Name: …………………………

CEA/C/D/PhD: Roll No: …………...

Department of Civil Engineering

NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

CE6101 THEORY OF ELASTICITY AND PLASTICITY

End Semester Examination – 9 Nov 2017

Time: 3 hours　　　　　　　　　　　　　　　　　　　　　　　　　　Maximum Marks: 50

Note: *Answer* **all** *questions*; Assume any missing data after stating clearly; and *Read the questions carefully*.

1. (a) *Write in index notation*: (i) equations of equilibrium, (ii) stress-strain relations, and (iii) strain-displacement relations. [3]

   (b) Combine the above and obtain Lamé-Navier equations. [2]

2. (a) *Write* the tensor transformation laws for *vectors* and *dyadics*. [2]

   (b) Consider two Cartesian coordinate systems x_i and x_i'. The direction cosines of x_1' and x_2' with respect to x_i are $\mathbf{n}_1$ = [0.6124, 0.5, –0.6124] and $\mathbf{n}_2$ = [0.3536, –0.866, –0.3536] respectively. *Find* the components of the transformation tensor a_{ij}. [3]

   (c) If the displacement components with respect to x_i at a point are: u_1 = 0.012mm, u_2 = –0.021mm, u_3 = 0.009mm, determine the displacement components u_i' with respect to x_i'. (d) Check the answer by comparing magnitudes of the two displacement vectors. [3]

3. A ring fixed at $r = b$ is subjected to a uniform circumferential shear at $r = a$ (where $a < b$) forming a couple M. Use the stress function $\phi = C\theta$ and *determine* the stress, strain and displacement fields inside the cylinder. Find an expression for the circumferential displacement v at $r = a$. The following relations may prove useful: [8]

$$\sigma_r = \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2};\ \sigma_\theta = \frac{\partial^2\varphi}{\partial r^2};\ \tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\varphi}{\partial\theta}\right);\ \varepsilon_r = \frac{\partial u}{\partial r};\ \varepsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial\theta} + \frac{u}{r};\ \gamma_{r\theta} = \frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial\theta} - \frac{v}{r}.$$

4. (a) Define *kinematically admissible displacement field* and *statically admissible stress field*. [2]

   (b) Write the *mathematical statement* of Clapeyron's theorem. Use it to derive *principle of virtual work* and *principle of stationary potential energy*. (c) State the *principle of stationary potential energy*. [5]

5. Write an expression for the *total potential energy* of a cantilever beam of span l, flexural rigidity EI, subjected to a uniformly distributed transverse load q and a tip concentrated load P. Obtain the resulting *Euler equation* and *natural boundary condition*(*s*) of the problem. [5]

6. (a) A prismatic shaft of *narrow rectangular* cross-section ($a\times b$) is subjected to a torque T. Use *approximate analysis* based on *membrane analogy* and determine *maximum shear stress* and *angle of twist* per unit length. [3]

   (b) Use the above to find the maximum allowable torque of a prismatic shaft of 2 m length made by bending a plate of 6 mm thickness into the cross-sectional shape shown in Fig. 1. Allowable shear stress = 120 MPa, allowable total angle of twist = 1 degree. [3]

   (c) If the cross-section in Fig. 1 is *welded* so that it becomes a *closed thin square tube* of same length 2 m, what is the maximum torque it can safely carry with the data given above. [3]

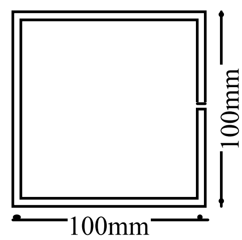

Figure 1

[3]

7. (a) Name the *three ingredients* of *theory of plasticity*. [1½]

   (b) Explain with aid of uniaxial stress-strain diagrams: (i) How you will distinguish between nonlinearly elastic and elastoplastic behaviours, and (ii) The difference between isotropic and kinematic hardening. [2½]

8. (a) Write the von Mises yield criterion and sketch the yield surface in three-dimensional principal stress space.

   (b) Write the von Mises criterion for plane stress problems and show the yield surface on σ_1-σ_2 space.

   (c) How does it get modified subsequently in the case of *isotropic* and *kinematic* hardenings? [6]

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