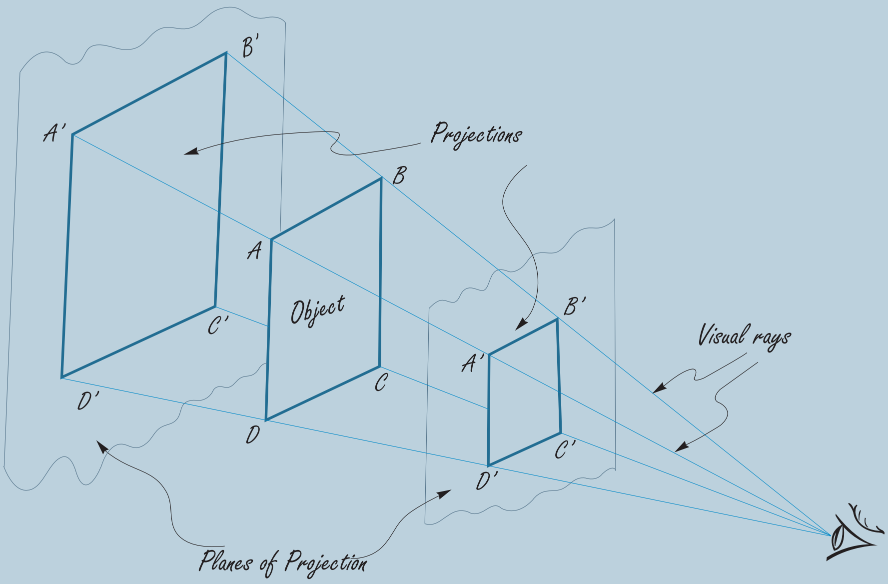
Engineering Graphics Self Taught



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PREFACE

Engineering graphics is a fundamental subject and every engineer, irrespective to the branch, should have thorough knowledge in this subject. The depth in graphics will improve visualizing capacity and confidence to learn any subject irrespective to its gravity.

Graphics will offer strength in communication skills. We have different mediums – gestic, verbal and graphics – for communication. In the first form, the ideas are communicated through gestures; for which training is required for even interpretation. Verbal medium can be either oral or written, which everyone uses consistently.

Graphics medium is very vast and it can be sub-divided into symbolic, multi-view representation and pictorial projection. In symbolic graphics, the matter is communicated through various symbols, which may not be easy to understand. Symbolic graphics was used by our ancestors to convey the story of their lives to the present world. Multi-view representations, popularly known as orthographic projections, are used by every technical hand for describing the design of any splendid structure or even any complex machinery. Sincere and regular training is essential for attaining profound knowledge in this. Pictorial projection has only one view and is commonly used by everybody including artists as it gives a three dimensional idea of the object. Computer graphics poses at a diverse dimension in the present era.

Every person uses graphics knowingly or unknowingly for communicating ideas. We must give due importance to special training in 'graphics' which will enhance the understanding of 'technical drawing' and can be combined with verbal medium to impart power.

This monogram titled 'Engineering Graphics Self Taught' is prepared for making the study of engineering graphics easy. All major topics in graphics are treated here giving importance to the basic theory and detailing the methodology of construction.

The motivation by Dr. M. A. Joseph for preparation of the monogram, the timely effort taken by Dr. Saleel Ismail and Dr. K. P. Murali for pointing out the discrepancies in the text and illustrations, and care taken for chapter-wise publication of the monogram in institute website by Dr. Vinay V. Panicker are acknowledged.

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Chapter 1

Principle of Projection

Chapter 1

Principle of Projection

1.1 Projection

Projection is defined as a geometrically represented image (visual image or figure) of an object obtained on a surface or plane.

1.2 Classification

Projections are basically classified in to two:

1. Perspective projection
2. Parallel projection

1.2.1 Perspective Projection

Perspective projection represents objects as perceived by the human eye(s) (refer Figure 1.1). It is a pictorial drawing by the intersection of observer's visual rays (lines of sight) converging on a plane (picture plane). The observer's eye - station point or point of sight - is located at a finite distance from the picture plane (refer Figure 1.1). Depending on the position of the picture plane, the size of the projection may vary.

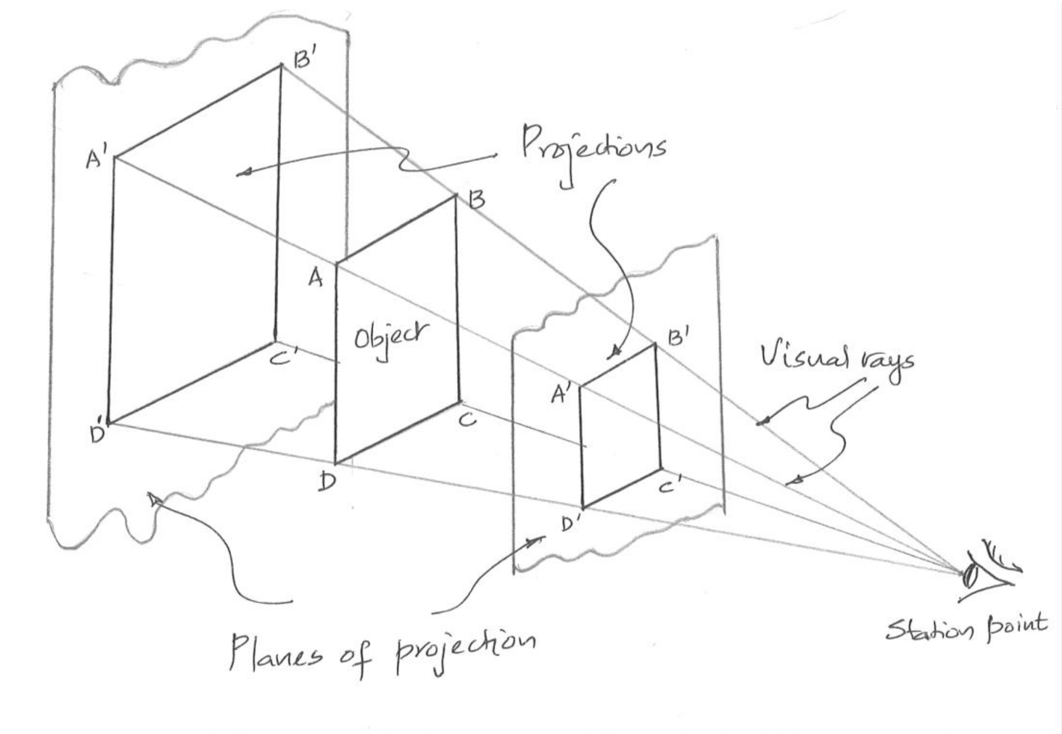


Figure 1.1 Principle of Perspective Projection

1.2.2 Parallel Projection

Parallel projection is obtained by assuming the observer at infinite distance from the object. Hence, the visual rays are considered as parallel to one another. These rays or lines of sight are used to project the object on a standard plane (refer Figure 1.2).

The object is projected to a plane by drawing straight lines from each and every point on the object. These lines used for projecting the object are 'projectors'. The plane to which the object is projected is the 'plane of projection'. All projectors are parallel to one another and perpendicular to the plane of projection. The image or view obtained on the plane is the 'projection'.

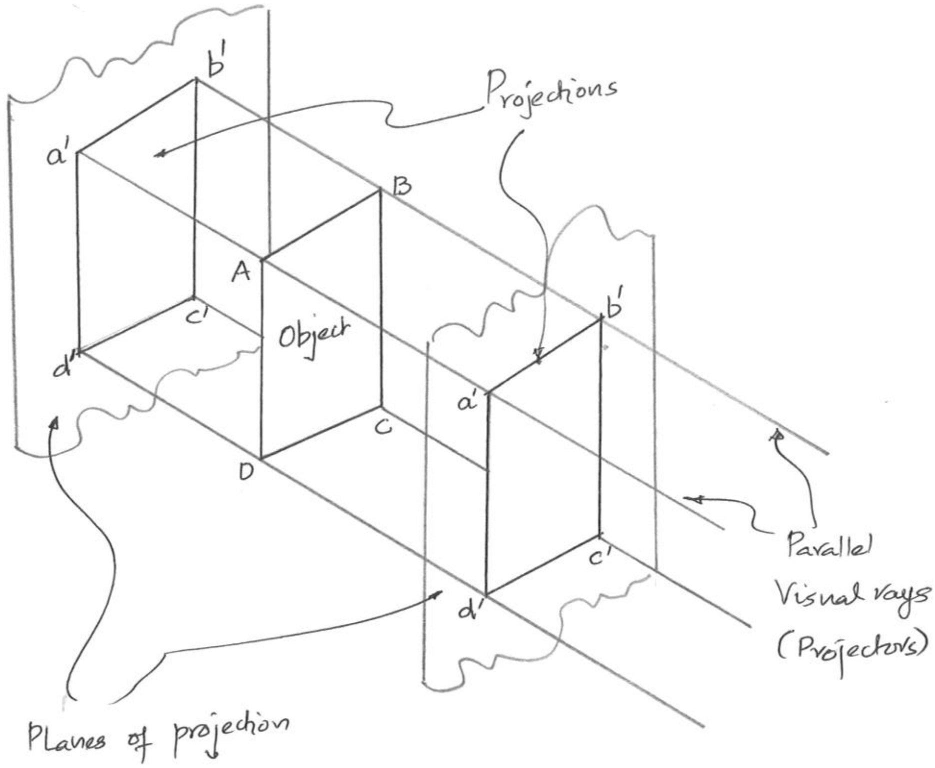


Figure 1.2 Parallel Projection

1.3 Orthographic Projections

In orthographic projection the visual rays (lines of sight) are parallel to one another and at right angle to the planes of projection.

1.4 Pictorial Projections

A pictorial projection is a single view representation, giving 3D idea of the object. They are:

1. Isometric projection
2. Perspective projection
3. Oblique projection

1.4.1 Isometric Projection

Isometric projection is a technique where three-dimensional objects are represented in two dimensional drawings. In isometric projection, the three coordinate axes appear equally shortened and the angle between any two of them is 120 degrees (refer Figure 1.3).

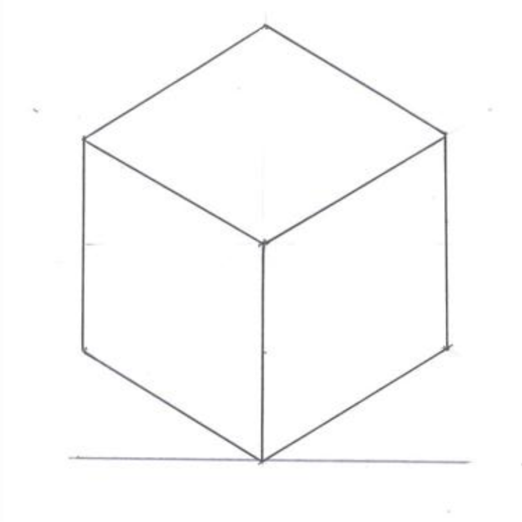


Figure 1.3 Isometric projection of a cube

1.4.2 Perspective Projection

Perspective projection represents objects as perceived by the human eye(s) (refer Figure 1.4).

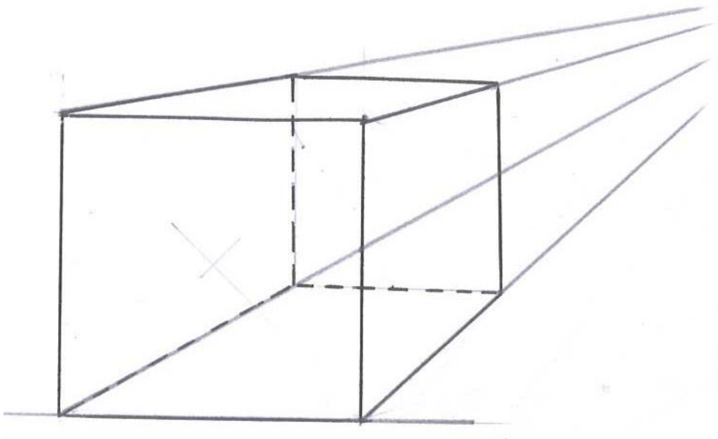


Figure 1.4 Perspective projection of a cube

1.4.3 Oblique Projection

Oblique projection is a method to represent 3D objects in 2D drawings in which the projection lines are drawn at 45°angle to the horizontal.

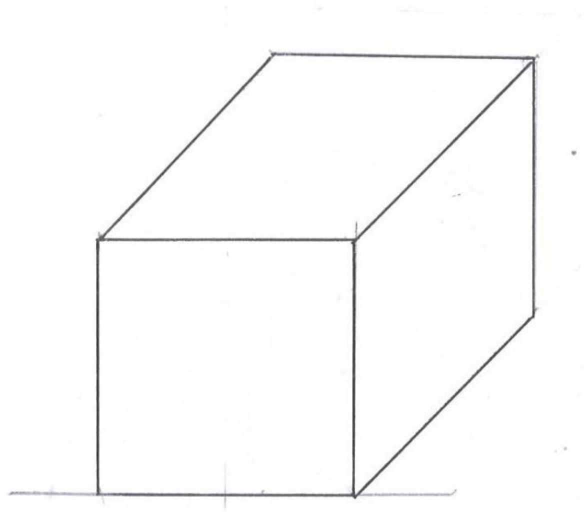


Figure 1.5 Oblique projection of cube

Chapter 2

Orthographic Projection

Chapter 2

Orthographic Projections

2.1 Orthographic Projections

In orthographic projection, an object is represented by projecting its views on imaginary orthogonal planes. Any object, irrespective to the dimensions, (1D, 2D or 3D objects) is converted to 2D drawings or projections.

2.1.1 Reference planes

Principal planes - horizontal plane and vertical plane –are the main reference planes used in orthographic projections.

Profile plane, auxiliary vertical plane, and auxiliary inclined plane are also used as reference planes when two views of the object are not sufficient.

2.1.2 Principal Planes

Horizontal and Vertical planes are the principal planes used in orthographic projections (refer Figure 2.1).

2.1.3 Horizontal Plane (HP)

A plane of reference which is assumed to be parallel to the plane along the horizon or a plane which is perpendicular to the gravity field at a place. In orthographic projection, there is only one horizontal plane.

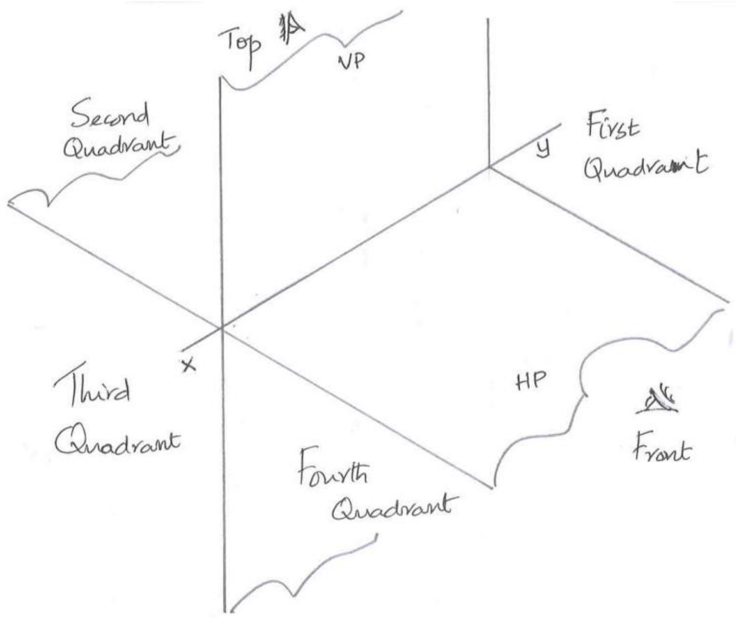


Figure 2.1 Principal Planes and Quadrants

2.1.4 Vertical Plane (VP)

A reference plane which is assumed to be along or parallel to the gravity field. This plane will be perpendicular to the horizontal plane.

2.1.5 Auxiliary Planes

Auxiliary Vertical Plane (AVP)

Planes perpendicular to HP but inclined to VP are AVPs. Projection on an AVP is called as auxiliary front view.

