigonometric polynomials and trigonometric series, Fourier series in $L^1(T)$, Riemann-Lebesgue lemma, convolutions, convergence, Fejer's theorem, Fourier series of continuous functions, Fourier series in $L^2(T)$, Bessel inequality, Parseval identity, Plancherel theorem.

Fourier transform on $L^1(R)$

L^p spaces, convolution of functions, Young's inequality, approximate identity, regularisation of functions, pointwise convergence, Fourier transform on $L^1(R)$, Riemann-Lebesgue lemma, multiplication formula, inversion, translations and dilations, multiplication and differentiation.

Fourier transform on $L^2(R)$

Abel means and Poisson kernel, uniqueness theorem for Fourier transform on $L^1(R)$, Fourier transform on $L^2(R)$, multiplication formula, Plancherel theorem, uniqueness theorem on $L^2(R)$, harmonic functions, Dirichlet problem for the upper half plane, point wise and norm convergence, Dirichlet problem for the disc.

Invariant and ultravariant subspaces

Eigen functions of Fourier transform, Gaussian, Hermite functions, Schwartz space, Paley-Wiener space, Paley-Wiener theorem, uncertainty principle, Hardy classes, Hardy's theorem.

References:

1. H. Dym & H. P. McKean, Fourier Series and Integrals, Academic Press, 1985.
2. Y. Katznelson, An Introduction to Harmonic Analysis, Cambridge University Press, 2004.
3. H. Helson, Harmonic Analysis, Hindustan Book Agency and Helson Publishing Co., 1995.
4. E. M. Stein & R. Shakarchi, Fourier Analysis: An Introduction, Princeton University Press, 2003.
5. C. Sadosky, Interpolation of Operators and Singular Integrals: An Introduction to Harmonic Analysis, Marcel Dekker, Inc., 1979.

MA7239E DIFFERENTAL GEOMETRY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Nil

Total Lecture Sessions: 39

Course Outcomes:

CO1: Understand the properties of regular curves and calculate the curvature and torsion.
CO2: Understand the properties of regular surfaces, find the parametrization for important surfaces, calculate the derivatives of functions defined on surfaces and the first fundamental form in local coordinates.
CO3: Calculate the second fundamental form in local coordinates and use it to calculate the Gaussian and mean curvatures of surfaces.
CO4: Understand the notions of parallel transport and covariant derivative and also verify the Gauss–Bonnet theorem for familiar surfaces.

Basic properties of curves

Parametrized curves, regular curves, reparametrization, arc length, curvature, plane curves and signed curvature, space curves, torsion and Frenet–Serret equation, fundamental theorems of plane and space curves.

Regular surfaces

Regular surfaces, differentiable functions on surface, tangent plane, orientability, ruled surfaces, surfaces of revolution, compact surfaces, first fundamental form, isometries of surfaces, equiareal and conformal maps.

Curvature of surfaces

Gauss map, Weingarten map, second fundamental form, Gaussian curvature, normal and geodesic curvatures, second fundamental form in local coordinates, geometric characterisations of Gaussian curvature, minimal surfaces, Gaussian curvature of compact surfaces.

Intrinsic geometry of surfaces

The exponential map, geodesics, Gauss' remarkable theorem, parallel transport, covariant derivative, Gauss-Bonnet theorem.

References:

1. Tapp, Kristopher. Differential geometry of curves and surfaces. Berlin: Springer, 2016.
2. Pressley, Andrew N. Elementary differential geometry. Springer Science & Business Media, 2010.
3. Manfredo P. Do Carmo, Differential Geometry of Curves and Surfaces, Prentice Hall, 1976.

MA7240E DISTRIBUTION THEORY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Functional Analysis

Total Lecture Sessions: 39

Course Outcomes:

CO1: Understand the topology of the space of compactly supported functions and the concept of a distribution.
CO2: Understand the notion of the support of distributions and properties of compactly supported distributions.
CO3: Understand the properties of Fourier transform on certain function spaces.
CO4: Understand the properties of Schwartz class functions and tempered distributions.

Test functions, partition of unity, distributions, order of distribution, convergence of distribution, derivative of distribution, multiplication of distribution by a function.

Local equality of distributions, support of distribution, singular support of distribution, compactly supported distributions,

Fourier transform in $L^1(R)$, Schwartz class functions, Riemann-Lebesgue Lemma, inversion, translations and dilations, multiplication and differentiation, Fourier transform in $L^2(R)$, Plancherel theorem.

Tempered distributions, convolution, Fourier transform, Paley-Weiner Theorems, distributions as solution to partial differential equations, fundamental solution, Malgrange–Ehrenpreis theorem.

References:

1. William Donoghue, Distributions and Fourier Transforms. Academic Press, 2014.
2. S. Kesavan, Topics in Functional Analysis and Applications, New Age International, 2008.
3. Walter Rudin, Functional Analysis, McGraw-Hill 1991.
4. Robert Strichartz, A Guide to Distribution Theory and Fourier Transforms, World Scientific, 2003.

MA7241E MULTIVARIABLE CALCULUS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Nil

Total Lecture Sessions: 39

Course Outcomes:

CO1: Understand the notion of Frechet derivative, its properties and prove the inverse function theorem and see its consequences.
CO2: Understand the notion of differential forms, evaluate integrals of differential forms on chains and prove Stokes' theorem for differential forms.
CO3: Understand the concept of differentiable manifolds, their tangent spaces and orientation and the meaning of differentiability of functions defined on manifolds.
CO4: Evaluate integrals of differential forms on manifolds, prove the generalised Stokes' theorem and derive classical integration theorems of vector calculus from it.

Calculus of several variables

Functions on Euclidean spaces, Frechet derivative, partial and directional derivatives, chain rule, mean value theorem, inverse function theorem, implicit function theorem, rank theorem.
Riemann integration in higher dimensions, Fubini's Theorem, change of variables, line and surface integrals, classical integration theorems of vector calculus.

Differential forms

Tensors, wedge product, differential forms, Poincaré Lemma, integration on chains, Stokes' Theorem for integrals of differential forms on chains, fundamental theorem of calculus.

Differentiable manifolds and generalised Stokes theorem

Differentiable manifolds (as subspaces of Euclidean spaces), differentiable functions on manifolds, tangent spaces, differential forms on manifolds, orientations, integration on manifolds, Stokes' theorem on manifolds.

References:

1. M. Spivak, Calculus on Manifolds, A Modern Approach to Classical Theorems of Advanced Calculus, CRC press, 2018.

2. J. R. Munkres, Analysis on Manifolds, CRC press, 2018.

3. W. Rudin, Principles of Mathematical Analysis, 3rd Edition, McGraw-Hill, 1984.

4. Gerald B. Folland, Advanced Calculus, Pearson Education, 2002.

MA7242E STATISTICAL DIGITAL SIGNAL PRPCESSING

L	T	P	O	C
3	0	0	6	3

Prerequisites: Probability and Statistics

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand random processes and their Spectral Factorization.
CO2: Apply techniques of Signal Modelling using Levinson Recursion.
CO3: Identify different types of filters in real life applications.
CO4: Handle problems involving Spectrum Estimation.

Random Processes

Discrete-Time Random Processes: Random Variables, Random Processes, Filtering Random Processes, Spectral Factorization, Special Types of Random Processes.

Signal Modelling & Levinson Recursion

Introduction, The Least Squares Method, The Pade Approximation, Prony's Method, Iterative Pre-filtering, Finite Data Records. The Levinson Recursion: Introduction, The Levinson-Durbin Recursion, The Levinson Recursion.

