Department of Civil Engineering

NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

Winter Semester 2023-24

End-semester exam, 22 Apr 2024

Name:_____

Roll No:_____

CE6111E FINITE ELEMENT METHOD

Duration: 3 hours | Maximum Marks: [50]

Note: *Answer* ***all*** *the questions, each after reading carefully. Assume missing data after stating clearly*

1. (a) *Obtain* the interpolation function N_7 of the 12-noded isoparametric serendipity element shown in Fig. 1. Sketch its shape. (b) *Describe* how you will obtain the interpolation function N_2 of the same element (no need to derive the intermediate details). Sketch its shape. [6]

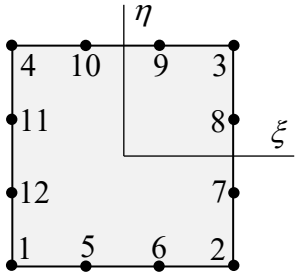
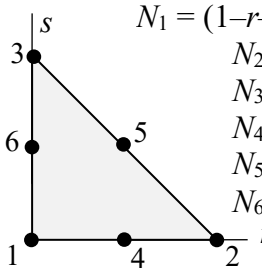


Figure 1



$N_1 = (1–r–s)(1–2r–2s)$

$N_2 = r(2r–1)$

$N_3 = s(2s–1)$

$N_4 = 4r\,(1–r–s)$

$N_5 = 4rs$

$N_6 = 4s\,(1–r–s)$

Figure 2a

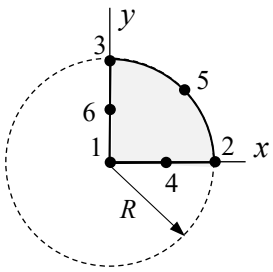


Figure 2b

2. For the isoparametric quadratic triangular element subtending 90^0 at the centre as shown in Fig. 2b, evaluate J_{12} of the Jacobian matrix at a point corresponding to $r = s = 0.25$. The nodal coordinates of the element are shown with R = 140 mm. Nodes 4, 5, 6 are at the middle of each edge. All N_i -s are given in Fig. 2a. [6]

3. Consider the triangular plane stress element depicted in Fig. 3. The local and global coordinates are also shown. Edge 1-2 makes an angle of 30° with respect to global X-axis. The element nodal displacement vector with respect to xy coordinates is obtained as: $\{u_1, v_1, u_2, v_2, u_3, v_3\}^T = [2\ \ -1\ \ 3\ \ 2.5\ \ 4\ \ -2]$ mm. The stress components (with respect to xy) are computed as: $\{\sigma_x\ \ \sigma_y\ \ \tau_{xy}\}^T = [85\ \ -30\ \ 15]$ MPa. (a) Find the displacement vector $\{U_1, V_1, U_2, V_2, U_3, V_3\}^T$ along the global coordinates. (b) Apply the stress transformation rule and find the stress components σ_X, σ_Y, τ_{XY} relative to the global coordinates. [6]

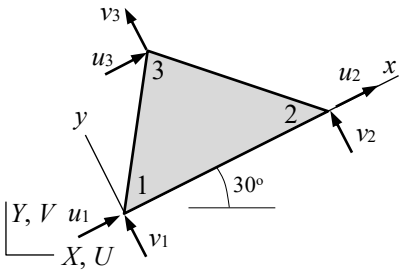


Figure 3

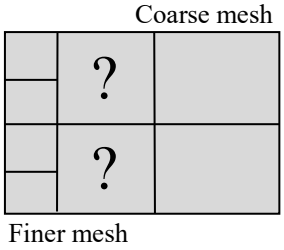


Figure 4

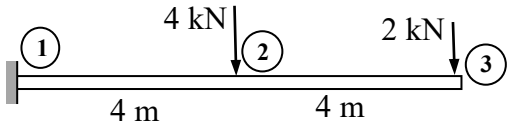


Figure 5

4. (a) Show a pattern by which you can move from a fine mesh to a coarse mesh using four-noded quadrilateral elements as shown in Fig. 4 (i.e., show mesh in the blank areas indicated by "?"). [3]

   (b) Show a possible mesh with at least 12 elements using four-noded quadrilateral elements to mesh a domain having the shape of a regular *pentagon*. [3]

5. The cantilever beam with constant flexural rigidity EI loaded as shown in Fig. 5 is to be analysed. Special constraints are kept at nodes 2 and 3 so that the transverse deflections v_2 and v_3 are equal, and the slopes θ_2 and θ_3 are also equal. Set up the reduced stiffness matrix and the corresponding load vector. Solve and find the deflections and slopes at nodes 2 and 3. [6]

6. Consider the following differential equation pertaining to a fixed-fixed beam

$$EI\frac{d^4v}{dx^4} = q(x),\ 0 < x < l,$$

and the associated boundary conditions $v(0) = v'(0) = 0$ and $v(l) = v'(l) = 0$. (a) Write the *weighted residual statement*. (b) Obtain the *weak form*. (c) Use the Galerkin's method and write expressions for the elements

of the stiffness matrix (K_{ij}). (d) Indicate the continuity requirement and show neat sketches of two of the shape functions if you discretise the beam into three elements. [6]

7. One end of a two-noded *beam element* is connected *rigidly* to a four-noded quadrilateral element on edge 2-3 as depicted in Fig. 6 at *quarter point*. The stiffness matrix of the beam element operates on the degrees of freedom $\mathbf{u}' = [u_5, v_5, \theta_5, u_6, v_6, \theta_6]^T$. Obtain the transformation matrix **T** which relates $\mathbf{u}'$ to $\mathbf{u} = [u_1, v_1, u_2, v_2, u_6, v_6, \theta_6]^T$. Also indicate how the same transformation matrix can be used to obtain the 7×7 stiffness matrix of the beam element with respect to the displacement vector **u**. [6]

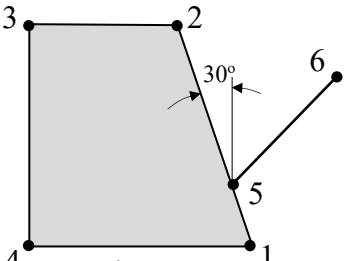


Figure 6

8. A three span continuous beam is loaded as shown in Fig. 7. Determine the slopes at *B*, *C* and *E* (centre) by using the *antisymmetric* nature of the beam and its loading. Given: $a = 6$ m, $EI = 2\times10^5$ kNm^2, $q = 10$ kN/m. (Hint: Analyse only one half of the structure) [6]

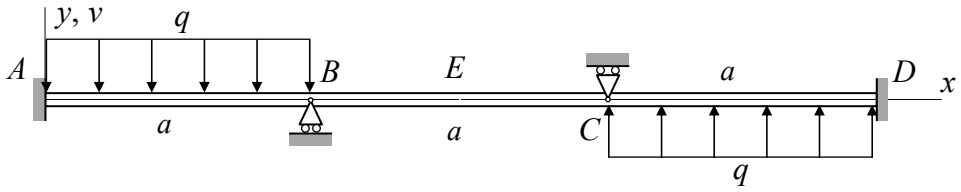


Figure 7

9. Briefly describe how a structure with cyclic symmetry can be analysed by analysing just one of the symmetric segments. [4]