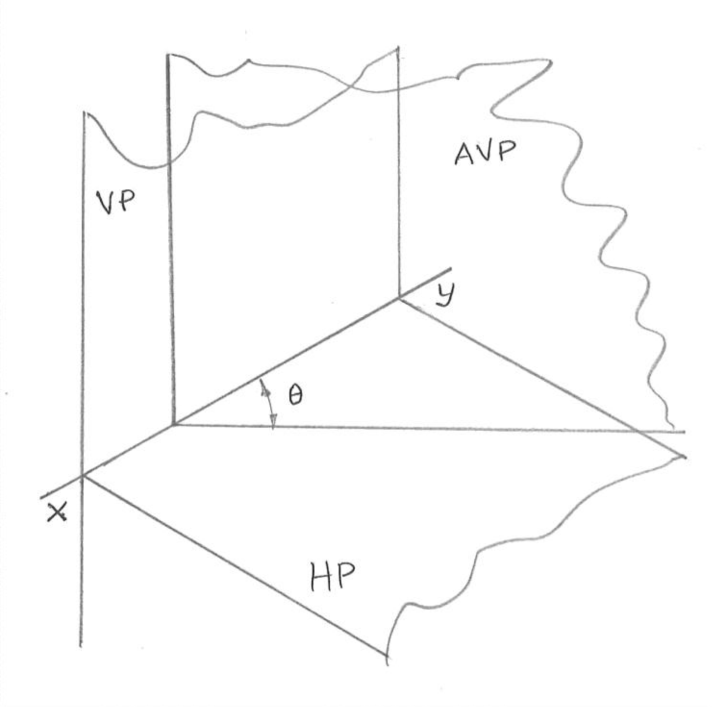


Figure 2.2 Auxiliary Vertical Plane

Auxiliary Inclined Plane (AIP)

Planes perpendicular to VP and inclined to HP are coming under this category. The projection on an AIP is auxiliary top view.

Profile planes (PP)

Planes perpendicular to both horizontal and vertical planes are profile planes. Projections on profile planes are known as side views.

2.1.6 Main Reference Line, x-y

The line of intersection of Horizontal plane (HP) and Vertical plane (VP) is the main reference line, x-y (refer Figure 2.1).

The lines of intersection of auxiliary planes with principal planes are auxiliary reference lines. While drawing auxiliary projections, auxiliary reference lines are used for representing projections.

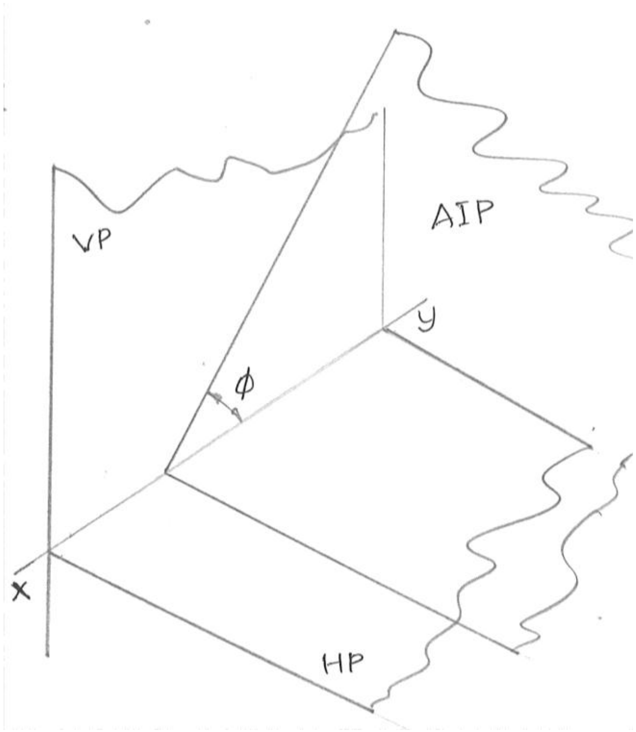


Figure 2.3 Auxiliary Inclined Plane

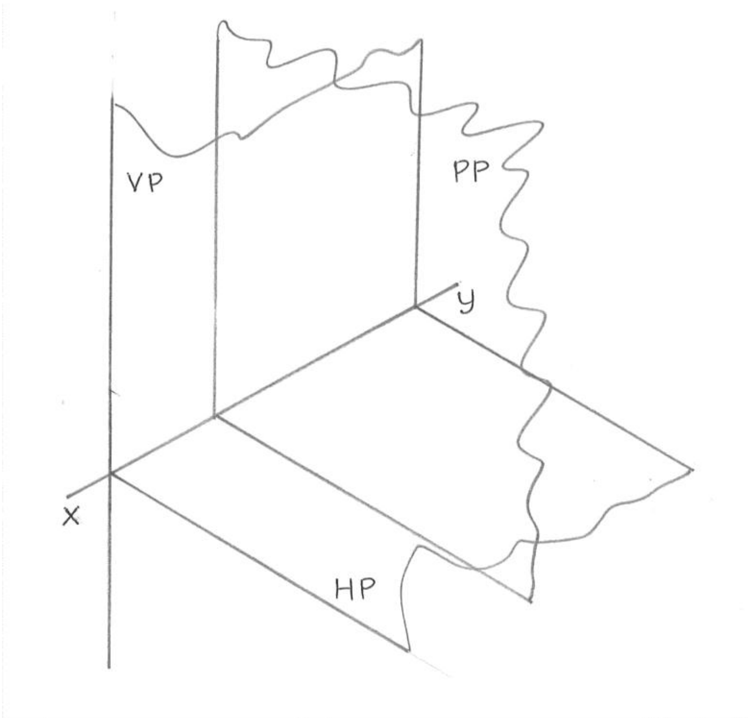


Figure 2.4 Profile Plane

2.1.7 Ground Plane (GP) and Ground Line (*g-l*)

A plane parallel to HP, assumed to be attached at the bottom most edge of VP is ground plane.

The line of intersection of GP with VP is called as ground line (*g-l*).

Basic characteristics of planes of projection

- ***Planes are assumed to have enormous area so that any object irrespective to the size can be projected to the plane.***
- ***Planes do have negligible thickness, such that they appear as lines while observing along the plane.***
- ***Planes are assumed to be transparent, so that irrespective to the quadrant where the object situates the observer can view it directly from both front and top.***
- ***Planes are not rigidly attached to the other plane(s), such that one plane can be rotated about the line of intersection to make coinciding with the other.***

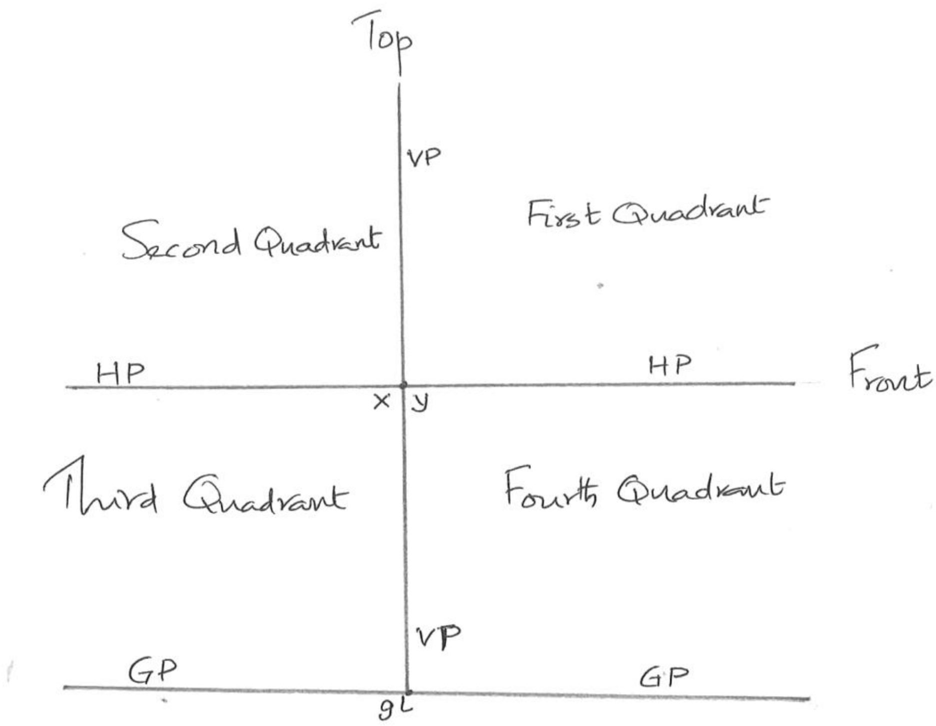


Figure 2.5 Principal Planes, Ground Plane and Quadrants viewing along x-y

2.1.8 Quadrants

The whole space available is divided in to four quadrants using HP and VP. Viewing along x-y, starting from above HP and in front of VP, the numbering of quadrants starts, which is the First quadrant. Moving in anticlockwise direction, above HP and behind VP is the Second quadrant; below HP and behind VP, the Third quadrant, and below HP and in front of VP, the Fourth quadrant (refer Figure 2.1, Figure 2.5)

As GP is attached to the bottom end of VP, there is no space below GP or objects are assumed to be situating either in the First or Second or Third or Fourth quadrant.

2.2 Multi-view Representations

Multi-view representations show more than one standard 2D views of an object. 3D objects are represented by different views or projections on imaginary planes.

In orthographic projections six views (Figure 2.6) of an object can be represented. They are: front view, top view, side view from left (side view-left), side view from right (side view-right), bottom view and back view (refer Figure 2.7). However most common views are top view and front view.

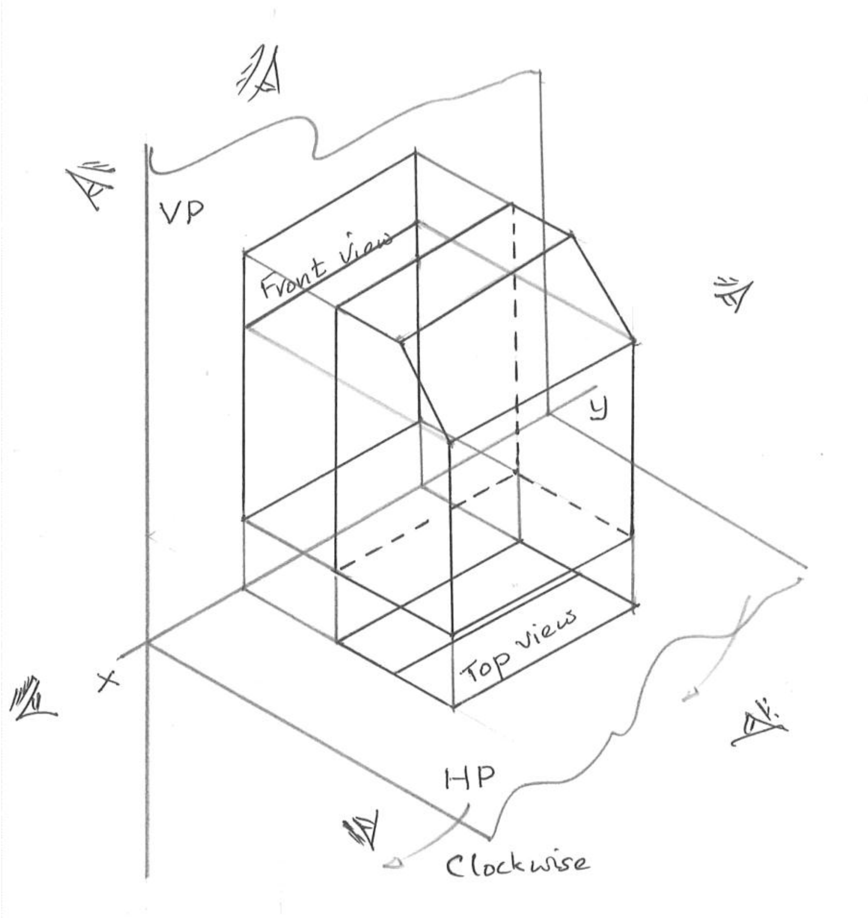


Figure 2.6 Directions of Observation of an Object in the First Quadrant

2.3 First Angle Projection

Projection of any object drawn assuming the object in the first quadrant is First Angle Projection. As per recommendation of Bureau of Indian Standards First angle projection is followed in India. Figure 2.7 shows the symbol for first angle projection (two views of the frustum of cone).

2.4 Third Angle Projection

Projection of any object drawn assuming the object in the third quadrant is Third Angle Projection. Figure 2.8 shows the symbol of third angle projection.

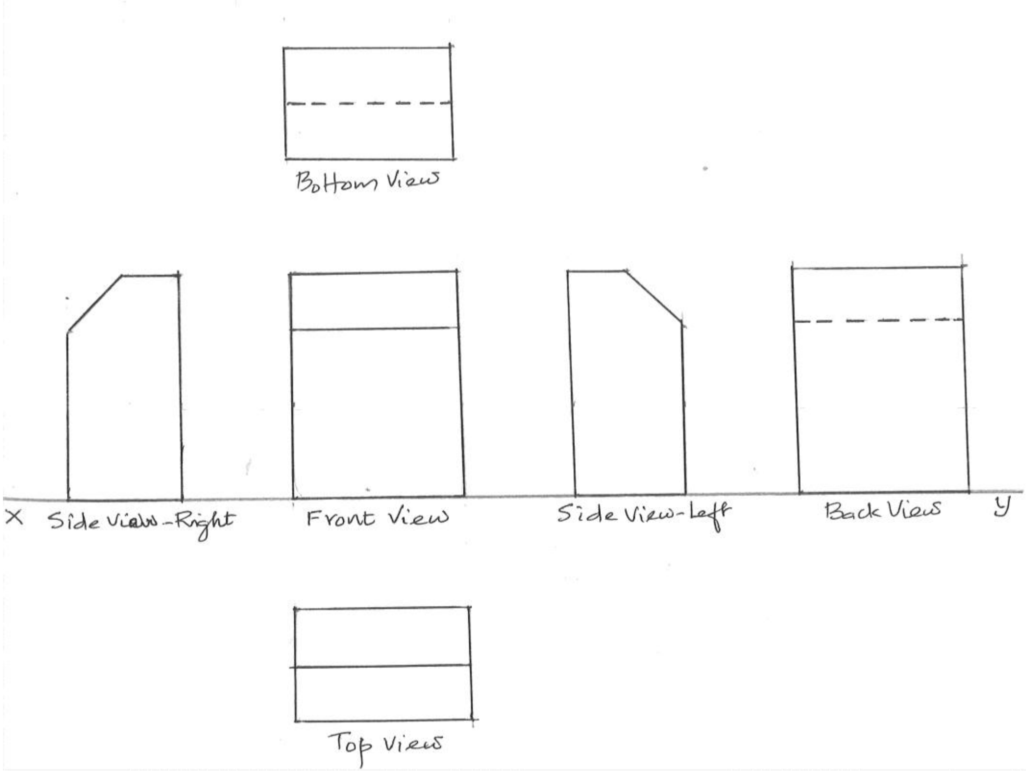


Figure 2.7 Six Views of the Object in Figure 2.6

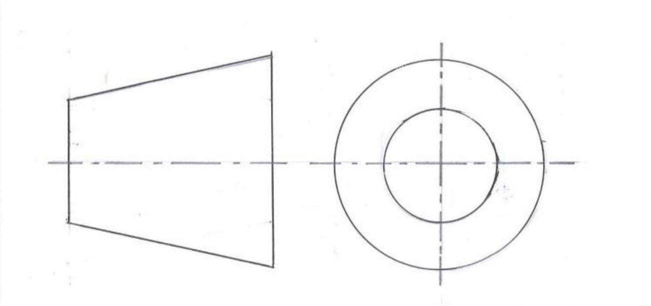
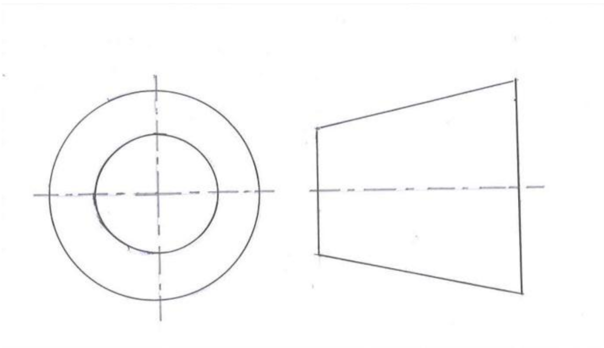


Figure 2.7 Symbol of First angle projection



Figur 2.8 Symbol of Third angle projection

Chapter 3

Projections of Points

Chapter 3

Projections of Points

3.1 Point, the simplest Object

A point will be the simplest object available. It can be treated as a zero dimensional object (no length, no breadth, and no height). A point can be denoted by any English alphabet in graphics.

A point can situate in any of the quadrant, say First or Second or Third or Fourth.

3.2 Point is in the First quadrant

A point is in the first quadrant (Figure 3.1). It is 50 mm above HP and 30 mm away from VP. Draw the projections.