Lattice Filters & Wiener Filtering

Introduction, The FIR Lattice Filter, Split Lattice Filter, IIR Lattice Filters, Lattice Methods for All-Pole Signal Modelling. Wiener Filtering: Introduction, The FIR Wiener Filter, The IIR Wiener Filter, Discrete Kalman Filter.

Spectrum Estimation

Introduction, Nonparametric Methods, Minimum Variance Spectrum Estimation, The Maximum Entropy Method, Parametric Methods.

References:

1. M. H. Hayes; "Statistical Digital Signal Processing and Modelling", John Wiley & Sons, 2004.

2. Clements, M. A. and Miao, G. J., "Digital Signal Processing & Statistical Classification"; Artech House, Inc., 2002.

3. Papoulis, A. and S. Unnikrishna Pillai, "Probability, Random Variables and Stochastic Processes, 4th Edition, Tata McGraw-Hill, 2002.

MA7243E TIME SERIES MODELLING

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Knowledge of elementary probability theory

Total Lecture sessions: 39

Course Outcomes:

CO1: Identify situations where stationary time series models are appropriate.
CO2: Estimate ARIMA models and use it for forecasting real world time series.
CO3: Use spectral theory of stationary processes for time series analysis.
CO4: Construct transfer function noise models and use it for forecasting.

Stationary Time Series and ARMA Models

Stationary Time Series and ARMA Models: Fundamental Concepts: Stochastic Process, the Auto covariance and auto-correlation functions. The Partial auto-correlation function, White Noise Process, Estimation of the Mean, auto-covariance, Auto-correlation, and partial auto-correlation - Stationary Time Series Models: Auto Regressive Processes, Moving Average processes, The Dual Relationship between AR(p) and MA(q) processes, Auto Regressive Moving Average (ARMA(p,q)) processes.

Identification of ARIMA Models

ARIMA Models And Its Identification - Nonstationary Time Series Models - Nonstationarity in the Mean, Auto Regressive Integrated Moving Average (ARIMA) Models, Nonstationarity in the Variance and Autocovariance - Model identification: Steps for Model Identification, Inverse Auto-correlation Function, Extended Sample Auto-correlation function and other Identification Procedures - Forecasting: Minimum Mean Square Error Forecasts, Computation of Forecasts, The ARMIA Forecasts as a weighted average of Previous Forecasts, The ARIMA Forecasts as a weighted Average of Previous Observations.

Parameter Estimation and Spectral Theory

Parameter Estimation and Spectral Theory - Parameter Estimation, Diagnostic Checking, and Model Selection: The Method of Moments, Maximum likelihood Method, Non-linear Estimation, Ordinary Least Square Estimation in Time Series Analysis, Diagnostic Checking and Model Selection Criteria. Examples - Seasonal Time Series Models: Introduction, Traditional Methods, Seasonal ARIMA Models, and Examples - Spectral Theory of Stationary processes: The Spectrum, The Spectrum of some Common Process, The Spectrum of Linear Filters - Estimation of the Spectrum: Periodogram Analysis, The Sample Spectrum, the Smoothed Spectrum, ARMA Spectral Estimation, and Examples.

Transfer Function Noise Models

Transfer Function Noise Models: Single-Input Transfer Function Models. The Cross Correlation Function and Transfer function models, Construction of Transfer Function Models, forecasting using Transfer Models, Bi-variate Frequency-Domain Analysis, The Cross Spectrum and Transfer Function Models, Multiple-Input Transfer Function Models, Examples.

References:

1. Wei, W. W. S., "Time Series analysis – Univariate and Multivariate Methods", Addison Wesley Publishing Company, Inc., 2006.
2. Box, G. E. P., Jenkins, G. M., Reinsel, G. C. and Ljung, G. M., "Time Series Analysis forecasting and Control", 5th Edn., Wiley, 2015.
3. Makrodakis, S., Wheelwright, S. C. and Mcgee, V. E., "Forecasting Methods and Applications", 2nd Edn., John Wiley and Sons, 1983.

MA7244E WAVELETS AND APPLICATIONS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Nil

Total Lecture sessions: 39

Course Outcomes:

CO1: Compute discrete Fourier transform and fast Fourier transform of sequences.
CO2: Construct wavelets on Zn.
CO3: Construct wavelets on Z.
CO4: Construct wavelets on R and use it for real life applications.

Discrete Fourier Transforms

Vector spaces and Bases, Linear transformation, Matrices and change of basis, Inner products, Hilbert Space, Fourier transforms, Basic Properties of Discrete Fourier Transforms, Translation invariant Linear Transforms, The Fast Fourier Transforms.

Wavelets on Zn

Construction of wavelets on Zn, The Haar system, Shannon Wavelets, Real Shannon wavelets, Daubechies's D6 wavelets on Zn, Examples.

Wavelets on Z

l2(Z), Complete orthonormal sets in Hilbert spaces, L2(-,) and Fourier series, The Fourier Transform and convolution on l2(Z) , First stage Wavelets on Z , Iteration step for Wavelets on Z, Implementation and Examples.

Wavelets on R

l2(R) and approximate identities, The Fourier transform on R, Multiresolution analysis, Construction of MRA, Wavelets with compact support, Condition number of a Matrix, Wavelet-Galerkin Methods for Differential Equations, Applications: Biomedical, Data Compression, Fingerprint Matching.

References:

1. Michael. W. Frazier, "An Introduction to Wavelets through Linear Algebra", Springer, New York, 1999.
2. Jaideva C. Goswami, Andrew K Chan, "Fundamentals of Wavelets: Theory, Algorithms and Applications", John Wiley and Sons, New York, 1999.
3. Yves Nievergelt, "Wavelets made easy", Birkhauser, Boston,1999.
4. M.V. Altaisky, "Wavelets: Theory, Applications, Implementation," Universities Press, 2005.
5. A. Aldroubi and M.Unser, "Wavelets in Biology and Medicine", CRC press,1996

MA7245E NUMERICAL LINEAR ALGEBRA

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Linear Algebra

Total Lecture sessions: 39

Course Outcomes:

CO1: Analyse the basic operations on vector spaces numerically.
CO2: Solve systems of linear equations and linear least squares problems using matrix factorization methods.
CO3: Apply numerical methods for computing eigenvalues.

Conditioning and Stability

Review Linear Algebra Basic Concepts, Conditioning and Stability; Condition numbers, Floating point arithmetic, Stability of various algorithms, Linear Equation Solving: Gaussian Elimination, Pivoting, Stability of Gaussian Elimination, Cholesky Factorization, Jordan canonical form and applications.

Matrix representation and decomposition

Positive definite systems, LU decomposition, Orthogonal Matrices, Projectors and QR Factorization, Gram-Schmidt Process, Householder Transformation, Least Square Problems, Numerical Computation of Eigenvalues and Eigenvectors, Gerschgorin's Method, Power method, Jacobi's and Givens method, Hessenberg form, inverse iteration, Schur factorization, QR algorithm, sensitivity of eigenvalues and eigenvectors.

