Department of Civil Engineering
NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

Name: ……
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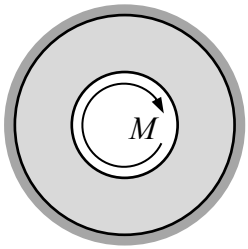
Sl. Number: ……

CE6101E THEORY OF ELASTICITY AND PLASTICITY
End Semester Examination – 18 Dec 2023

Time: 3 hours | Maximum Marks: 50

Note: Answer all questions; Assume missing data after stating; Read the questions carefully

1. Consider the problem of a thick tube $a < r < b$, fixed at $r = b$ (outer periphery) and free at $r = a$. Along the free (inner) surface, the tube is subjected to uniform shear $\tau_{r\theta}$ which forms a couple M as shown in figure. Use the stress function $\phi = C\theta$ and determine the stress field. Also, *describe* how you will find the strain and displacement fields inside the cylinder. Consider a unit length of the tube. Given:



$$\sigma_r = \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2};\ \sigma_\theta = \frac{\partial^2 \phi}{\partial r^2};\ \tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \phi}{\partial \theta}\right);\ \varepsilon_r = \frac{\partial u}{\partial r};\ \varepsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r};\ \gamma_{r\theta} = \frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{v}{r}.$$ [6]

2. (a) Define *kinematically admissible displacement filed* and *statically admissible stress field*. [2]
   (b) Starting from the equations of equilibrium, derive the *principle of virtual work*. Use it to arrive at the *mathematical statement* of the *principle of stationary potential energy*. State the principle of stationary potential energy *in words*. [5]
   (c) Write the total potential energy of a simply supported beam of span l and flexural rigidity EI, subjected to a distributed transverse load $q(x)$. Obtain the *Euler equation* and *natural boundary condition*(*s*) of the problem. [5]

3. (a) What are the assumptions used in Saint Vénant's semi-inverse method? (b) What is the displacement field he arrived at? (c) What is the resulting stress field? [5]

4. (a) Derive the equilibrium equation of a thin membrane under uniform in-plane tension, subjected to a uniform lateral pressure and mention how this equation is useful in solving problems of prismatic bars under torsion. [3]
   (b) A prismatic bar of narrow rectangular section is subjected to a torque T. Use the membrane analogy to determine expressions for the maximum shear stress developed and the angle of twist per unit length.
   (c) Use the above to determine the maximum shear stress and the angle of twist per unit length of the channel section shown in Fig. 2 when subjected to a torque of 1.0 kNm. G = 20 GPa. [4+4]

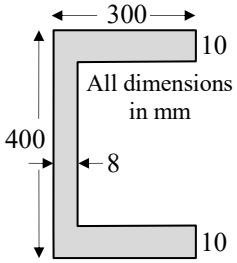


Figure 2

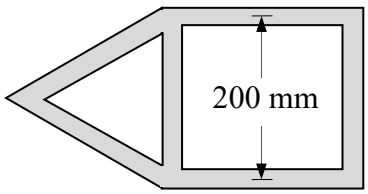


Figure 3

5. A thin walled stiffened shaft has a cross-section consisting of a square and an equilateral triangle joined together as shown in Fig. 3. It is subjected to a torque T = 10 kN m. If the wall thickness of the box is 10 mm throughout, determine the shear stresses in the walls and the stiffener and the angle of twist per unit length. [6]

6. (a) Name the *three ingredients* of *theory of plasticity*.
   (b) Explain with aid of uniaxial stress-strain diagrams: How you will distinguish between nonlinearly elastic and elastoplastic behaviours, and the difference between isotropic and kinematic hardening. [4]

7. (a) Write the von Mises yield criterion and sketch the yield surface in three-dimensional principal stress space. What is the diameter of the corresponding circle in the deviatoric plane?
   (b) Write the von Mises criterion for plane stress problems and sketch the yield surface on σ_1-σ_2 space.
   (c) How does it get modified subsequently in the case of isotropic and kinematic hardening? [8]