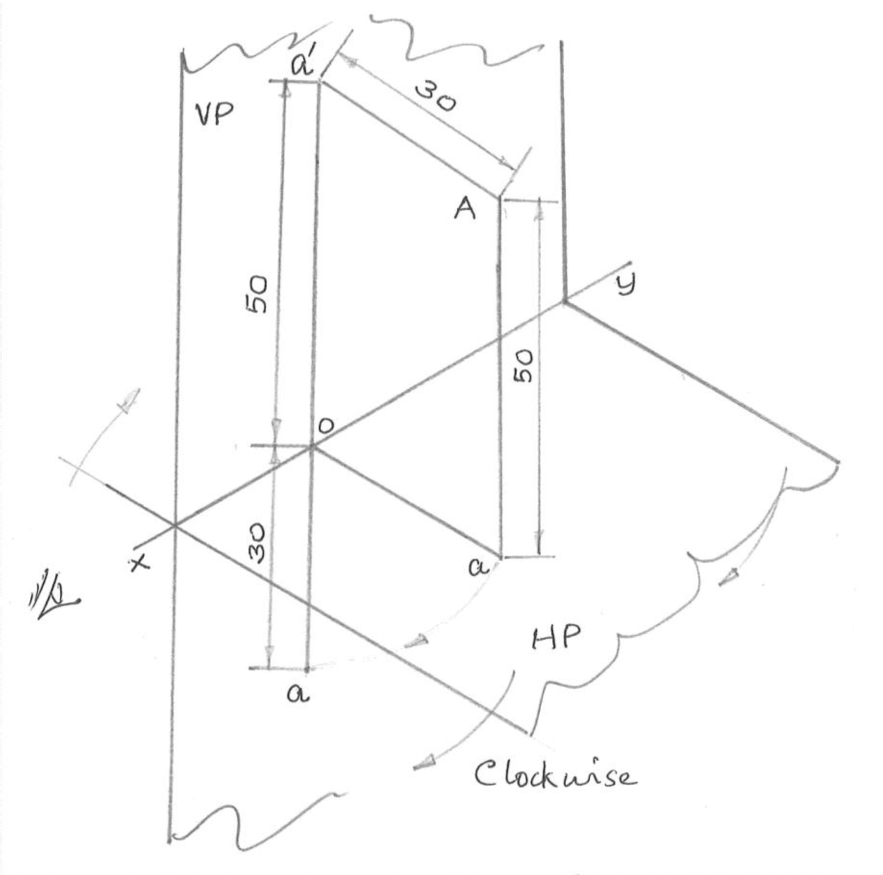


Figure 3.1 Point in the first quadrant

Solution

Draw projectors – straight lines – from point A to both HP and VP. The point at which the projector meets on HP is denoted as ***a***, known as the projection of A on HP, the top view or plan of A.

The point at which the projector meets on VP is denoted as ***a′***, known as the projection of A on VP, the front view or elevation of A.

Being Aa and Aa′ are projectors drawn from A to HP and VP, respectively ∟Aao and ∟Aa′o are right angles. ∟aoa′ is also a right angle as it is the angle between HP and VP. In a four sided figure, if three angles are right angles, the fourth angle will also be a right angle. Or, Aaoa′ is a rectangle.

The projections on HP and VP are obtained. These projections are on two different planes which are at right angle. Observing along the x-y, VP may be assumed to be stationary and HP may be rotated in the clock-wise direction about x-y, so that HP can be made coinciding with VP (Alternatively, HP may be made stationary and VP may be rotated in the anti-clockwise direction will give the same results). The final positions of projections, top view will be below x-y and front view will be above x-y (refer Figure 3.2).

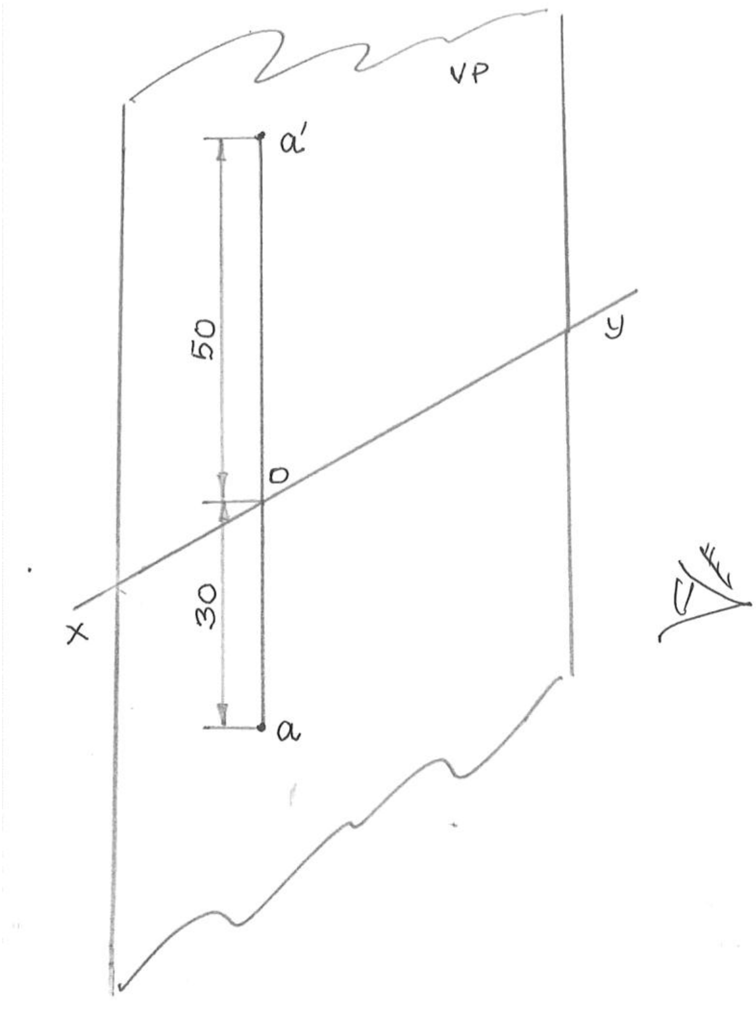


Figure 3.2 Relative position of top view and front view with respect to x-y after making HP conciding with VP.

The distance of A from HP and distance of ***a′*** (front view) from x-y are equal, and the distance of A from VP will be equal to the distance of ***a*** (top view) from x-y. [Referring Figure 3.1, Aaoa′ is a rectangle and hence opposite sides are equal. Distance Aa=distance a'o and Aa'=ao]

Hence, it is concluded that when the point (object) A is in the first quadrant, with respect to x-y, its top view will be below x-y and front view will be above x-y.

When a point is on the HP (distance from the point to HP is zero), its front view will be on x-y (the distance from front view to x-y is zero) and if the point is on the VP its top view will be on x-y.

- ***The distance of the front view to x-y will be equal to the distance of the point from HP, and the distance of the top view from x-y will be equal to the distance of the point from VP.***
- ***The top view, the projection of a point on HP, is indicated by the small letter of the name of the point (if point is A by a) and the front view, the projection on VP, by the small letter with a dash (if point is A by a').***
- ***When a point is in the first quadrant, the top view will be below x-y and front view, above x-y.***
- ***If a point is on the HP, its front view will be on x-y line.***
- ***If a point is on the VP, its top view will be on x-y line.***

Construction of projections (Figure 3.3)

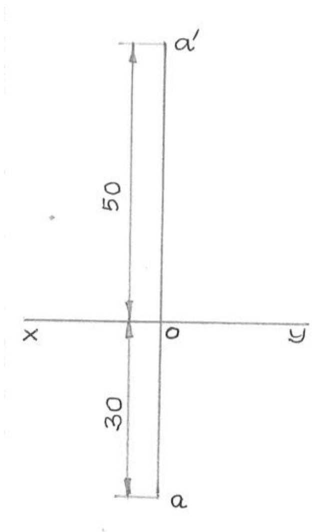


Figure 3.3 Projection of point A when it is in the first quadrant

Draw x-y taking a convenient length. Draw a perpendicular line (assumed to be projector) to x-y. Indicate a point ***a′*** on the projector at a distance of 50 mm above x-y. Indicate the point ***a*** on the projector 30 mm below x-y. Now the projections are ready. Show the dimensions as shown in Figure 3.3 using aligned system of dimensioning. Really in Figure 3.3, ***a'o*** and ***ao*** are projections of the projectors Aa and Aa′, respectively. But these are also treated as projectors only.

3.3 Point is in the Second quadrant

A point B is 50 mm above HP and 30 mm behind VP. Draw the projections.

When the point is in the second quadrant (Figure 3.4), both the projections will be above x-y (Figure 3.5). The distance of front view ***a′*** from x-y is 50 mm while that of top view ***a*** from x-y is 30 mm. These are respectively the distances of the point from HP and VP.

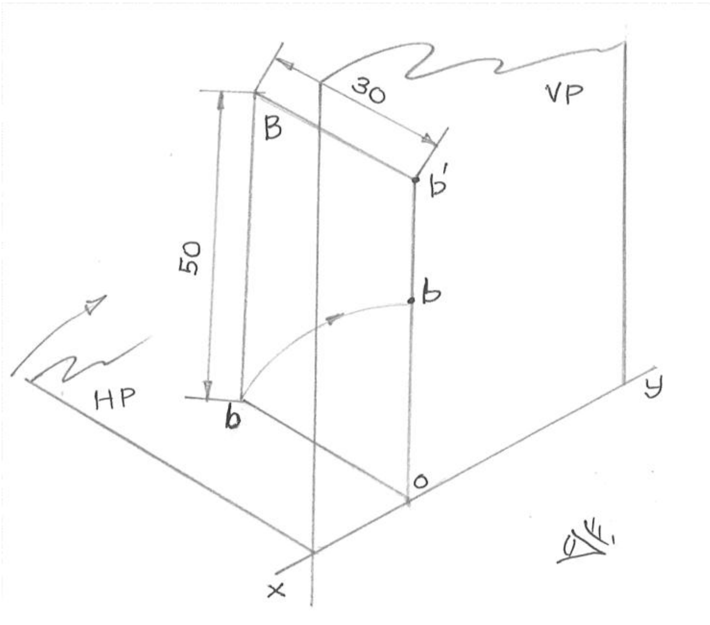


Figure 3.4 Point is in the second quadrant

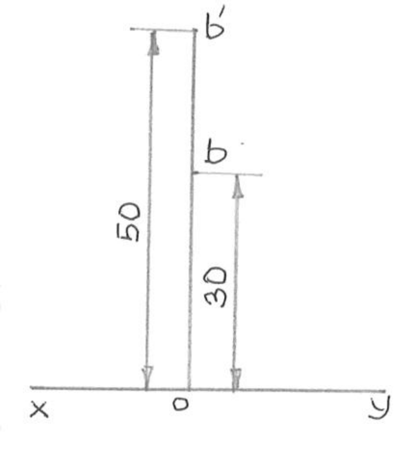


Figure 3.5 Projections of B when it is in the second quadrant

- ***When a point is in the second quadrant, both the views, the front view and top view, will be above x-y.***

3.4 Point is in the Third quadrant

A point C is 50 mm below HP and 30 mm behind VP. Draw the projections.

When the point is in the third quadrant (Figure 3.6), the top view will be above x-y and front view, below x-y.

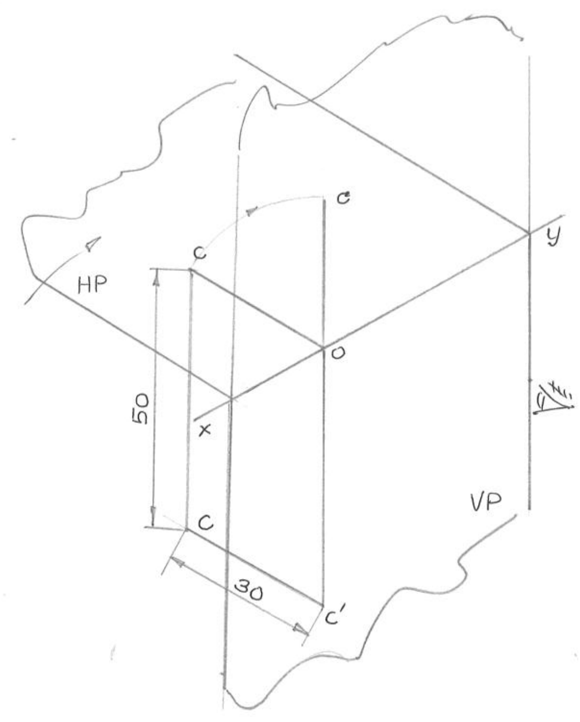


Figure 3.6 Point is in the third quadrant

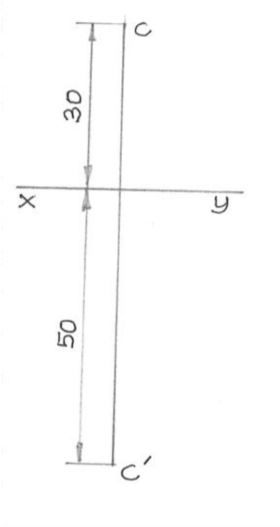


Figure 3.7 Projections when the point is in the third quadrant

The point C is 50 mm below HP and 30 mm behind VP. After obtaining the projections on HP and VP independently, both the projections are made on the same plane (Refer 3.7).

The distance of the front view to x-y will be equal to the distance of the point from HP, and the distance of the top view from x-y will be equal to the distance of the point from VP.

- ***When a point is in the third quadrant, the top view will be above x-y and front view, below x-y.***

3.5 Point is in the Fourth quadrant

A point D is 50 mm below HP and 30 mm in front of VP. Draw the projections.

When the point is in the fourth quadrant (Figure 3.8), both the views, front and top view, will be below x-y (Figure 3.9).

The distance of the front view to x-y will be equal to the distance of the point from HP, and the distance of the top view from x-y will be equal to the distance of the point from VP.

- ***When a point is in the fourth quadrant, both the views, front view and top view, will be below x-y.***
- ***Irrespective to the quadrant, the distance of the front view to x-y will be equal to the distance of the point from HP, and the distance of the top view from x-y will be equal to the distance of the point from VP.***

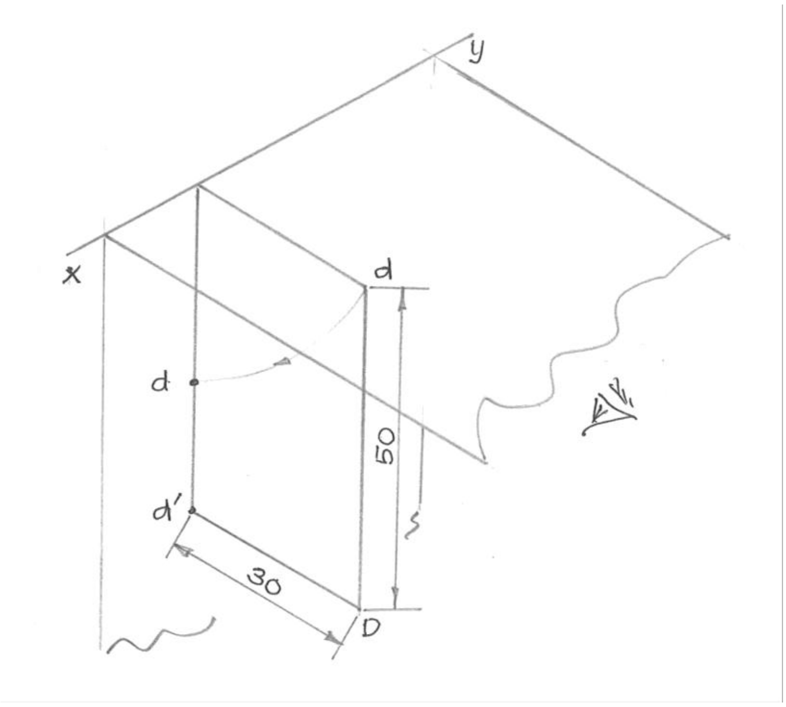


Figure 3.8 Point is in the fourth quadrant

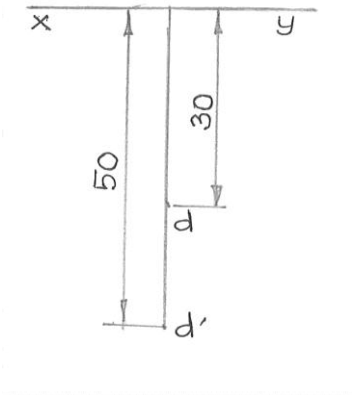


Figure 3.9 Projections of a point when it is in the fourth quadrant

Chapter 4

Projections of Straight Lines

Chapter 4

Projections of Straight Lines

4.1 Straight Line

A straight line is a collection of points which are arranged through the shortest between the end points. A straight line is a single dimensional object. Straight lines are referred based on end points; named by English alphabets usually.

4.2 Bureau of Indian Standards recommends use of First angle projection in India. Based on this, every object is assumed to be situating in the first quadrant. But, if an object is given as situating in a particular quadrant it should be considered as in that quadrant. Or if the quadrant is not specified in a particular case it is assumed as the object is in the first quadrant.

4.3 Classification of straight lines

Straight lines are classified according to their position in the space with respect to the principal planes.

1. Straight lines parallel to both HP and VP
2. Straight lines perpendicular to one principal plane and parallel to the other
3. Straight lines inclined to one principal plane and parallel to the other
4. Straight lines inclined to both HP and VP

4.4 Straight line parallel to both HP and VP

Problem 4.1

A straight line AB is parallel to both HP and VP. The distance from HP to the line is 30 mm and that of VP is 40 mm. Draw the projections of the line.