Singular Value decomposition and Pseudo inverse of a Matrix

Singular Value Decomposition(SVD), Computing the SVD, Determining the rank of a matrix using SVD, Applications of SVD, QR algorithm for SVD, Pseudoinverse and the SVD

References:

1. L.N. Trefethen & David Bau III, Numerical Linear Algebra, SIAM, 1997.
2. D. S. Watkins, Fundamentals of Matrix Computations, John Wiley & sons, 2rd Edn., 2002.
3. G. Golub & C.V. Loan, Matrix Computations, John Hopkins University Press, 3rd Edn., 1996
4. K. Hoffman & R. Kunze, Linear Algebra, Second Edition, Pearson Education, 2015.
5. V. Sundarapandian, Numerical Linear Algebra, PHI,2008.,

MA7246E SPECTRAL THEORY OF HILBERT SPACE OPERATORS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Linear Algebra, Functional analysis

Total Lecture sessions: 39 hours

Course Outcomes:

CO1: Apply the properties of bounded and compact operators to operators on Hilbert spaces
CO2: Analyse the spectral properties of bounded and compact operators
CO3: Investigate the spectrum and spectral properties of various type of operators

Operators on Hilbert spaces

Elements of Hilbert space theory, bounded linear operators on Hilbert spaces, Bounded linear functionals, projection, Riesz representation theorem, Invertible operators, Adjoint, Self-adjoint, Unitary, Normal operators.

Spectrum of operators on Hilbert spaces

Spectral properties of bounded linear operators, resolvent and spectrum, resolvent operator, spectral theory, Complex analysis in spectral theory, Gelfand-Mazour theorem

Theory of compact operators

Compact linear operators, spectral theory of compact self-adjoint operators, formula for the inverse operator, Minimum-maximum Properties of eigenvalues, compact normal operators, Operator equations, Fredholm operators, Fredholm alternative.

Spectrum of special operators

Spectral properties of bounded self-adjoint linear operators, Positive operators, Square root of an operator, Projection operators, spectral family and spectral family of bounded self-adjoint linear operators, spectral representation of bounded self-adjoint linear operators.

References:

1. J. B. Conway, A course in Functional Analysis, Springer, 2nd Edn., 1990.
2. B. V. Limaye, Functional Analysis, New Age Publishers, 3 Edn., 2014.
3. I. Gohberg & S. Goldberg, Basic operator Theory, Birkhauser, 2014.
4. R. Courant & D. Hilbert, Methods of Mathematical Physics, Interscience, 1996.
5. M. Reed, Methods in Mathematical Physics: Functional Analysis, Elsevier Science,2012.

MA7247E OPERATOR THEORY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Functional Analysis

Total Lecture sessions: 39

Course Outcomes:

CO1: Analyse the properties of compact and Fredholm operators
CO2: Apply the properties of bounded linear operators to special class operators
CO3: Analyse the properties of unbounded operators and their spectra

Compact operators and Fredholm theory

Compact operators, Fredholm operators and their properties, semi-Fredholm operators, characterization of Fredholm operators, Banach algebras, The Calkin Algebra.

Theory of bounded linear operators

Hilbert Space Operators, Parts of Spectrum, Orthogonal Projections, Invariant Subspaces, Reducing Subspaces, Shifts, Decompositions of Operators. Compact linear operators, Spectral properties of compact bounded linear operators, spectral theorem and functional calculus for compact normal operators.

Spectral Theory of operators

Spectral projections, spectral decomposition theorem, spectral theorem for a bounded normal operator, Measures of operators. Perturbation classes, strictly singular operators, Spectral theory of integral operators: Hilbert Schmidt theorem, Mercer's theorem, Trace formula for integral operators, integral operators as the inverse of differential operators. Sturm- Liouville systems.

Theory of unbounded operators

Unbounded operators: Basic theory of unbounded self-adjoint operators, unbounded Fredholm operators and its properties, essential spectrum, unbounded semi-Fredholm operators, Spectral theorem for unbounded self-adjoint operators.

References:

1. M. Schechter, Principles of Functional Analysis, AMS,2th Edn., 2002.
2. I. Gohberg & S. Goldberg, Basic operator Theory, Birkhauser,1981.
3. B.V. Limaye, Functional Analysis, New Age Publishers, 3rd Edn., 2014
4. Israel Gohberg, Seymor Goldberg, Marinus Kaashoek, Classes of Linear operators, Vol I, Birkhauser, 1990.

5. F. Riesz & B. Sz.- Nagy, Functional Analysis, Reprint Edn, Dover Publications, 1990.
6. V. S. Sunder, Functional analysis' spectral theory, Birkhauser,1998.
7. G.J. Murphy, C*-Algebras and Operator Theory, Academic Press Inc., 1990.
8. R. G. Douglas, Banach Algebra Techniques in Operator Theory, 179 (Graduate Texts in Mathematics), Springer, 1998.

MA7248E ADVANCED COMPLEX ANALYSIS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Complex Analysis

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand and know the importance of Schwarz lemma in complex analysis.
CO2: Get acquainted with various techniques of proving Monodromy theorem, Hurwitz theorem, Inverse function theorem and Mittag-Lefler theorem.
CO3: Get expertise in the concept of Infinite products.

Analytic Continuation, Monodromy theorem, Hurwitz' theorem, Inverse function theorem, Winding number, Simply connected domains.

Automorphisms of the upper half plane, the unit disc, Schwarz-Pick Lemma, Montel's theorem, Riemann mapping theorem.

The Poisson Integral formula, Characterization of harmonic functions, Schwarz Reflection principle, Runge's theorem, Mittag-Lefler theorem.

Infinite products, Weierstrass' product theorem, Gamma and Zeta functions – a brief introduction.

References:

1. S. Kumaresan, A Pathway to Complex Analysis, Techno world Publications, 2021.
2. T.W. Gamelin, Complex Analysis, Springer-Verlag, 2001.
3. S. Ponnusamy and H. Silverman, Complex Variables with Applications, 2006, 524 pp, Birkhaeuser, Boston.
4. L. Ahlfors: Complex Analysis, 2nd ed., McGraw-Hill,New York, 1966.
5. J.B. Conway, Functions of One Complex Variable, 2nd ed., Springer, 2002.
6. H. Priestely, Introduction to Complex Analysis, Oxford India, 2008.

MA7249E SPECIAL FUNCTIONS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Basic complex analysis and differential equations

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand various kinds of special functions, their properties and relations in detail.
CO2: Solve basic ordinary differential equations using analytical techniques.
CO3: Understand how special functions are useful in differential equations.

Infinite product of complex numbers, Infinite product of analytic functions, Gamma function, Beta function, Power series solution of differential equations, **Hypergeometric Functions:** Integral form; The contiguous function relation; Hypergeometric differential equation; Logarithmic solution; Relation between functions of z and 1-z

Bessel's Functions

Definition; Bessel's differential equation; Recurrence relation; A generating function; Bessel's integral; Modified Bessel's function.

Generating Functions

Functions of the form G(2xt-t^2); Functions of the form exp(t) ψ(xt); Functions of the form

A(t) exp(-xt/(1-t))

Orthogonal Polynomials

Legendre polynomial; Hermite polynomial; Laguerre polynomial; Jacobi polynomial.

References:

1. Earl D. Rainville, Special Functions, Chelsea Pub. Co. NY, 1971. ISBN: 978-0828402583
2. G.E. Andrews, R. Askey, and R. Roy, Special Functions, Cambridge University Press, 1999. ISBN: 978-0521623216
3. R. Beals and R. Wong, Special Functions: A Graduate Text, Cambridge University Press, 2010. ISBN: 978-0521197977

MA7250E UNIVALENT FUNCTION THEORY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Basic complex analysis

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand the analytic and geometric properties of univalent functions.
CO2: Discuss the behaviour of the coefficients of univalent function.
CO3: Understand the properties of univalent functions using its subclasses.