Analysis

As the quadrant is not specified, the line is assumed to be in the first quadrant. The straight line in 3D is shown in Figure 4.1. The line AB is parallel to both HP and VP. The distance from HP to the line is 30 mm and that from VP is 40 mm. Being a straight line end points can be taken for analysis. Projectors may be drawn from A and B to both the principal planes, HP and VP. Projector from A will meet HP at ***a*** and that on VP at ***a′***. Similarly, projectors from B to HP and VP will meet at ***b*** and ***b′***, respectively on HP and VP. Joining ***a*** and ***b***, and ***a′*** and ***b′***, the top view and front view of AB are available. Observing along x-y, HP may be rotated about x-y in the clockwise direction and HP may be made coinciding with VP. Now both the projections are on a single plane.

In the four sided figure ABba, Aa and Bb are projectors from A and B, respectively to HP and are equal in length. ∟Aab and ∟Bba are right angles. Hence ABba is a rectangle. Length of ***ab***, the top view is equal to length of the straight line, AB being opposite sides.

When a straight line is parallel to a plane, length of its projection on that plane will give true length. Since the straight line AB is parallel to VP also the length of front view, ***a'b'*** will be equal to true length of the line.

Over and above, both the projections are parallel to x-y.

The 3D view after making both planes coinciding and final projections are shown in Figure 4.2 and Figure 4.3, respectively.

- ***When a line is parallel to a plane its projection on that plane will be equal to true length of the line.***
- ***If a line is parallel to a principal plane, the projection on the other principal plane will be parallel to x-y.***

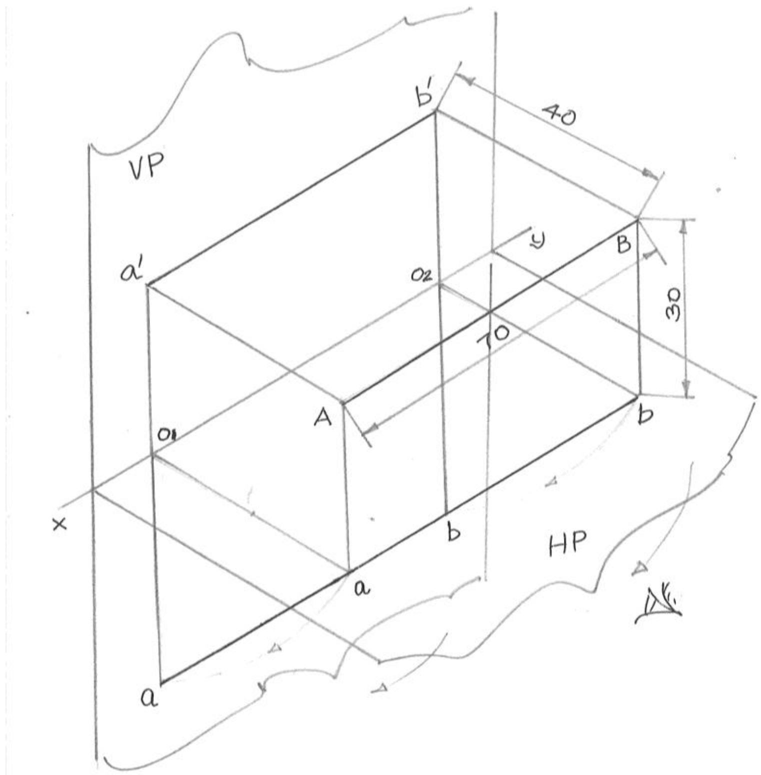


Figure 4.1 Straight parallel to both HP and VP

Solution

All problems from projections of straight line are solved after fixing one end or at least a point on the straight line. Fixing one end is nothing but both top view and front view of a point will be known and can be drawn as the first step.

Here the line is parallel to both HP and VP. The distance from the line to HP is 30 mm and that from VP is 40 mm. All points on the line are equi-distant from HP, and all the points are equi-distant from VP also. The solution can be completed easily by drawing the projections of end points.

Draw x-y conveniently taking a distance more than 70 mm, which is the length of the line. The projections of one end, say the point A, may be drawn following the projections of points. The front and top views of A may be indicated as 30 mm above x-y and 40 mm in front of x-y, respectively. It is already proved that the projections, ***ab*** and ***a'b'*** are of equal in length which is the true length of the line.

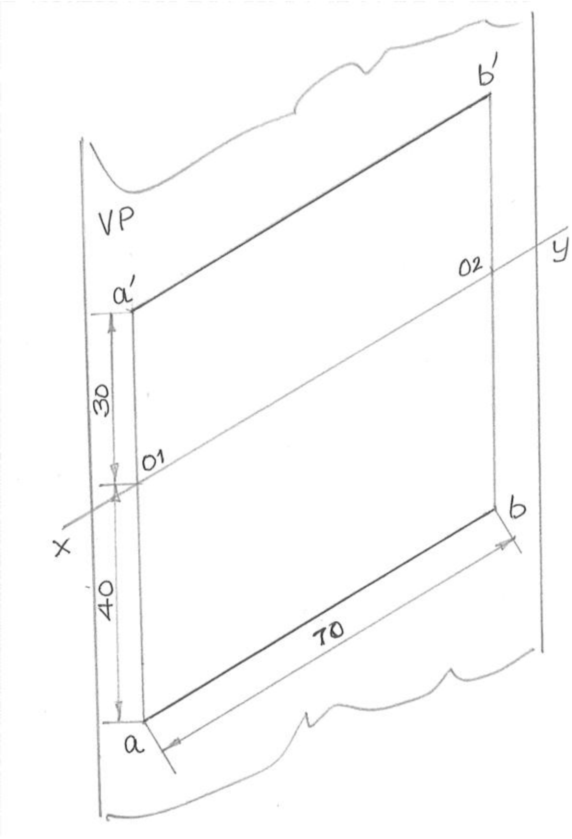


Figure 4.2 Both planes are made coinciding after obtaining the projections

Through ***a*** and ***a'*** lines may be drawn parallel to x-y. ***b*** and ***b'*** may be marked on these lines 70 mm (true length) away from ***a*** and ***a'***, respectively. The top view, ***ab*** and front view ***a'b'*** may be finished (refer Figure 4.3) properly as those are the required projections.

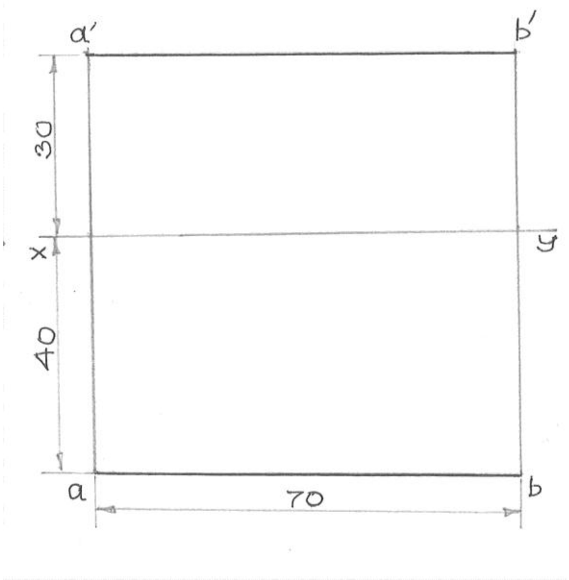


Figure 4.3 Projections of line AB which is parallel to both HP and VP

4.5 Straight lines perpendicular to one plane (and parallel to the other)

Problem 4.2

Straight line AB, 60 mm long is perpendicular to HP and parallel to VP. Its distance from VP is 20 mm and one end (end B) of the line is on HP.

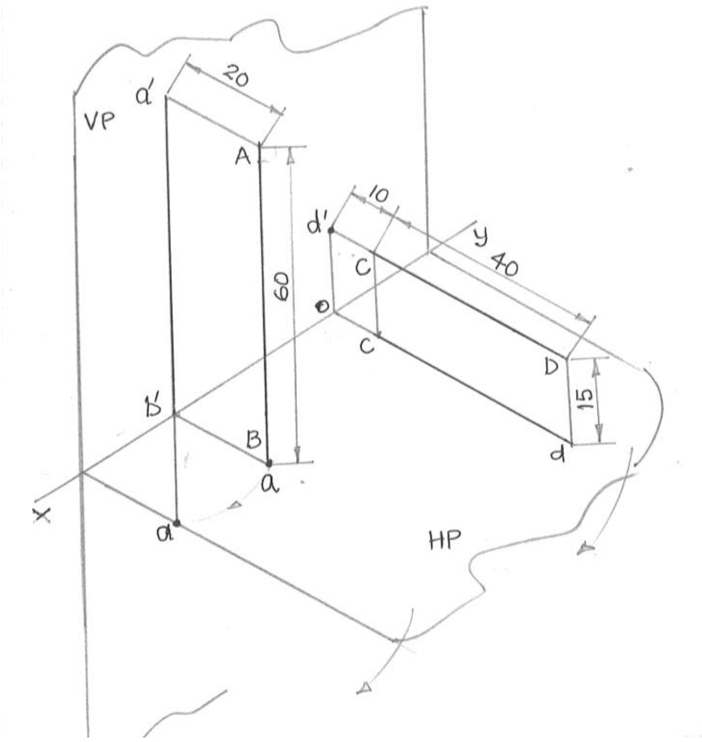


Figure 4.4 Straight lines perpendicular to one plane

Analysis

Projectors may be drawn from end A and B to both HP and VP. It may be noted that as B is on HP the length of the projector from B to HP is zero. The projector drawn from A to HP coincides the line and it meets at the same point where the projector from B meets HP or that is B only as per the 3D figure. The top view of the line is a mere point as ***a*** and ***b*** are coinciding. While drawing projectors from A and B to VP, the rectangle ABb′a′ is available. The length of a′b′ and AB are equal (already proved as AB is parallel to VP).

- ***When a straight line is perpendicular to a plane, its projection on that plane will be a point.***

Solution

x-y may be drawn taking a convenient length. Draw a line perpendicular to x-y which is the projector. Indicate ***b*** and ***b′*** on the projector at a distance of 20 mm below x-y and on the x-y, respectively. The top view of A, ***a*** may be marked coinciding with ***b***. ***a′*** may be marked on the projector at a distance of 60 mm above x-y. Front view ***a′b′*** may be finished properly (refer Figure 4.5).

Problem 4.3

The projections of the straight line CD can also be drawn following the same procedure. Its length is 40 mm and it is 15 mm above HP. One end C is 10 mm in front of VP.

The line CD is perpendicular to VP and parallel to HP. The top view will have true length and front view will be a mere point. The projections are shown in Figure 4.5.

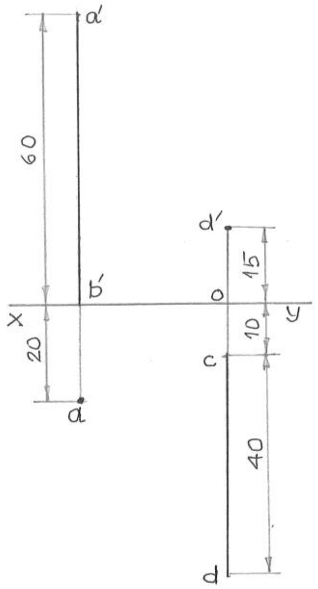


Figure 4.5 Straight lines perpendicular to HP or VP

4.6 Straight line inclined to one principal plane and parallel to the other

Problem 4.4

A straight AB, 70 mm long, is inclined at 45^o to VP and parallel to HP. The end A is 20 mm away from HP and 10 mm away from VP. Draw the projections.

Analysis

The straight line is inclined to VP and parallel to HP (refer Figure 4.6 for 3D representation). As the straight line is parallel to HP, length of top view will be equal to true length of the line, 70 mm. Assume the point of intersection of projectors ***aa'*** and ***bb'*** with x-y are o_1 and o_2, respectively. The trapezia ***ABb'a'*** and abo_2o_1 are identical. Hence the inclination of ***ab*** with x-y is 45^o, which is the true inclination of AB with VP.

It can also be observed that ***a'b'*** has a reduced length which is commonly known as projected length of AB on VP. But it is parallel to x-y, as the line is parallel to HP.

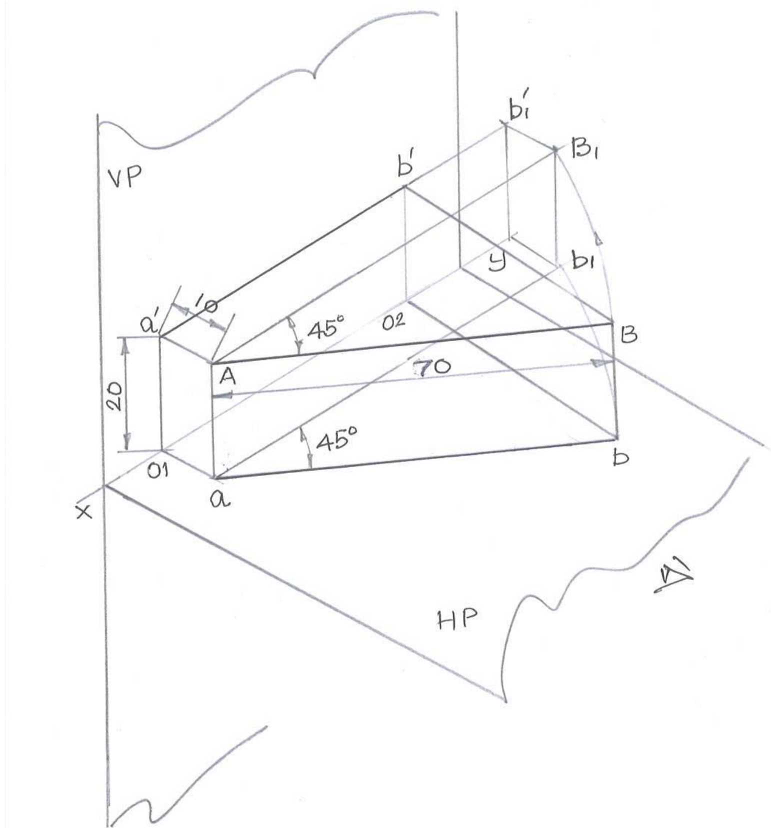


Figure 4.6 Straight line inclined to one plane and parallel to the other

- ***When a straight line is parallel to HP and inclined to VP, the top view will be having true length of the line, its inclination with x-y will be the true inclination of the line with VP and front view will be parallel to x-y.***
- ***When a straight line is parallel to VP and inclined to HP, the front view will be***

equal to true length of the line, its inclination with x-y will be the true inclination of the line with HP and top view will be parallel to x-y.

Solution

The line AB is parallel to HP and inclined to VP. Assuming the line is parallel to both HP and VP, without changing the position of end A, projections can be drawn (refer Figure 4.6).

Draw x-y taking a convenient length which is more than 70 mm. The projections of A, ***a*** and ***a***', may be indicated after drawing the projector. Draw $\boldsymbol{ab_1}$ and $\boldsymbol{ab_1'}$ taking a length of 70 mm (true length) parallel to x-y. AB_1 is rotated about A to make the straight line inclined to VP keeping parallel to HP. B_1 moves parallel to HP or its front view, $\boldsymbol{b_1'}$ moves along a line parallel to x-y (refer Figure 4.6).

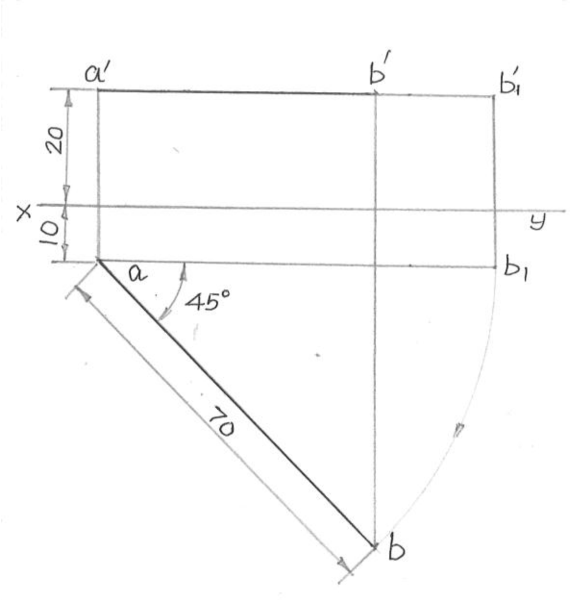


Figure 4.7 Projections of line inclined to VP and parallel to HP

Correspondingly ***ab***=70 mm will make 45^o with x-y. From **b** a projector may be drawn to meet with the front view $\boldsymbol{a'b_1'}$ at ***b′***. The front view ***a'b'*** and top view ***ab*** may be finished properly (refer Figure 4.7).

The above construction can be simplified by avoiding the step in which the line is assumed to be parallel to both HP and VP. Keeping this step in mind, draw the projections of A. At ***a*** an angle of 45^o to x-y is constructed and ***ab*** is drawn with a length of 70 mm. Through ***a′*** a line is drawn parallel to x-y on which front view of the line falls. A projector is drawn from ***a*** to intersect with the parallel line drawn through ***a′***. The point of intersection of the projector with the parallel through ***a′*** is ***b′***. The front view ***a'b'*** and top view ***ab*** may be finished properly (refer Figure 4.8).

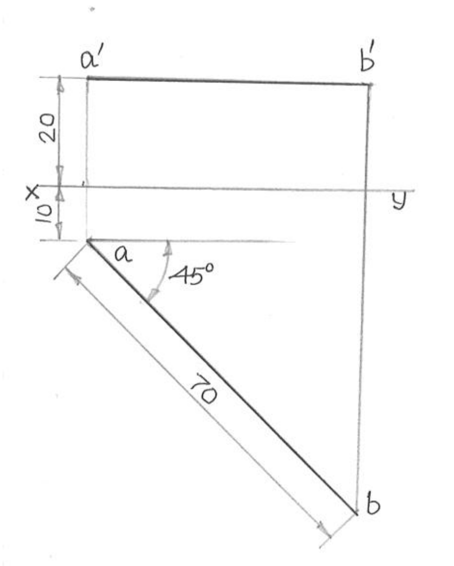


Figure 4.8 Projection of line inclined to VP and parallel to HP

Problem 4.5

A straight line AB, 75 mm long, is inclined at 30^o to HP and parallel to VP. Draw the projections if end A is on the HP and 20 mm in front of VP.

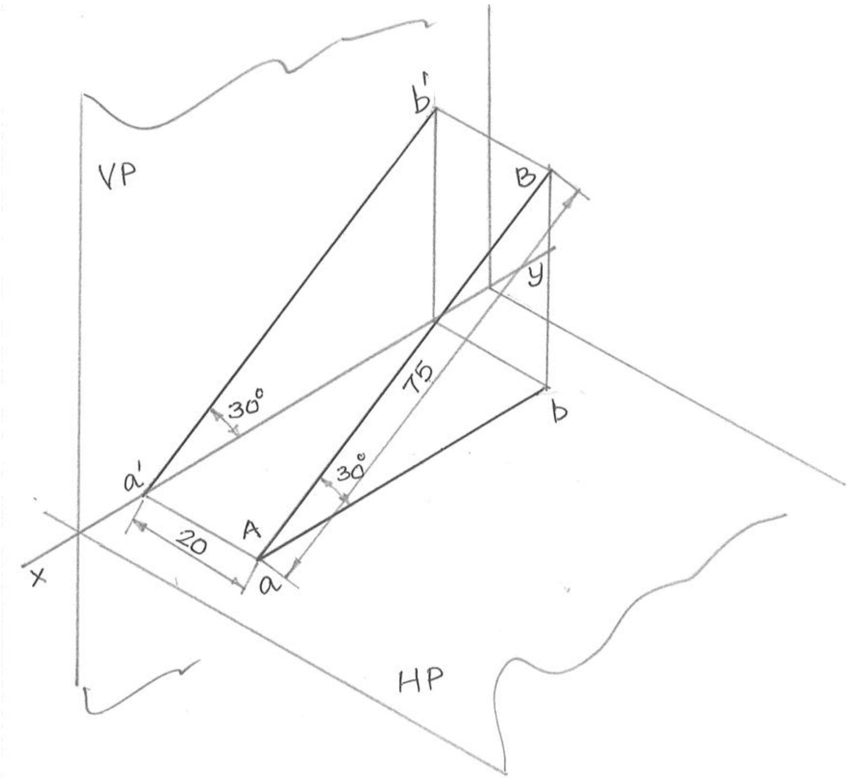


Figure 4.9 Line inclined to HP and parallel to VP

Solution

x-y line is drawn taking a convenient length. Being end A on the HP and 20 mm in front of VP, ***a′*** will on the x-y and ***a***, 20 mm below x-y. Draw a projector and mark

a' on x-y and ***a***, 20 mm below x-y. At ***a'*** construct an angle of 30^o to x-y. As the line is parallel to VP, the front view will have true length. ***b'*** may be marked on this line such that length of ***a'b'*** is 70 mm. A parallel line is drawn to x-y through ***a***. The point of intersection of the projector drawn from ***b***' and the parallel line through ***a*** is ***b***. The front view, ***a'b'*** and top view, ***ab*** of the line may be finished properly (Figure 4.10).

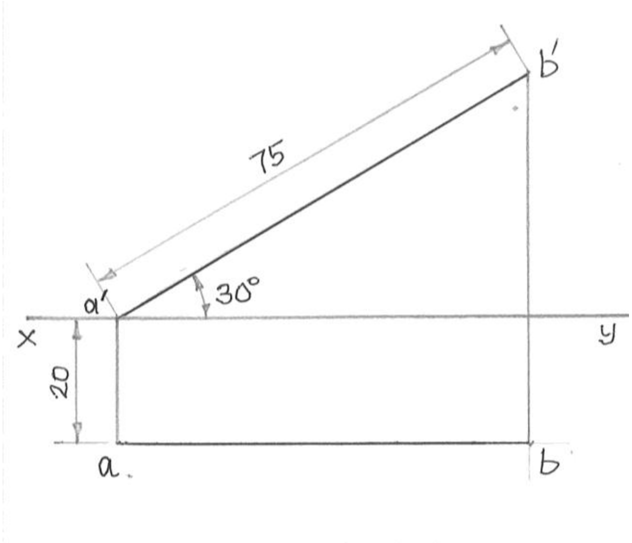


Figure 4.10 Projections of line inclined to HP

Problem 4.6

A straight line AB, 115 mm long, is inclined 30^o to VP and parallel to HP. Its one end A is in the first quadrant and the other end B, in the second quadrant. Draw the projections if the point C on the straight line 70 mm from A is on the VP and 20 mm away from HP (refer Figure 4.11).

Solution

Basically the straight line AB (ACB) with AC=70 mm, which is in the first quadrant and CB=45 mm, in the second quadrant.

Sufficiently long x-y line is drawn. ***c***, on x-y and ***c***', 20 mm above x-y may be marked after drawing a projector. A line ***ca***=70 mm is drawn from ***c*** at an angle of 30^o. ***c'a'*** may be drawn parallel to x-y by dropping projector from ***a*** and getting intersected on the parallel line through ***c'***.

ac is extended through 45 mm and ***b*** may be marked. While dropping a projector from **b** and getting it intersected with extended line ***a'c***', ***b'*** is available. Both front view and top view may be finished (refer Figure 4.12).

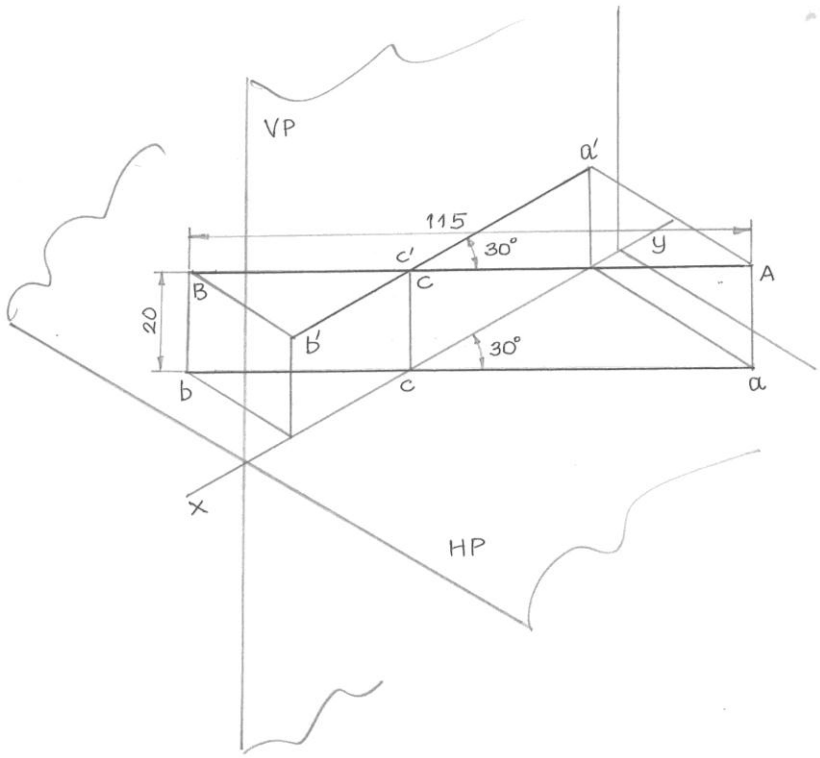


Figure 4.11 Straight line contained by more than quadrant

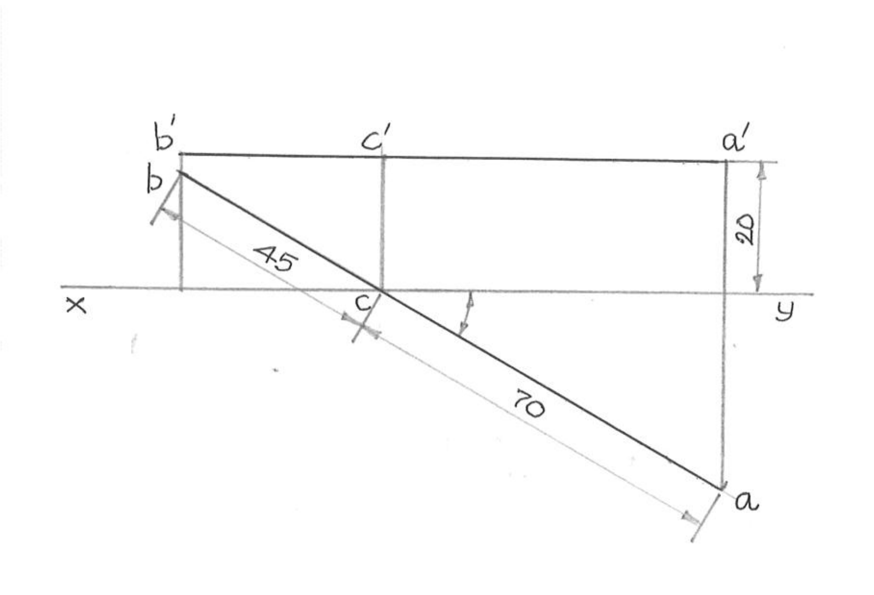


Figure 4.12 Projections of line contained by two quadrants

Problem 4.7

A straight line AB, 70 mm long, is inclined to HP and parallel to VP. The end A is 10 mm away HP and 20 mm away from VP. Draw the projections of AB if its top view measures 60 mm. Also find out its inclination with HP.

Solution

After drawing x-y line, the top view and front view of A may be drawn (refer

Figure 4.13). ***a'*** is 10 mm above x-y and ***a***, 20 mm below x-y. Since the line is parallel to VP and inclined to HP, the top view will be parallel to x-y and front view will have true length, it is inclined 30^o to x-y. Draw a parallel to x-y through ***a*** and mark ***b*** such that ***ab***=60 mm. ***a'*** as centre and 70 mm (true length of line) as radius an arc may be cut on the projector drawn from ***b***. Mark the point of intersection of the arc and projector as ***b'***. ***ab*** and ***a'b'*** are finished properly.

Inclination ($\theta=29^o$) of ***a'b'*** with x-y may be measured and neatly written as the required inclination of the line with HP.

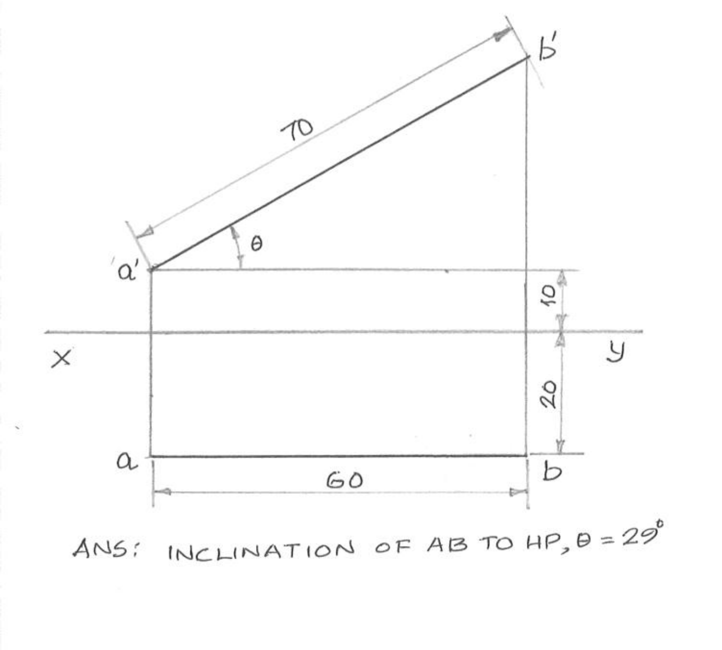


Figure 4.13

Problem 4.8

Two oranges on a tree are 2.0 metres from the ground. One orange is 4.0 metres from a 50 centimetres thick vertical wall while the other is 2.5 metres away from the wall, but on the opposite side. Determine graphically the true distance between the oranges if the distance between the oranges is 10 m while measuring parallel to the wall.

Solution

As all dimensions given in the problem are relatively large, suitable scale may be employed. Assume a scale of 1:100. After using the scale in drawing, actual dimensions may be indicated during dimensioning.

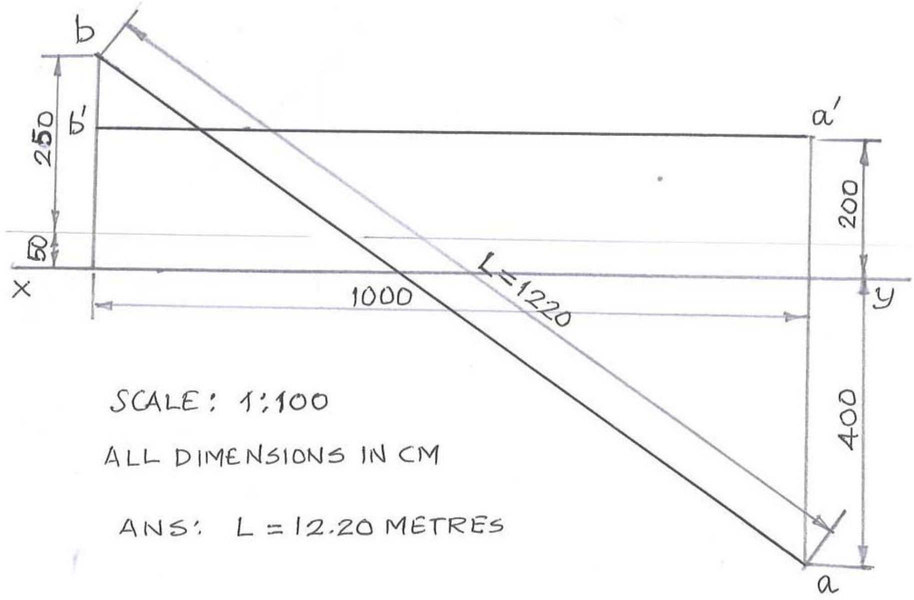


Figure 4.14 Projections of oranges

The oranges may be named as A and B. In the first angle projection HP will replace the ground. As oranges are on opposite sides of the wall, one orange may be assumed to be in the first quadrant and the other, in the second quadrant. Both the oranges are at equal distance from the ground. Hence the line joining these oranges in the front view appears to be parallel to x-y. It is also given that the distance between projectors of these points (oranges) is 10 m.

x-y may be drawn. ***a′*** and ***a*** may be marked on the same projector taking 20 mm above x-y and 40 mm below x-y. At a distance of 100 mm from the projector of A, draw the projector of B. ***b′***, at 20 mm above x-y and ***b***, 30 mm (25 + 5=30 mm) above x-y, may be marked on the projector. ***a*** and ***b,*** and ***a′*** and ***b′*** are respectively joined.

The top view ***ab*** will give the true distance between the oranges. The distance may be measured and written as shown in Figure 4.14.

4.7 Straight lines inclined to both principal planes

Straight lines inclined to both HP and VP are commonly known as oblique lines. The solution for projections of such straight lines is discussed here through illustrations.

Problem 4.9

A straight line AB, 80 mm long, is inclined 40^{o} to HP and 30^{o} to VP. Draw the projections if end A is 25 mm away from HP and 15 mm away from VP.

Analysis

Since the straight line is inclined to both principal planes it can not be solved directly.

If the straight line is inclined to one plane and parallel to the other, the projection on the plane to which it is parallel will give true length and the projection appears to be inclined to x-y at true inclination to which it is inclined.

With out much difficulty it can be understood that if a line is inclined to HP at 40^o, whether it is parallel to VP or inclined at any angle to VP, the length of the top view will be same. It is purely depending upon the distance between the points where the end projectors are meeting with HP.