Normal families, extremal problems, The Riemann mapping theorem, Analytic continuation, Harmonic and Sub-harmonic functions, Green's functions, Positive Harmonic function.

Univalent function: Definition of univalent function and elementary properties, the Area theorem, Growth and Distortion theorems, Coefficient estimates for univalent functions, The maximum modulus of univalent functions, Bieberbach Conjecture.

Subclasses of Univalent functions: Classes of convex, Starlike, and Close-to-convex functions and their properties in the unit disk, Spirallike functions, Typically Real functions, Growth of Integral means, Odd univalent functions.

References:

1. P. Duren, Univalent Functions, New York, Springer, 1983.
2. A. W. Goodman, Univalent functions, Vol. 1-2, Mariner, 1983.
3. Ch. Pommerenke, Univalent functions, Vandenhoeck and Ruprecht, Gottingen, 1975.

MA7251E NUMBER THEORY AND CRYPTOGRAPHY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Nil

Total Lecture sessions: 39

Course Outcomes:

CO1: Effectively express the concepts and results of number theory.
CO2: Demonstrate knowledge and critical understanding of the well-established principles within number theory.
CO3: Apply the concept of number theory in cryptography and primality testing.
CO4: Collect and use numerical data to form conjectures about the integers.

Prime Numbers

Well ordering principle, Principle of mathematical induction, Divisibility, Division Algorithm, Greatest Common Divisor, Euclidean Algorithm, Linear Diophantine Equation, Pythagorean triples, Primes and their basic properties, Fundamental Theorem of Arithmetic, Infinitude of Prime Numbers, Primes in arithmetic progressions.

Congruences

Modular arithmetic: Basic properties, Residue classes, Reduced residue system, Fermat's Little Theorem, Euler's theorem linear congruences, Simultaneous linear congruences, Chinese Remainder Theorem, Polynomial congruences, Quadratic residues, Legendre symbol, Quadratic reciprocity law.

Cryptography

Arithmetic functions: Multiplicative functions, Mobius function, Euler totient function, Mobius inversion formula, Sum and divisor function, Greatest integer function, Primitive roots, Introduction to Cryptography, RSA algorithm, Primality testing.

References:

1. G. H. Hardy and E. M. Wright, An Introduction to the Theory of Numbers, Oxford University Press, 6th Edition, 2008.
2. M. B. Nathanson, Elementary Methods in Number Theory, Springer, 2000.
3. I. Niven, H.S. Zuckerman and H.L. Montogomery, An Introduction to the Theory of Numbers, Wiley, 5th Edition, 2008.
4. T.M. Apostol, Introduction to Analytic Number Theory, Springer International Student Edition, 1998.
5. K. Ireland and M. Rosen, Classical Introduction to Modern Number Theory, Springer-Verlag (GTM), 1990.

MA7252E ALGEBRAIC NUMBER THEORY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Abstract Algebra

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand the concept (definition and significance) of algebraic numbers and algebraic integers.
CO2: Define, describe and analyse more advanced concepts such as ideals, ideal classes, unit groups, norms, traces and discriminant.
CO3: Find the factorisation of ideals, the ring of integers, the class number and ideal class group of a number field.
CO4: Solve certain Diophantine equations by applying tools from the course.

Algebraic Integers

The Gaussian integers, Integrality, Ideals, Noetherian and Principal Ideal Domains, Dedekind domain, Lattices, Algebraic numbers and number fields.

Theory of Valuations

The p-adic numbers, the p-adic absolute value, Valuations, Completions, Local fields, Ostrowski's theorem, Hensel's lemma, Unramified, Totally ramified and tamely ramified extensions of p-adic fields.

Class Groups

Binary quadratic forms, Forms and ideals, Ideal class group, Finiteness of the ideal class group, Units in number rings, Dirichlet’s unit theorem.

References:

1. J. Neukirch, Algebraic Number Theory, Springer, 1999.

2. R. A. Molin, Algebraic Number Theory, 2nd Edition, CRC Press, 2014.

3. K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, 2nd Edition, Springer-Verlag, Berlin, 1990.

4. S. Lang, Algebraic Number Theory, Addison-Wesley, 1970.

5. D. A. Marcus, Number Fields, Springer-Verlag, 1977.

MA7253E INTRODUCTION TO ALGEBRAIC GEOMETRY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Abstract Algebra

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand the concepts of basic objects like affine and projective varieties, and the maps between them.
CO2: Solve problems involving varieties by converting them into problems in abstract algebra.
CO3: Pursue further studies in this and related areas.

Plane Curves

The affine plane, The projective plane, Plane projective curves, Tangent lines, Resultants and discriminants, Nodes and cusps, Hensel's lemma, Bézout's theorem.

Affine Algebraic Geometry

The Zariski topology, Some affine varieties, The Nullstellensatz, The spectrum, Morphisms of affine varieties, Localization, Finite group actions.

Projective Algebraic Geometry

Projective varieties, Homogeneous, Ideals, Product varieties, Rational functions, Morphisms, Algebraic function fields, Dimension of varieties, Rational maps.

References:

1. W. Fulton, Algebraic Curves: An Introduction to Algebraic Geometry, Addison Wesley Longman Publishing Co, 1989.

2. M. Reid, Undergraduate Algebraic Geometry, Cambridge University Press, Cambridge, 1988.

3. R.J. Walker, Algebraic Curves, Springer-Verlag, 1978.

4. S. S. Abhyankar, Algebraic Geometry for Scientists and Engineers, American Mathematical Society 1992.

MA7254E INTRODUCTION TO DYNAMICAL SYSTEMS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Algebraic Number Theory

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand basic classical complex dynamics.
CO2: Understand arithmetic dynamics of rational maps with coefficients in a local field.
CO3: Understand arithmetic dynamics over global fields.

Classical Dynamics

Rational function, Riemann sphere, Riemann mapping theorem, Montel's theorem, Arzela-Ascoli theorem, Hyperbolic metric, Classifications of fixed points, Attracting fixed points, Repelling fixed points, Super-attracting fixed points, The Julia set and the Fatou set.

Dynamics over Local Fields

The non-Archimedean Chordal Metric, Periodic points, Reduction of Points, The Resultant of a Rational Map, Rational maps with Good Reduction, Periodic Points and Good Reduction.

Dynamics over Global Fields

Height Functions, Canonical Heights and Dynamical Systems, Local Canonical Heights, Integral Points in Orbits, Periodic Points and Galois Groups, Equidistribution and Preperiodic Points.

References:

1. L. Carleson and T. W. Gamelin, Complex Dynamics, Universitext: tracts in Mathematics, Springer-Verlag, New York, 1993.

2. J. Milnor, Dynamics in One Complex variable, Friedr. Vieweg & Sohn, Braunschweig, 1999.

3. Joseph H. Silverman, The Arithmetic of Dynamical Systems, GTM, Springer, 2007.

MA7255E GALOIS THEORY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Abstract Algebra

Total Lecture sessions: 39

Course Outcomes:

CO1: Identify various types of field extensions.
CO2: Understand how Galois theory is applied to the question of solvability of the quantic.
CO3: Describe the Galois group of a given field extension, and to find correspondences between subgroups and intermediate fields.
CO4: To explain insolubility of certain classes of equations or impossibility of certain geometric constructions.