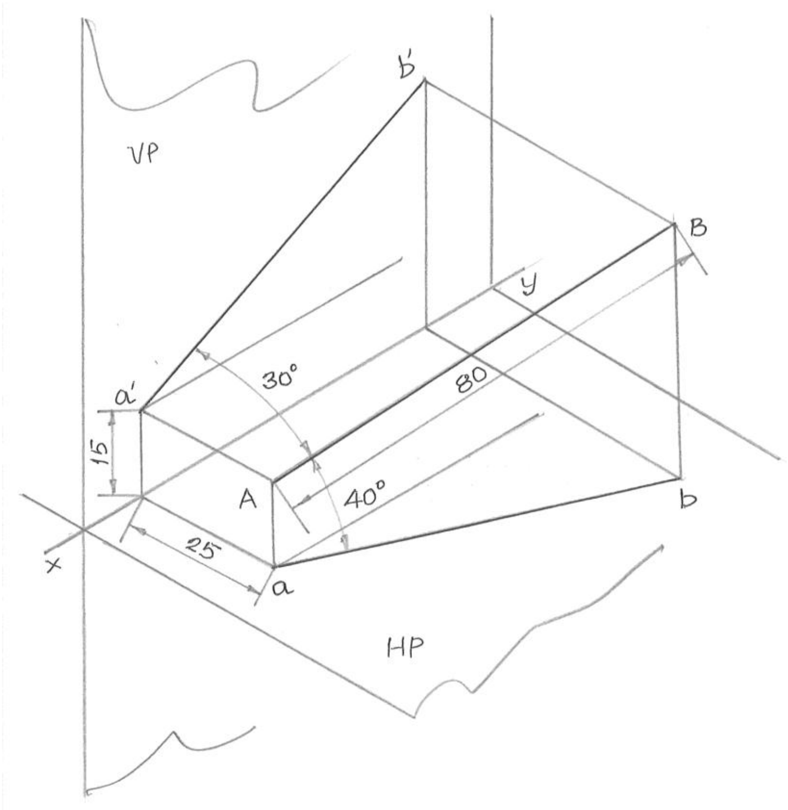


Figure 4.16 Straight line inclined to 40^o HP and 30^o VP

Similary irrespective to the inclination to HP (or it is parallel to HP), if the line is inclined 30^o to VP, the length of front view will be uniform.

Using this principle, the line AB is assumed that: (i) it is inclined at 40^o to HP and parallel to VP, and then (ii) the line is inclined to VP at 30^o and parallel to HP (refer Figure 4.17). Independently the length of top view and front view are obtained.

Keeping the iclination to HP 40^o while making inclination to VP the end B will take a path parallel to HP, which appears to be a straight line parallel to x-y in the front view (locus of ***b'***) and a circular path having the radius of the top view in the top view. Similar process will happen when the line is inclined to VP and later on making inclined to HP also. The point B will appear as moving along a line parallel to x-y in the top view (locus of ***b***) and a circular path having the radius of front view in the font view.

With the help of a simple construction the problem can be completed as detailed in the solution.

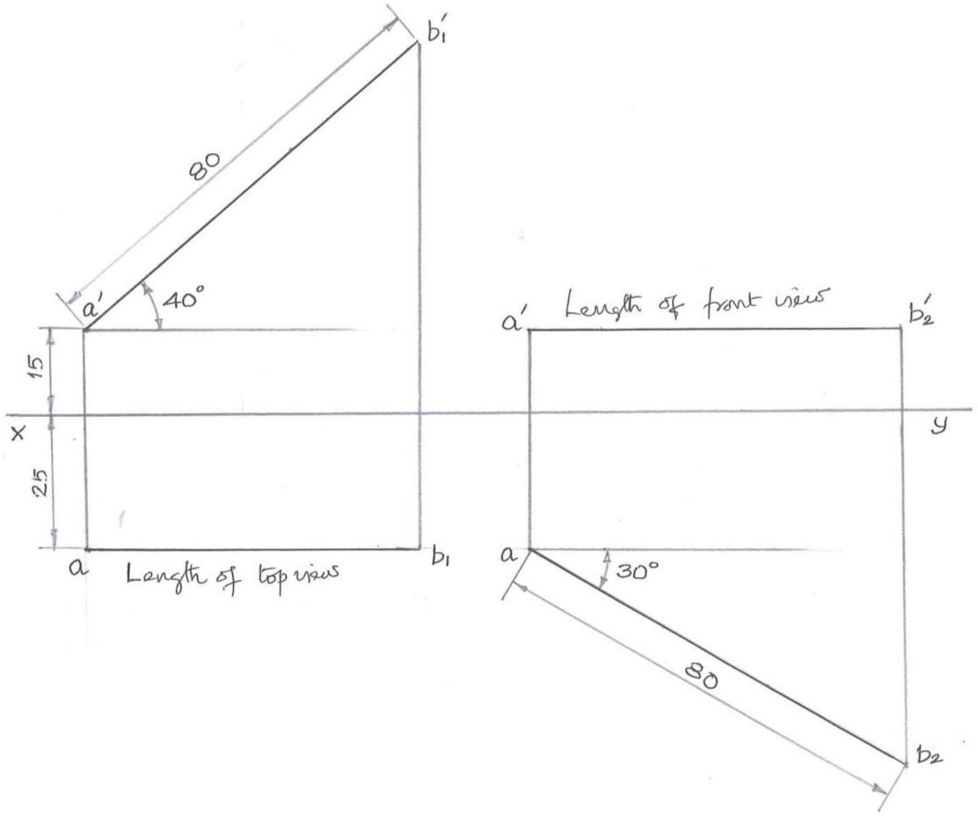


Figure 4.17 Line AB is assumed that: (i) it is inclined at 40^o to HP and parallel to VP, and (ii) the line is assumed to be inclined to VP at 30^o and parallel to HP

Solution

Step 1: The line is assumed to be inclined to HP at 40^o and parallel to VP.

x-y is drawn. After drawing a projector, ***a*** is marked 25 mm below x-y and ***a′***, 15 mm above x-y. An angle of 40^o is constructed at ***a′*** to x-y; b_1' is marked taking $a'b_1'$ as 80 mm. Draw ***ab*** parallel to x-y dropping a projector from b_1'. ab_1 is the length of the top view of the line AB.

Step 2: The line is assumed to be parallel to HP and inclined to VP at 30^o.

After drawing x-y, line ab_2***=80 mm*** is constructed at ***a*** at an angle of 30^o to x-y. Its front view $a'b_2'$ is drawn by drawing a projector from b_2 and intersecting the line through ***a′*** and parallel to x-y. ***a′b₂′*** is the length of front view of the line AB.

Step 3: A line is drawn parallel to x-y through b_1', which is the locus of ***b′,*** when the line is made inclined to VP also. Another line is drawn through b_2, which is the locus of ***b,*** when the line is rotated to make inclined to HP also.

Step 4: Considering the front view; ***a′*** is fixed, it is one end of front view, $a'b_2'$ is the length of front view and ***b′*** falls on the locus drawn through b_1'. ***a′*** as centre $a'b_2'$ as radius, an arc is cut to the locus of ***b′***. This arc is nothing but the locus of B in the front view. The point of intersection of the arc with the locus is ***b′***. ***a′***and ***b′*** may be joined and finished.

Considering the top view, ***a*** its one end, is fixed, ab_1 is the length of top view and ***b*** falls on the locus drawn through b_2. ***a*** as centre ab_1as radius an arc is cut to the locus of ***b***. This arc is the locus of B in the top view. The point of intersection of the arc with the locus is ***b***. ***a*** and ***b*** may be joined. Also confirm whether ***b*** and ***b′***are falling on the same projector. ***ab*** and ***a′b′*** may be finished.

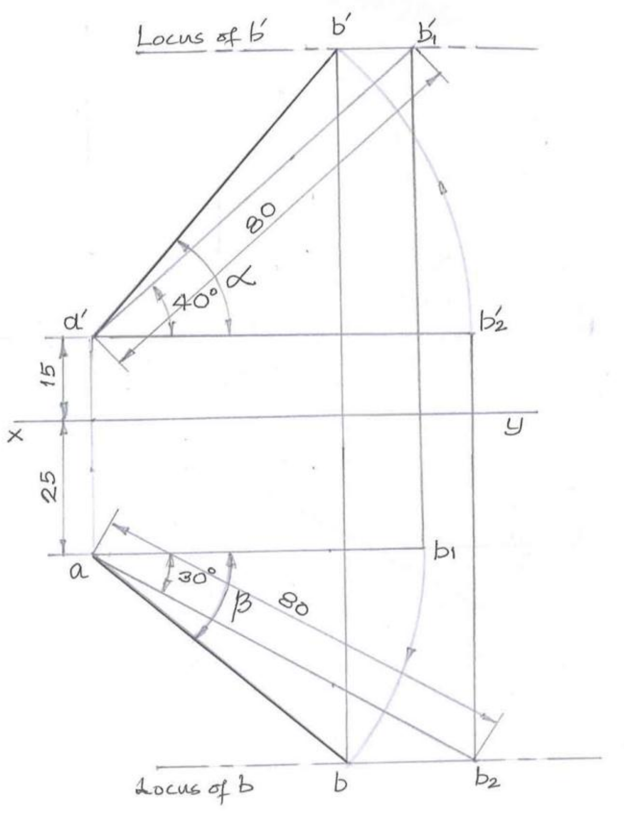


Figure 4.18 Solution of line AB inclined 40° to HP and 30° to VP

Angles α and β (refer Figure 4.18) are apparent angles. The line appears to be inclined at an inclination of α with HP and β, the visible inclination with VP.

Problem 4.10

A straight line AB, 70 mm long, is inclined to both HP and VP. The length of the top view of the line is 65 mm and that of front view is 50 mm. Draw the projections of the straight line. Also find out the true inclinations with HP and VP. C is on VP, 10 mm above HP.

Analysis

The 70 mm long line CD which is inclined to both HP and VP, has its top view 65 mm long and front view 50 mm long.

The straight line may be assumed as inclined to only HP and parallel to VP. The front view will give true length and front view appears to be inclined to x-y at true inclination with HP. The top view will be parallel to x-y.

Assume the line is inclined to VP alone and parallel to HP. The top view will have true length, true inclination with VP is visible in the top view and front view will be parallel to x-y.

After these two independent steps the final position of the line can be achieved easily as explained in the solution.

Solution

c on x-y and ***c***'10 mm above may be marked on the projector after drawing x-y. d_1 may be marked on x-y by taking 65 mm from ***c***. A projector is drawn from d_1. Taking a radius of 70 mm, an arc is cut with ***c'*** as centre to this projector. The point of intersection is d_1'. Now $c'd_1'$ and cd_1 are respectively the front view and top view of the line CD when it is inclined to HP and parallel to VP.

d_2' may be on the line parallel to x-y 50 mm from ***c'***. A projector is drawn from d_2'. ***c*** as centre 70 mm as radius an arc is cut to the projector drawn. The point is marked as d_2. cd_2 and $c'd_2'$ are respectively the top and front views of CD when it is inclined to VP and parallel to HP.

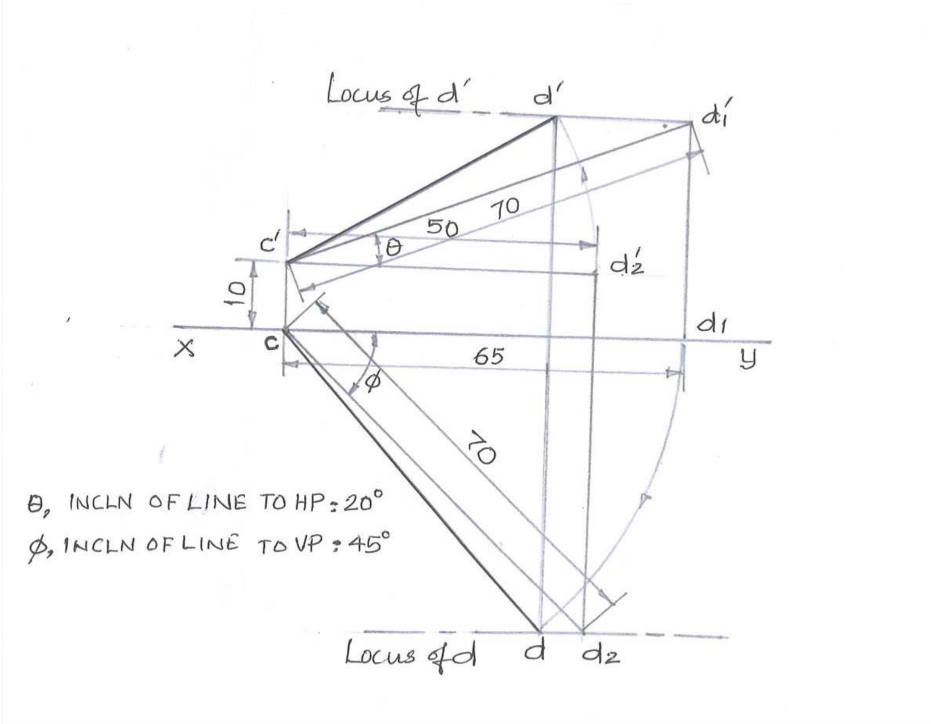


Figure 4.19 Projections of Line CD inclined to HP and VP whose Top and Front views are known

Draw lines parallel to x-y, through $\boldsymbol{d_1'}$ and $\boldsymbol{d_2}$. These are the respective loci of D in the front view and top view. With $\boldsymbol{c'}$ as centre $\boldsymbol{c'd_2'}$ as radius an arc is to th locus of $\boldsymbol{d_1'}$ to get $\boldsymbol{d'}$. Similarly an arc of radius $\boldsymbol{cd_1}$ may be constructed with $\boldsymbol{c}$ as centre to the locus of $\boldsymbol{d_2}$. $\boldsymbol{c'}$ and $\boldsymbol{d'}$, and $\boldsymbol{c}$ and $\boldsymbol{d}$ may be joined to get the final front view and top view.

Angle $\boldsymbol{d_2cd_1}$ is the true inclination of the line with VP. Angle $\boldsymbol{d_1'c'd_2'}$ is the true inclination with HP (refer Figure 4.19).

Problem 4.11

A straight line PQ, 65 mm long, is inclined 50^o to HP and 40^o to VP. Draw the projections. End P is 15 mm in front of VP and 10 mm above HP.

Analysis

The straight line PQ is inclined 50^o to HP and 40^o to VP. As these are complementary angles, the straight line is lying on a profile plane. As the profile plane appears to be straight lines perpendicular to x-y both in top view and front view, the projections of the line will be perpendicular to x-y (refer Figure 4.20).

The line may be assumed to be inclined to HP and parallel to VP, and projections may be drawn to get the length of top view.

Simalrly the length of front view may be obtained by assuming the line inclined 50^o

to VP and parallel to HP.

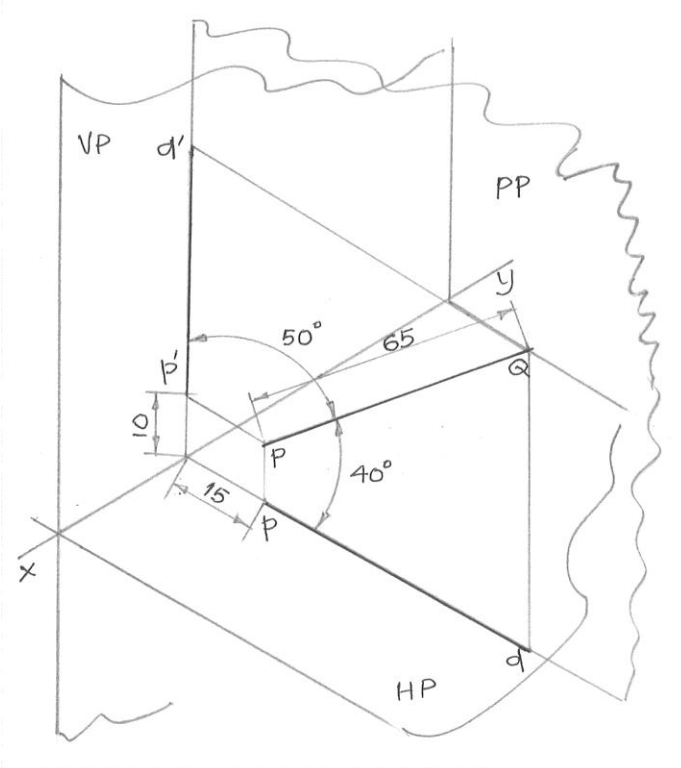


Figure 4.20 Line PQ is on a profile plane

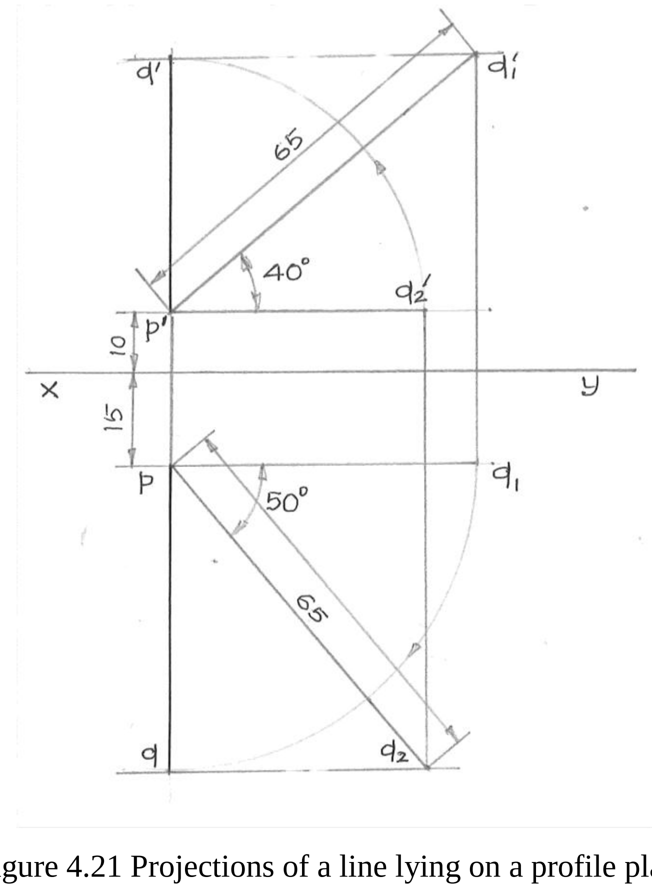


Figure 4.21 Projections of a line lying on a profile plane

Solution

p' and ***p*** may be marked on the projector 10 mm above and 15 mm below x-y, respectively. ***p'q₁'***= 65 mm may be drawn at 40^o inclined to x-y. The top view $\boldsymbol{pq_1}$ may be drawn when the line is inclined to only HP and parallel to VP.

$\boldsymbol{pq_2}$=65 mm may be drawn at 50^o to x-y. The front view $\boldsymbol{p'q_2'}$ may be drawn when the line is inclined to VP and parallel to HP. Lines may be drawn through $\boldsymbol{q_1'}$ (locus of ***q'*)** and through $\boldsymbol{q_2}$ (locus of ***b***) for x-y. With ***p'*** as centre $\boldsymbol{p'q_2'}$ as radius an arc is cut to the locus of ***q'*** and mark the point of intersection as ***q'.*** With ***p*** as centre, $\boldsymbol{pq_1}$ as radius arc is drawn to the locus of ***q*** and the point of intersection is ***q. pq*** and ***p'q'***, respectively the top view and front view of PQ, may be finished (refer Figure 4.21).

Problem 4.12

A straight line AB, 90 mm long, is icnlined 45^o to HP and 20^o to VP. The end A is on HP and B, on VP. Draw the projections (refer Figure 4.22).

Analysis

The line AB is inclined to both HP and VP. No end of the line is given here specifically, or both projections of an end can not be drawn. The distance of A from VP or distance of B from HP must be known to complete the solution.

The line may be assumed to be parallel to VP, keeping the end B on VP and A on HP, with the same inclination 45^o to HP or the line is lying on VP with 45^{o}inclined to HP. The projections can be drawn easily. The distance of ***b***' from VP is available from this assumption or projections of end B, ***b*** and ***b'*** are known now. The solution may be completed following standard procedure.

Solution

$\boldsymbol{a_1}$ and $\boldsymbol{a_1}$' may be marked conveniently on x-y line. $\boldsymbol{a_1'b'}$=90 mm is constructed at 45^o inclined to x-y. Th corresponding top view, $\boldsymbol{a_1b}$ may be drawn. ***b*** and ***b'*** are the required projections of end B and $\boldsymbol{a_1b}$ is the length of top view of AB.

$\boldsymbol{ba_2}$=90 mm may be drawn at 20^o inclined to x-y. The corresponding front view is $\boldsymbol{b'a_2'}$. Locus of ***a'*** is x-y and locus of $\boldsymbol{a_2}$ may be drawn parallel to x-y through $\boldsymbol{a_2}$. ***b***' as centre and $\boldsymbol{b'a_2'}$ as radius an arc is cut to x-y (locus of ***a'***). ***b*** as centre and $\boldsymbol{ba}_1$ as radius arc is cut to the locus of ***a***. The points of intersection are respectively ***a***' and ***a***. ***ab*** and ***a'b'*** may be finished to get final projections of AB (refer Figure 4.23).

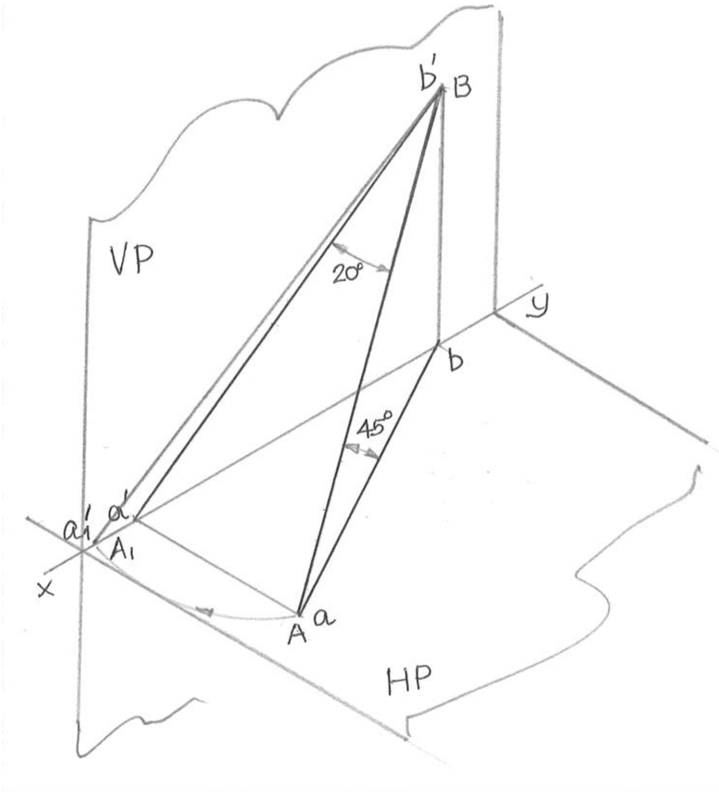


Figure 4.22 Line is inclined to both HP and VP with end on different planes

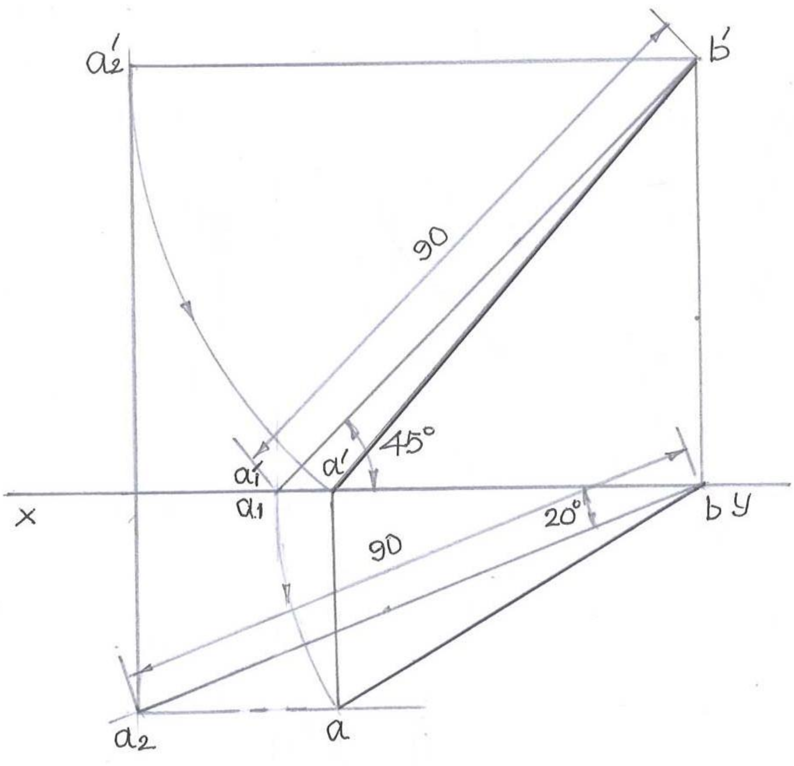


Figure 4.23 Line inclined to both HP and VP with end points on principal planes

Problem 4.13

A straight line AB, 120 mm long, is inclined 45^o to HP and 30^o to VP. Draw the projections if the end A is in the first quadrant, B is in the third quadrant and a point C, such that AC=50 mm, is on VP 15 mm above HP.

Analysis

The straight line is contained in more than one quadrant. The straight line AB is divided into two parts-AC and CB, of lengths 50 mm and 70 mm, respectively. The part AC=50 mm of the straight line AB is inclined 45^{o} to HP and 30^{o} to VP, and is in the first quadrant. After drawing the projections of AC, the remaining part can be completed by the extending the length.

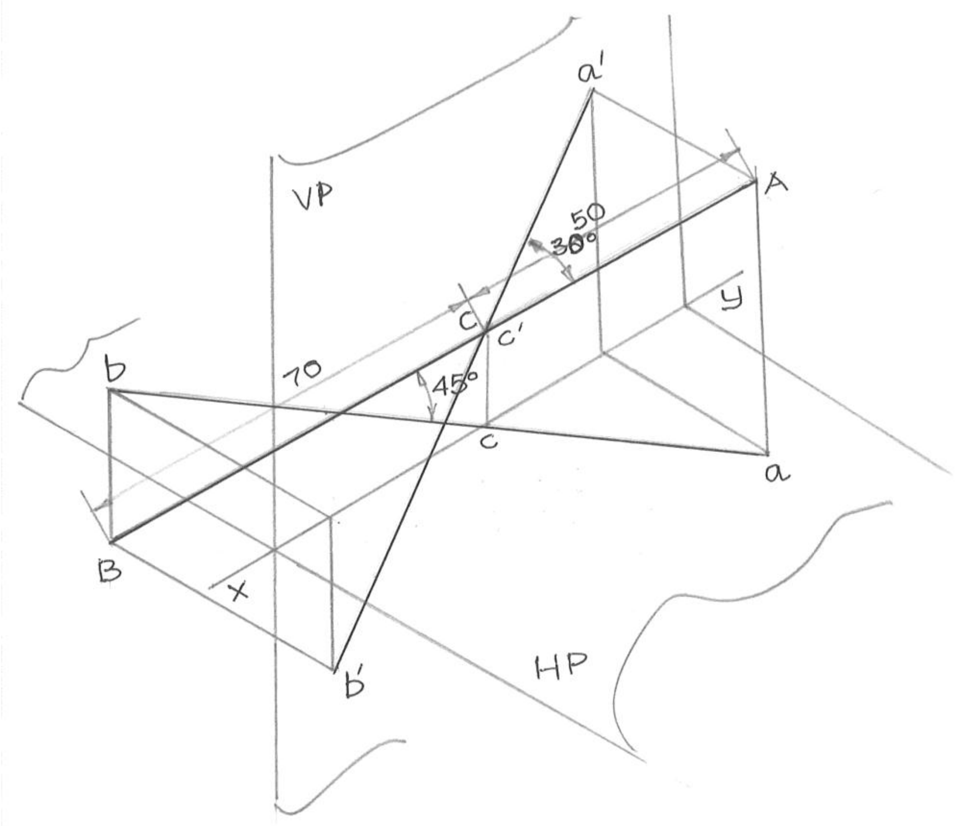


Figure 4.24 Line contained in more than one quadrant

Solution

c and ***c'*** may be marked respectively on x-y and 15 mm above x-y on the projector. $\boldsymbol{c'a_1'}$=50 mm may be drawn at 45^{o} inclined to x-y. $\boldsymbol{ca_1}$, top view of CA may be obtained. $\boldsymbol{ca_2}$=50 mm may be drawn 30^{o} inclined to x-y. Corresepondingly $\boldsymbol{c'a_2'}$ may be obtained. Locus of ***a*** through $\boldsymbol{a_2}$ and locus of ***a'*** through $\boldsymbol{a_1'}$ are drawn. $\boldsymbol{c'a_2'}$ as radius, ***c'*** as centre an arc is drawn to the locus of ***a'*** and $\boldsymbol{ca_1}$ as radius, ***c*** as centre another arc to the locus of ***a***. ***c'a'*** and ***ca*** will be the projections of CA.

$\boldsymbol{a_1'c'}$ is extended to $\boldsymbol{b_1'}$ such that $\boldsymbol{c'b_1'}$=70 mm. $\boldsymbol{cb_1}$, the top view is drawn. $\boldsymbol{a_2c}$ is extended to $\boldsymbol{b_2}$, such that $\boldsymbol{cb_2}$=70 mm. The corresponding front view, $\boldsymbol{c'b_2'}$ is also drawn. The loci of ***b'*** and ***b*** are drawn respectively through ***b_1'*** and ***b_2***. Arcs are cut with radii ***c'b_2'*** and ***cb_1*** with ***c'*** and ***c*** respectively to the loci of ***b'*** and ***b***. ***c'b'*** and ***cb*** are the projections of the part CB.

a'b' and ***ab*** are finished to get the final projections of AB (Figure 4.25).

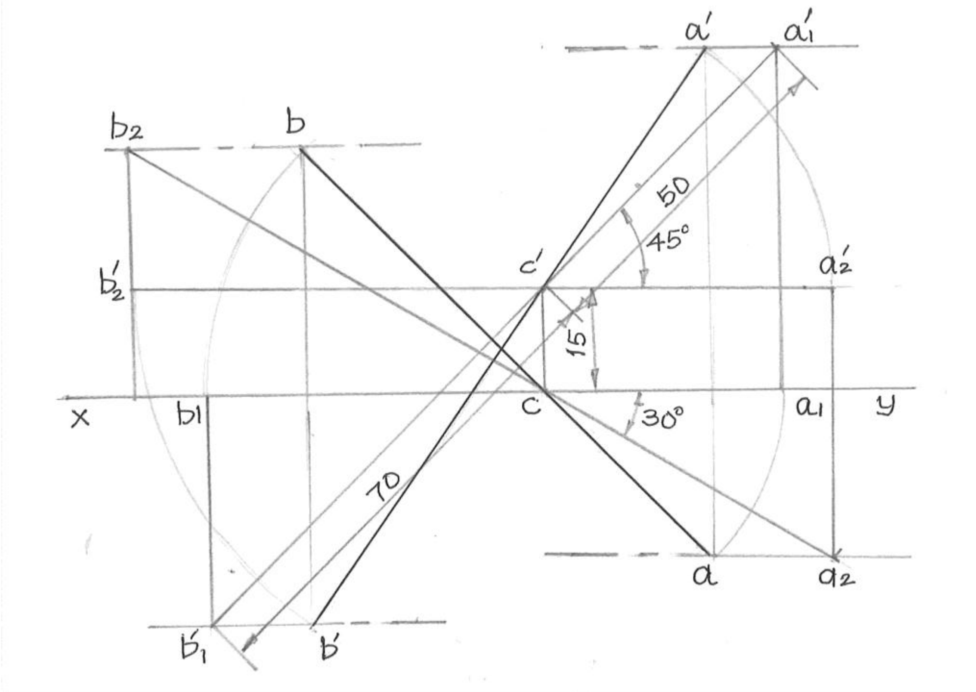


Figure 4.25 Projections of line conataining in three quadrants

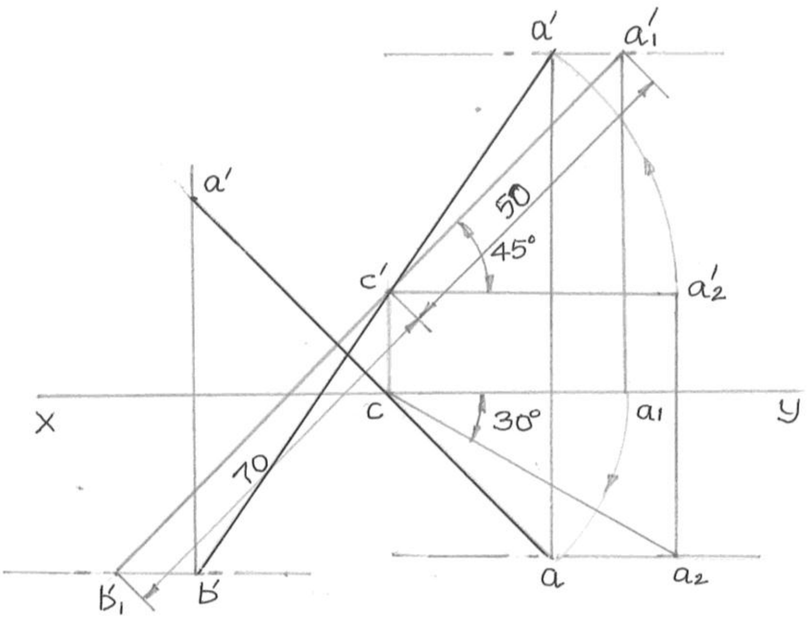


Figure 4.26 Alternative solution for Problem 12

Alternative Solution

c and ***c'*** may be marked respectively on x-y and 15 mm above x-y on the projector. $c'a_1'$=50 mm may be drawn at 45^o inclined to x-y. ca_1, top view of CA may be obtained. ca_2=50 may be drawn 30^o inclined to x-y. Corresepondingly $c'a_2'$ may be obtained. Loci of ***a*** through a_2 and ***a'*** through a_1' are drawn. $c'a_2'$ as radius, ***c'*** as centre an arc is cut to the locus of ***a'*** and ca_1 as radius, ***c*** as centre another arc to the locus of ***a***. ***c'a'*** and ***ca*** will be the projections of CA.