Field Theory

Fields, Characteristic and prime subfields, Field extensions, Algebraic and finitely generated field extensions, Classical ruler and compass constructions, Splitting fields and normal extensions, Algebraic closures, Finite fields, Cyclotomic fields, Separable and inseparable extensions.

Galois Group

Galois groups, Fundamental theorem of Galois theory, Composite extensions, Examples (including cyclotomic extensions and extensions of finite fields).

Insolvability of the Quintic

Galois Group of polynomials, Solvability by radicals, Galois' theorem on solvability, Cyclic extensions, Abelian extensions, Polynomials with Galois groups, Transcendental extensions.

References:

1. D. S. Dummit and R. M. Foote, Abstract Algebra, 3rd Edition, Wiley, 2003.

2. S. Lang, Algebra, 3rd Edition, Springer, 2010.

3. S. Lang, Undergraduate Algebra, 3rd Edition, Springer, 2005.

4. M. Artin, Algebra, Prentice Hall of India, 1994.

MA7256E SOBOLEV SPACES WITH APPLICATIONS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Basic Functional Analysis, Basic Partial Differential Equations

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand the theory of Sobolev spaces and its importance to study partial differential equations.
CO2: Apply the Lax-Milgram theorem to establish the existence and uniqueness of weak solutions.
CO3: Learn the weak and strong convergence in L^p- and Sobolev spaces.
CO4: Characterize eigenvalues and eigenfunctions of linear elliptic equations and their regularity properties.

Integration by parts, Gauss-Green identities, Fundamental solutions, Green's function, non-existence of classical solution, Weak convergence, Test Functions, distributions, Weak and Distributional derivative, Compact support, convolution, Fourier transform, mollifiers, partition of unity.

Sobolev spaces: definition and basic properties, approximation by smooth functions. Extension theorems, Poincare inequality, Sobolev inequality, Embedding theorems, Compactness theorems, traces, dual space, fractional Sobolev spaces, Hardy's inequality. Characterization of Sobolev spaces.

Second-order elliptic equations: Laplace and Poisson equation, weak solutions, existence of weak solutions, Lax-Milgram lemma, weak formulations of elliptic boundary value problems, Interior and boundary regularity Maximum principles, Eigenvalue problems.

References:

1. L. C. Evans, Partial differential equations, Second edition, Graduate Studies in Mathematics, 19, American Mathematical Society, Providence, RI, 1998.
2. H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer New York, 2011.
3. R.A. Adams and J.J.F. Fournier, Sobolev spaces, Second edition, Academic Press, 2003.
4. S. Kesavan, Topics in Functional Analysis and Applications, Third Edition, New Age International Publishers, 2019.
5. D. Gilbarg and N.S. Trudinger, Elliptic partial differential equations of second order, Second edition, Springer Berlin, Heidelberg, 2001.
6. G. Leoni, A first course in Sobolev spaces, Second edition, American Mathematical Society, Providence, RI, 2017.

MA7257E CALCULUS OF VARIATIONS AND PDES

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Basic Analysis and Differential Equations

Total Lecture sessions: 39

Course Outcomes:

CO1: Construct Euler-Lagrange equations corresponding to a given partial differential equation.
CO2: Understand the theory of Sobolev spaces and its importance to study partial differential equations.
CO3: Construct energy functions and obtain existence of minimizers via direct method of calculus of variations.
CO4: Apply variational techniques together with the classical methods to study the existence of weak solutions to PDEs.

First variation, Second variation, Euler-Lagrange equation, Constrained problems and Lagrange multipliers, Existence of minimizers: Classical method, Dirichlet integral and p-Dirichlet Integral, Existence and nonexistence of minimizers: examples and counterexamples, Wierstrass function, weak form of the Euler-Lagrange equations.

Sobolev spaces: definition and basic properties, Poincare inequality, Sobolev inequality, Embedding theorems, Compactness theorems, fractional Sobolev spaces, direct method of calculus of variations, Dirichlet principle, convexity, coercivity, lower semicontinuity, local minimizers, nonlinear eigenvalue problems, Pohozaev identity for non-existence of solutions.

Critical points, saddle points, Palais-Smale sequence, mountain pass lemma, weak convergence method, Schauder's fixed point theorem, method of subsolutions and supersolutions. Weyl lemma, L^2 regularity, Moser iteration, Schauder' estimate.

References:

1. L. C. Evans, Partial differential equations, Second edition, Graduate Studies in Mathematics, 19, American Mathematical Society, Providence, RI, 1998.
2. M. Struwe, Variational Methods: Applications to Nonlinear Partial Differential Equations and Hamiltonian Systems, Fourth edition, Springer Berlin, Heidelberg, 2010.
3. H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer New York, 2011.
4. R.A. Adams and J.J.F. Fournier, Sobolev spaces, Second edition, Academic Press, 2003.
5. D. Gilbarg and N.S. Trudinger, Elliptic partial differential equations of second order, Second edition, Springer Berlin, Heidelberg, 2001.
6. G. Leoni, A first course in Sobolev spaces, Second edition, American Mathematical Society, Providence, RI, 2017.

MA7258E FIXED POINT THEORY AND APPLICATIONS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Real Analysis

Total Lecture sessions: 39

Course Outcomes:

CO1: Appreciate how the study of fixed point theory helps to solve problems which are theoretical as well as practical.
CO2: Realize contraction and contractive maps have elegant results on the existence and uniqueness of fixed points.
CO3: Appreciate the generalizations of Brouwer's fixed point theorem, viz., Schauder, and its use in analysis and differential equations.
CO4: Thoroughly understand the idea behind Michael's selection theorem.

Banach's contraction principle - Further extensions- Caristi - Ekeland principle, Equivalence of Caristi- principles.

Tarsiki's Fixed point theorem, Hyperconvex spaces - Properties, the intersection of hyperconvex spaces - Isbell's convex hull.

Uniformly convex Banach spaces - Fixed point theorem of Browder, Gohde, and Kirk, Reflexive Banach spaces.

Generalized Banach Fixed-point theorem, Upper and lower semi-continuity of multivalued maps, Generalized Schauder Fixed point theorem, Variational Inequalities and the Browder Fixed-Point theorem, Extremal Principle, Applications to Game Theory, Michael's selection theorem.

References:

1. E. Zeidler, Nonlinear Functional Analysis and its Applications, Vol. I Springer – Verlag New York,(1986).
2. 2. M. A. Khamsi & W. A. Kirk, An introduction of Metric spaces and Fixed point theory, John Wiley & Sons, (2001).
3. D.R. Smart, Fixed point theory, Cambridge University Press, (1974).
4. Sankatha Singh, Bruce Watson, and Pramila Srivastava, Fixed Point Theory and Best Approximation: The KKM - map Principle, Kluwer Academic Publishers, 1997.
5. Kim C. Border, Fixed point theorems with applications to economics and game theory, Cambridge, University Press, 1985.
6. S. Kumaresan and D. Sukumar, Functional Analysis: A First Course, Narosa, 2020.

MA7259E APPROXIMATION THEORY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Real analysis

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand the concept of Approximation and study the fundamental theorems.
CO2: Acquire knowledge on polynomials and the modern approximation theory, including the standard theorems.
CO3: understand the idea behind the Chebyshev Approximations.

Approximation of Periodic Functions, Approximation by Algebraic Polynomials, Convergence of Bernstein Polynomials, Korovkin's Theorem, The Stone- Weierstrass Theorem.

Approximation in Normed Linear Spaces, Linear Chebyshev Approximation of Vector-valued Functions, Uniqueness Results, and Haar Subspaces.

Strong Uniqueness and Continuity of Metric Projection, Upper Semicontinuity of Metric Projections, Lower Continuity, Continuous Selections, and Lipschitz Continuity of Metric Projections.

References:

1. N. Mhaskar and Devidas V.Pai, Fundamentals of Approximation Theory, Revised Edition Narosa, 2007.
2. S. Kumaresan and D. Sukumar, Functional Analysis: A First Course, Narosa, 2020.
3. M. T. Nair, Functional analysis: A First Course, PHI-Learning, New Delhi, 2014.
4. C.D. Kubrusly, Elements of Operator Theory, Birkhauser, 2001.
5. S. Kumaresan, Topology of Metric Spaces, 2nd Edition, Narosa Publishing House, New Delhi, 2011.

MA7260E GEOMETRIC FUNCTION THEORY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Complex analysis, Measure theory.

Total Lecture sessions: 39

Course Outcomes:

CO1: Introduce the concept of Hardy spaces of analytic functions and study the basic theorems.
CO2: Characterize the space A^{p} in terms of derivatives of functions in A^{p}.
CO3: Understand classifications of Bergman Kernel.

Review of Complex Analysis, Subharmonic Functions, and their properties, Hardy spaces of analytic functions, Zero Set of H^{p}, Mean Convergence to Boundary Values, Applications of H^{p} Spaces.

Bergman Spaces, Growth of A^{p} functions, Bergman Kernel, Density Properties of Bergman Space.

The Bergman Projection on A^{p}, A Bounded Projection of L^{1} onto A^{1}, A Characterization of A^{p} in terms of derivatives.

References:

1. Duren, P. L., Theory of H^{p} Spaces. Academic Press, New York and London,1970.
2. Duren, P. L., Univalent Functions. Springer-Verlag, 2001.
3. S. Kumaresan and D. Sukumar, Functional Analysis: A First Course, Narosa, 2020.
4. Hakan Hedenmalm, Theory of Bergman Spaces. Springer-Graduate Texts in Mathematics,2000.
5. S. Kesavan, Measure and Integration, TRIM, 2019.
6. W. Rudin, Real and Complex Analysis, Third edition, McGraw-Hill, International Editions, 1987.
7. V. S. Sunder, Functional Analysis: Spectral Theory, Hindustan Book Agency (TRIM Series), 1997.

MA7261E INTRODUCTION TO LIE ALGEBRAS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Linear algebra, Abstract algebra.

Total Lecture sessions: 39

Course Outcomes:

CO1: Have a complete picture of some low dimensional Lie algebras.
CO2: Find the root systems of finite dimensional complex simple Lie algebras.
CO3: Understand the approach to classify all finite dimensional complex simple Lie algebras.

Lie algebras

Lie Algebras-Motivation and Definition, Examples of Lie Algebras-Linear Lie algebra, Lie algebras of Derivations, Abstract Lie algebras, Subalgebras, Ideals, Homomorphisms and Representations, Initial Classification-Rough classification of Lie algebras, Nilpotent Lie algebra-Engel's theorem, Solvable Lie algebra-Lie's theorem, Semisimple Lie algebras, Simple Lie algebras, Lie algebras of dimension one, two and three, Representations of sl_2(C).

Root Systems

Jordan-Chevelley decomposition, Cartan's Criterion, Killing Form, Simple ideals, Inner derivations, Abstract Jordan Decomposition, Complete Reducibility of representations, Schur's Lemma, Casimir element of representation, Weyl's Theorem, Levi's theorem, Preservation of Jordan decomposition, Root space decomposition using Killing form, Cartan subalgebra, Root system-orthogonal Properties, Integrality properties, rational properties.

Classification of Simple Lie Algebras

Abstract Root System-simple roots, Weyl group, Irreducible root system, Classification of complex simple Lie algebras-Cartan matrix, Coaxter Diagrams, Dynkin Diagram associated to semisimple Lie algebras, Classifying Dynkin Diagrams, Recovering a Lie algebra from its Dynkin Diagram.

References:

1. James E. Humpherys, Introduction to Lie algebras and Representation Theory, Springer-Verlag,1973.
2. Alexander Kirillov Jr., Introduction to Lie Groups and Lie algebras, Cambridge University Press, 2008.
3. J. P. Serre, Complex Semisimple Lie algebras, Springer, 2001.
4. A. L. Onishchik, E. B. Vinberg(Eds), Structure of Lie Groups and Lie Algebras-III, Springer-Verlag, 1993.

MA7262E INTRODUCTION TO LIE SUPERALGEBRAS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Linear and Multilinear Algebra, Introduction to Lie algebras.

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand classifications of finite dimensional simple Lie superalgebras.
CO2: Understand highest weight theory of classical Lie superalgebras.
CO3: Learn the difference between Lie Algebras and superalgebras.

Classification of Lie superalgebras

Lie superalgebras, the general linear Lie superalgebras and special linear Lie superalgebras, Ortho Symplectic Lie superalgebras, queer, periplectic and exceptional Lie superalgebras, The Cartan Series, solvable, nilpotent, semi-simple Lie superalgebras, Classification theorem.

Structure Theory

Killing form-degenerate and non-degenerate, Cartan and Borel Subalgebras, Root space decomposition, Root Systems, Weyl group for classical Lie superalgebras, Non-conjugate positive system, Odd reflections, Contragradient Lie superalgebras, Serre relations, invariant bilinear form.

Representation Theory

The Poincare- Birkhoff-Witt(PBW) theorem, Free Lie superalgebras, Filtered and graded rings, representation of solvable Lie superalgebras, induced and produced representations, Highest weight theory of basic Lie superalgebras and queer Lie superalgebra.

Finite Dimensional Modules

Classification of finite dimensional simple modules, Virtual Character Formula, Simple modules for queer, Super Symmetrization, Central Characters, Harish-Chandra homomorphisms for basic Lie superalgebras, Typical finite dimensional irreducible character, Central Characters, Harish-Chandra homomorphisms, typical finite dimensional characters for queer.

References:

1. M. Scheunert, The Theory of Lie superalgebras: An Introduction, Lecture Notes in Mathematics, Springer-Verlag, 1979.

2. Ian M. Musson, Lie superalgebras and Enveloping Algebras, AMS Providence, GTM, Vol-131, 1953.

3. Shun-Jen Chen and Weiqiang Wang, Dualities and Representations of Lie superalgebras, AMS Providence, Vol-144, 1963.

4. L. J.R. Frappt, A. Sciarrino and P. Sorba, Dictionary of Lie algebras and Lie superalgebras, Academic press Inc., 2000.

MA7263E KAC-MOODY ALGEBRAS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Introduction to Lie Algebras

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand a class of infinite dimensional Lie algebras namely Kac-Moody algebras, their construction and properties. Also how the theory is developed close to the classical theory of simple Lie algebras.
CO2: Understand a subclass namely affine algebras, their root systems and Weyl group.
CO3: Understand the highest weight theory of Kac-Moody algebras.

Kac- Moody Algebras- Basic definitions and examples, The invariant Bilinear Form, Generalized Casimir operator, Integrable representations of Kac-Moody algebras, Weyl Group, Generalized Cartan matrices- classification.

Real and Imaginary roots, Affine algebras- invariant form, root System, and the Weyl group. Affine algebra as a central extension of loop algebras Twisted affine algebras and finite order automorphisms.

Highest weight modules over Kac-Moody algebras- Category O, Character Formula, Integrable Highest weight module- the weight system, Unitarizability.

References:

1. Victor G. Kac, Infinite dimensional Lie algebras, 3rd Edition, Cambridge University Press, 2003.
2. Roger Carter, Lie algebras finite and affine type, Cambridge University Press, 2005.
3. V. G. Kac and Ashok K. Raina, Bombay Lectures on Highest Weight representations of infinite dimensional Lie algebras, 2nd Edition, World Scientific; Advanced Series in Mathematical Physics, Vol 29, 2013.

MA7264E REPRESENTATION THEORY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Abstract Algebra, Introduction to Lie algebras.

Total Lecture sessions: 39

Course Outcomes:

CO1: Find a complete picture of finite dimensional irreducible representations of classical Lie algebras.
CO2: Have an understanding of how to construct highest weight modules (Verma modules).
CO3: Find out necessary and sufficient conditions for the existence and uniqueness of finite dimensional highest weight modules.

Representation of Finite Groups

Definitions, complete reducibility-Schur's Lemma, Abelian Groups, symmetric group S_3, Characters- examples, Induced Representations, Frobenius Reciprocity, Representations of S_n-Young Diagram, Frobenius's Character Formula, Weyl's construction.

Basic Representations of Lie algebras

Lie Groups, Lie algebras, The Exponential map, Definitions-representation of Lie groups and Lie algebras, adjoint representation, Indecomposable representation, Irreducible representation, Weyl's theorem, representation of sl_2(C), sl_3(C), representation of arbitrary semisimple Lie algebras, Universal enveloping algebras.

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Highest Weight Theory

Weights, maximal vectors, weight spaces, Standard cyclic module, Varma module, finite dimensional module-necessary and sufficient conditions, Weight strings and weight diagrams.

Characters

Invariant polynomial functions, Standard cyclic modules and characters, Harish-Chandra's theorem, Weyl character formula, Kostant's multiplicity formula, Steinberg's formula.

References:

1. William Fulton, Joe Harris, Representation Theory, A First Course, Springer, 3rd Edition, 2009.
2. A. W. Knapp, Lie Groups Beyond an Introduction, Birkhauser Boston, MA, 2002.
3. Brian C. Hall, Lie Groups, Lie algebras & Representations: An Elementary Introduction, GTM Vol 222, Springer, 2003.

MA7265E BANACH ALGEBRAS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Functional Analysis, complex analysis, algebra

Total Lecture sessions: 39

Course Outcomes:

CO1: Analyse the algebraic properties of operators in an abstract setting.
CO2: Apply the functional analysis concepts to Banach algebras.
CO3: Investigate the spectral properties of Banach algebras.

Banach Algebras

Preliminaries on functional analysis, normed algebras, Banach algebra, homomorphism, spectrum, basic properties of spectra, Gelfand-Mazur theorem, spectral mapping theorem, group of invertible elements.

Gelfand Theory

Commutative Banach Algebras and Gelfand Theory: Ideals, maximal ideals and homomorphism, semisimple Banach Algebra, Gelfand topology, Gelfand Transform, involutions

Banach* algebras

Banach* algebras: Definition and examples, Gelfand-Naimark theorem, applications to non-commutative algebras, characterization of Banach* algebras.

References:

1. Allan, G. R., Introduction to Banach Spaces and Algebras, Oxford Graduate Texts in Mathematics 20, Oxford University Press, 2011.

2. Sunder, V. S., Functional Analysis: Spectral Theory, TRIM Series, No. 13, Hindustan Book Agency, Delhi, 1997,

3. Rudin, W., Functional Analysis, second edition, McGraw-Hill, 1991.

4. G. F. Simmons, Introduction to Topology and Modern Analysis, Tata McGraw-Hill Education Private, 2004.

5. R. G. Douglas, Banach Algebra Techniques in Operator Theory, 179 (Graduate Texts in Mathematics), Springer, 1998.

MA7266E FINITE ELEMENT METHODS AND APPLICATIONS

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Partial Differential Equations, Numerical analysis, Functional Analysis, Measure Theory

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand the basic concepts of finite element method.
CO2: Get knowledge of existence and uniqueness and various theorems related to existence and uniqueness.
CO3: Derive weak formulation, construct stiffness matrix and calculate the weak derivatives for a given function.
CO4: Solve 1D and 2D problems by using finite element methods.

Introduction to Finite Element Method (FEM), basic concepts, Variational Formulation of elliptic and parabolic boundary value problems, Lax Milgram Lemma, Existence and uniqueness of solutions, Galerkin and Ritz variational formulations.

Discretization concepts, choice of elements, derivation of shape functions, Lagrangian and Hermite in physical coordinates, Isoparametric mapping, Numerical Integration, Construction of element stiffness matrices, Assembly and solution techniques, 1-dimensional problems.

Formulation of eigen value problems, Time dependent problems, Applications, Non-linear problems, Finite element error analysis, Automatic mesh generation, Finite element problems; conforming and non-conforming methods, Application to PDE: Solutions of two dimensional partial differential equations under different geometric conditions.

Solution of model problems and computer implementation procedures, Asymptotic error estimate results, Eigenvalue problems of Laplace operator. Applications to Partial Differential Equations.

References:

1. Reddy, J.N.: “Introduction to the Finite Element Methods”, Tata McGraw-Hill, 4th Edition, 2020.
2. Bathe, K.J., “Finite Element Procedures”, Prentice-Hall, New Jersey, 2nd Edition, 2015.
3. Thomas, J.R. Hughes: “The Finite Element Method: Linear Static and Dynamic Finite Element Analysis”, Dover Publication, 2012.
4. George R Buchanan, “Finite Element Analysis”, McGraw Hill India, 2020.

MA7267E TRANSFORM TECHNIQUES

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Partial Differential equations, Complex Analysis.

Total Lecture sessions: 39

Course Outcomes:

CO1: Learn the concept of Fourier Series and ability to apply it for Periodic and Non-Periodic Signal.
CO2: Understand the concept of Fourier transforms and apply it to solve differential and integral equations.
CO3: Solving the differential and integral equations by using Laplace transform.
CO4: Understand the concept of Z-transforms and apply it to solve finite difference equations.

Fourier Series

Introduction to Fourier Series, Examples, Complex Fourier Series, Conditions for the Convergence of Fourier Series, Use of Delta Function in the Fourier Series Convergence, More Examples on Fourier Series of a Periodic Signal, Gibb's Phenomenon, Properties of Fourier Transform of a Periodic Signal, Parseval's Identity.

Fourier Transforms

Definition of Fourier Transforms, Fourier Transform of a Heaviside Function, Use of Fourier Transforms to Evaluate Some Integrals, Properties of Fourier Transforms, Fourier Integral Theorem, Application of Fourier Transform to ODEs, Application of Fourier Transforms to Differential and Integral Equations, D'Alembert's Solution by Fourier Transform, Solution of Heat and Laplace Equations by Fourier Transform.

Laplace Transform

Introduction to Laplace Transform, Properties of Laplace Transforms, Inverse Laplace Transform, Heaviside Expansion Theorem, Inverse Laplace Transform by Contour Integration, Application of Laplace Transform – ODEs, Solving First Order PDEs, Solution of Wave Equation, Solving Hyperbolic Equations, Solving Heat Equation, Solution of Integral Equations by Laplace Transform.

Z-Transforms

Introduction to Z-Transforms, Properties of Z-Transforms, Evaluation of Infinite Sums by Z-Transforms, Solution of Difference Equations by Z-Transforms, Inverse Z-Transforms.

References:

1. Lokenath Debnath, Dambaru Bhatta, Integral Transforms and Their Applications, Second Edition, Chapman & Hall/CRC, 2007.
2. I.N. Sneddon, The Use of Integral Transforms, Tata Mc-Graw Hill, 1974.
3. J.L. Schiff, The Laplace Transform, Springer, 1999.
4. M.R. Spiegel, Laplace Transforms (Schaum's Series), McGraw-Hill, 1965

MA7268E ALGEBRAIC TOPOLOGY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Topology

Total Lecture sessions: 39

Course Outcomes:

CO1: Understand simplicial homology and determine homology groups
CO2: Compute algebraic invariants associated to topological spaces and maps between them
CO3: Prove topological results by using algebraic methods
CO4: Apply methods from algebraic topology to problems in a broader mathematical context.

Introduction and motivating Examples, Geometric Complexes and Polyhedra , Orientation of Geometric Complexes, Simplicial Homology Groups - Chains, cycles, Boundaries, Homology groups, examples of Homology Groups, The structure of Homology Groups, The Euler Poincare Theorem, Pseudo manifolds and the Homology Groups of Sn.

Simplicial Approximation- Introduction, induced Homomorphisms on the Homology groups, the Browder Fixed point theorem and related results.

The Fundamental group - Introduction, Homotopic paths and the fundamental group, the covering homotopy property for S1, Examples of Fundamental group, the relation between H1(K) and π1([K])

Covering Spaces – Definition and examples, basic properties of Covering spaces, Classification of covering spaces, Universal covering spaces, and applications.

References:

1. Fred H. Croom, Basic Concepts of Algebraic Topology, Springer – Verlag, 1978.
2. Satya Deo, Algebraic Topology A Primer, Hindustan Book Agency, New Delhi, 2003.
3. Allen Hatcher, Algebraic Topology, Cambridge University Press,2001,
4. James R. Munkres, Elements of Algebraic Topology, Addison Wesley Publishing Company,1993,

MA7269E ADVANCED TOPOLOGY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Topology

Total Lecture sessions: 39

Course Outcomes:

CO1: Prove basic results about separation, compactness and connectedness
CO2: Apply the theory in the course to solve a variety of problems at an appropriate level of difficulty.
CO3: Demonstrate skills in communicating mathematics orally and in writing
CO4: Prove topological results by using algebraic methods

Topological spaces: definition, basis, sub basis, Fundamental examples, subspace - Closure: closed sets, limit points, Hausdorff spaces. Continuity: equivalent definitions, homeomorphisms and embeddings. Product topology: basis and subbasis, box product.

Separation Axioms and Covering Properties - Separation axioms: Hausdorff, regular, Tychonoff, and normal topological spaces, Covering properties: Compactness, Lindelofness, paracompactness, metacompactness, Relations between covering properties and separation axiom, Normality of paracompact spaces, paracompactness of Lindel¨of spaces, Preservation of separation and covering properties.

Metrizability and Connectedness - The metrization theorems of Urysohn and Bing, Smirnov,Nagata. Connectedness and total disconnectedness: Definitions and examples of connectedness, total disconnectedness, zero-dimensionality. Local properties: Local compactness, local connectedness

Fundamental Group - Homotopy: homotopy of paths, Fundamental group and covering spaces, simply connected spaces, Fundamental group of the circle, Deformation retraction: fundamental group of the punctured plane

References:

1. James R Munkres; Topology - A first course, Prentice Hall of India, 2000.
2. K.D Joshi; Introduction to general Topology, New age International Ltd.,New Delhi, 2022
3. George F Simmons; Introduction to Topology and Modern Analysis, Mc Graw Hill International, 2017.
4. James Dugundji; Topology, Universal Book stall, New Delhi, 1995.
5. Willard, S; General Topology, Addison-Wesley, 1970.

MA7270E GENARALIZED SET THEORY

L	T	P	O	C
3	0	0	6	3

Pre-requisites: Nil

Total Lecture sessions: 39

Course Outcomes:

CO1: Distinguish the difference between crisp set and fuzzy sets
CO2: Apply multisets and rough sets in decision systems
CO3: Analyse soft sets and its hybridizations
CO4: Study algebraic and topological structures in various generalized set contexts

An overview of basic operations on Fuzzy sets, Intuitionistic fuzzy sets, Hesitant fuzzy sets, Multisets, Multiset relations, Compositions, Equivalence multiset relations and partitions of multisets, Multiset functions, Fuzzy Multisets.

Rough sets, Knowledge representation, Information systems, Exact sets, rough sets, approximations, Set-algebraic structures, Topological structures, Decision systems, Knowledge reduction, Reducts via Boolean reasoning, discernibility approach. Reducts in decision systems, Rough membership functions

Soft sets, Tabular representation of a soft set, Operations with Soft sets: soft subset, complement of a soft set, null and absolute soft sets, AND and OR operations, Union and intersection of soft sets, De-Morgan laws, Applications and soft analysis.

Fuzzy soft sets and soft fuzzy sets, Intuitionistic Fuzzy Soft Sets and Soft Intuitionistic Fuzzy Sets, Hesitant Fuzzy Soft Sets, Soft Rough Sets and Rough Soft Sets, Fuzzy rough sets and rough fuzzy sets.

References:

1. Bing-Yuan Cao, Fuzzy Information and Engineering, Springer, 2007.
2. Girish K P, John S J, Multiset Relations and Topologies on Multisets, Lap Lambert Pub., 2012.
3. Lech Polkowski, Rough Sets: Mathematical Foundations.Springer, 2002 ,
4. Sunil Jacob John, Soft Sets; Theory and Applications, Springer International Pub, 2021.
5. Zimmerman H.J., Fuzzy set Theory and its Applications, Allied Publishers Ltd, 2000.

MA7294E COMPUTER PROGRAMMING

L	T	P	O	C
1	0	3	6	3

Pre-requisites: Nil

Total Lecture sessions: 13

Course Outcomes:

CO1: Understand data types, operators and control statements.
CO2: Develop C programs using arrays, pointers, functions and structures.
CO3: Understand basics in C++.
CO4: Develop C++ programs using functions, function and operator overloading, inheritance.

Constants, variables. data types, operators, expressions, control statements, branching, looping, arrays, multidimensional arrays, character arrays, strings, functions, recursion, passing arrays and strings to functions

Pointers, pointers and functions, pointers and one dimensional arrays, pointers and strings, dynamic memory allocation, operations on pointers, pointers and multi-dimensional arrays, arrays of pointers, pointers as function arguments, pointers to functions. Structures, structures and functions, structures and arrays, structures and pointers.

Basics in C++, object oriented programming, classes and objects, constructors and destructors, constructors with default arguments, dynamic arrays in C ++.

Friend functions, In-line functions, function overloading, operator overloading: overloading arithmetic, relational, increment, decrement operators, inheritance, virtual functions.

References:

1. Brian W. Kernighan, Dennis M Ritchie, The C Programming Language, 2nd edition, PHI Learning Private Limited, 2012.
2. Yashavant P. Kanetkar, Let Us C, 17th Edition, BPB Publications, 2020.
3. John R Hubbard, Programming with C++, 2nd edition, Schaum's outlines, 2000.
4. E. Balagurusamy, Object-Oriented Programming with C++, 8th Edition, Tata McGraw Hill Education Pvt. Ltd., 2020.