$a_1'c'$ is extended to b_1' such that $c'b_1'$=70 mm. ***a'c'*** may be extended to meet the line drawn parallel to x-y through b_1' at ***b'***. ***ac*** may be extended to intersect with the projector drawn from ***b'*** at ***b***. The expected solution is availabe after finishing ***a'b'***and ***ab*** (refer Figure 4.26).

Problem 4.14

A point A in the first quadrant is 40 mm away from VP and 20 mm away from HP, while B is in the third quadrant 40 mm away from VP and 30 mm away from HP. The distance between the projectors of A and B is 80 mm. Draw the projections of the line joining A and B. Also find out its true length and inclinations to HP and VP.

Solution

x-y may be drawn sufficiently long. ***a'*** and ***a*** may be marked on a projector at 20 mm above x-y and 40 mm below, resepectively. Another projector may be drawn 80 mm away from the previous projector. ***b*** and ***b'*** may be marked on the projector 40 mm above x-y and 30 mm below x-y, respectively. ***ab*** and ***a'b'*** may be joined and finished to get the projections of the line AB.

About ***a***, ***ab*** may be rotated so that it may be made parallel to x-y (ab_1). A projector may be drawn from b_1 to meet the locus of ***b'***. $a'b_1'$ is the true length of AB and inclination of $a'b_1'$ with x-y will be true inclination to HP.

About ***a'***, ***a'b'*** may be rotated and ***a'b'*** is made parallel to x-y ($a'b_2'$). A projector is drawn from b_2' to the locus of ***b***. ab_2 will be equal to $a'b_1'$ which is the true length of line. Inclination of ab_2 will be true inclination to VP (refer Figure 4.27).

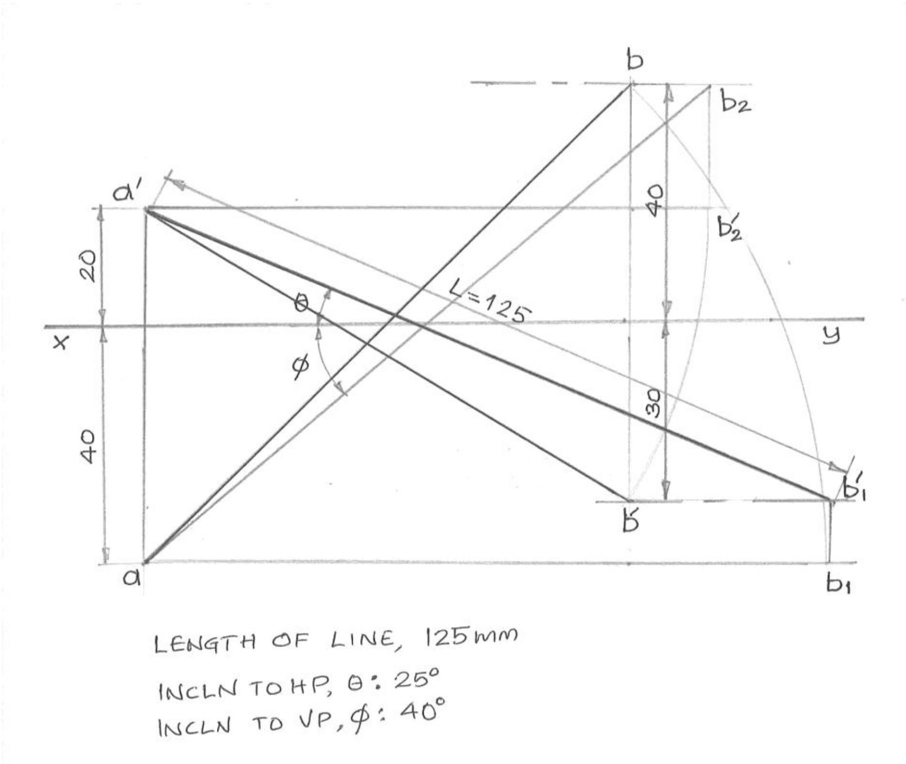


Figure 4.27 True length and inclinations of lines contained in more than one quadrant

4.8 Traces of Straight Lines

A trace is a point at which the straight line meets with a principal plane. The point at which the straight line intersects with HP is horizontal trace (HT) and that at which the line meets VP is vertical trace (VT).

If a straight line is not meeting the principal planes directly, the line may be extended to meet either HP or VP or both as per its orientation.

If a line is inclined to both HP and VP it can have both HT and VT.

If a line is parallel to a principal plane the line does not have a trace on that plane.

Determination of traces of a line

A straight line PQ (refer Figure 4.28) is inclined to both HP and VP. Its projections are also shown on HP and VP. The line PQ is a part of AB which is a typical line meeting both HP and VP. The traces of line AB can easily be determined as shown in figure. PQ is lying on AB, and not meeting directly either HP or VP. But, while extending it meets both HP and VP, at the same points where AB meets. PQ and AB are having common traces.

After drawing the projections, the construction required for determining traces will be clear (refer Figure 4.29).

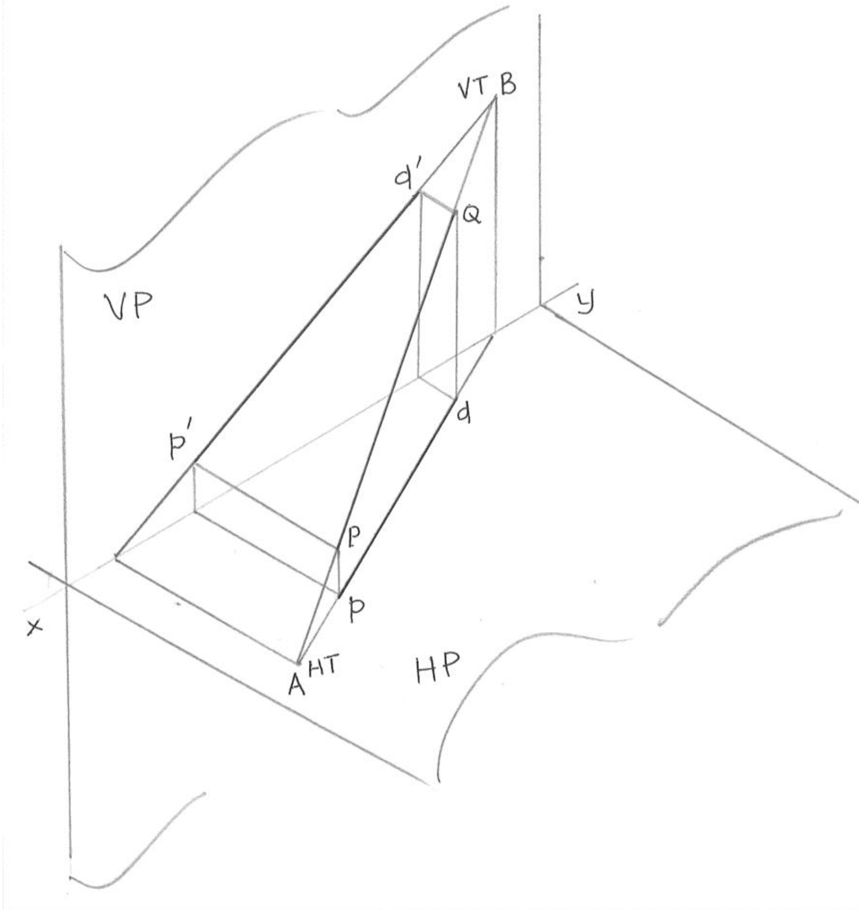


Figure 4.28 Traces of a typical straight line

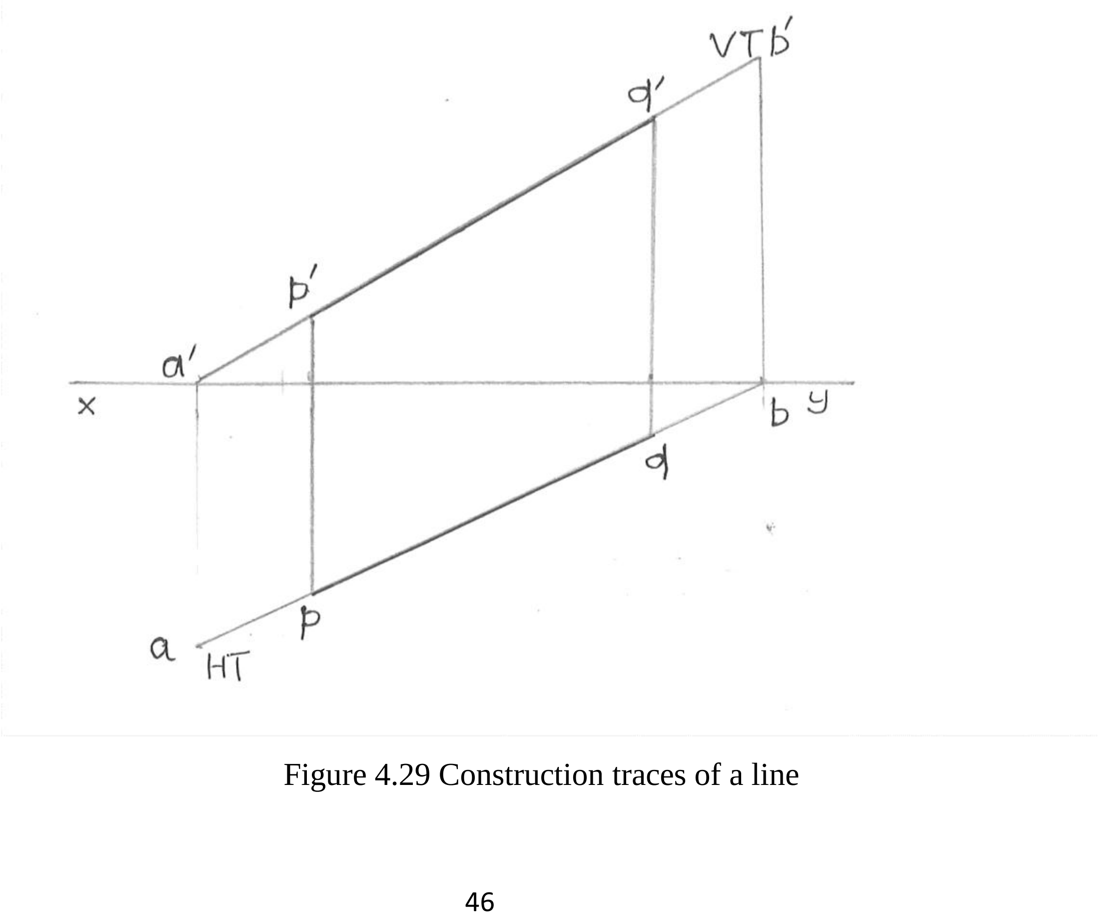


Figure 4.29 Construction traces of a line

The point of intersection of top view and the projector drawn from the point at which the front view meets x-y is the Horizontal Trace (HT).

Similarly, the point of intersection of the front view and the projector drawn from the point where the top view meets x-y is the Vertical Trace (VT).

Determination of traces of a line – Alternative method

The method given in the previous section for determining the traces will not work if the straight line is lying on a profile. In such cases, an alternative method, Trapezoidal method can be employed.

The line PQ shown in Figure 4.30 is lying on a profile plane. θ and ϕ are complemntary angles. PQ is a portion of line AB which is directly meeting with HP and VP. The line PQ and AB have common traces as shown in Figure 4.30.

θ is the inclination of AB (or PQ) with HP, which is also equal to the inclination of the line with its top view. The point of intersection of the line with its top view is HT. The inclination of the line with its front view is equal to the inclination of the line with VP, ϕ. The point of intersection of the line with its front view is VT.

Using this concept the true length, true inclination and traces can be obtained.

PQq'p' is a trapezoidal plane perpendicular to both HP and VP, attached to VP through ***p'q'***. The trapezoidal plane may be rotated about ***p'q'*** and it may be made coinciding to VP as shown in Figure 4.30. The final position of the plane is shown in the figure as $\boldsymbol{p'q'Q_1P_1}$. $\boldsymbol{P_1Q_1}$ gives the true length of the line. While extending $\boldsymbol{P_1Q_1}$ and ***p'q'***, they will meet at VT only. The inclination between ***p'q'*** and $\boldsymbol{P_1Q_1}$ is true inclination of line with VP.

PQqp is another trapezoidal plane perpendicular to both HP and VP, attached to HP through ***pq***. The trapezoidal plane may be rotated about ***pq*** and it may be coincided to HP. The final position of the plane is shown in the figure as $\boldsymbol{pqQ_2P_2}$. $\boldsymbol{P_2Q_2 = P_1Q_1}$ which is the true length of the line. While extending $\boldsymbol{P_2Q_2}$ and ***pq***, they will meet at HT only. The inclination between ***pq*** and $\boldsymbol{P_2Q_2}$ is true inclination of line with HP. The complete construction is shown in Figure 4.31.

Trapezoidal method can be employed to any straight line to find true length, true inclinations and traces.

- ***The inclination of a line with HP is equal to the inclination of the line with its top view and the point of intersection of the line with its top view is HT.***
- ***The inclination of a line with its front view is equal to the inclination of the line***

with VP and the point of intersection of the line with its front view is VT.

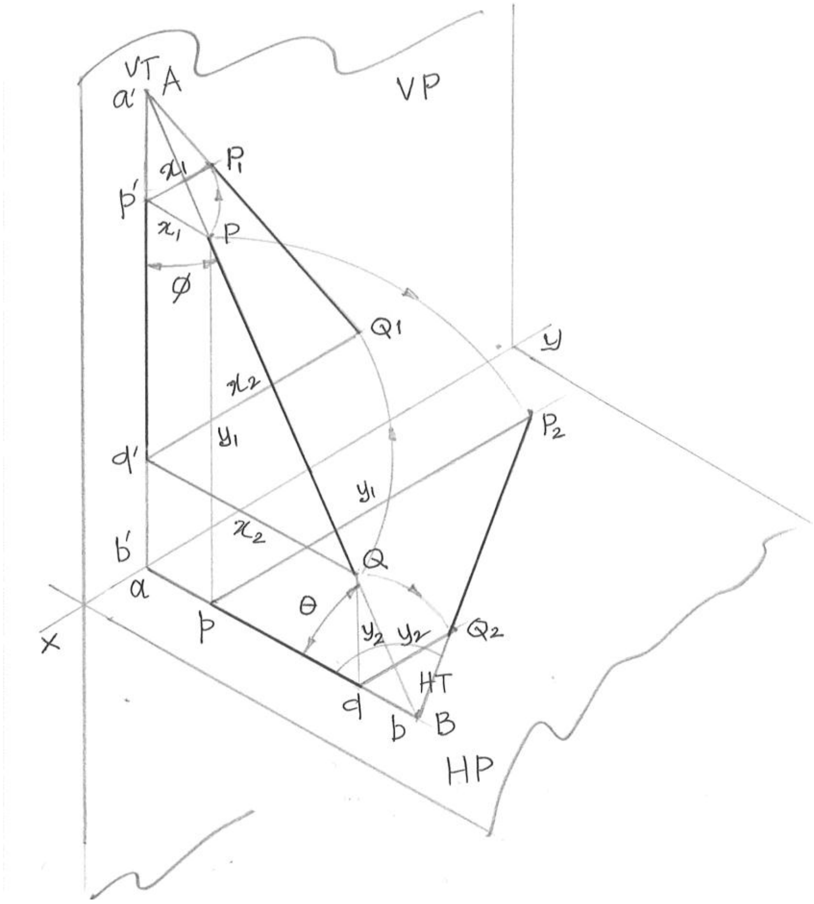


Figure 4.30 Trapezoidal method to determine the true length, true inclination and traces of a line

Solution

The projections ***pq*** and ***p'q'*** of the line PQ are drawn. At ***p***' and ***q'*** perpendiculars are drawn (on the same side of ***p'q'***) for ***pq***. At ***p***', the distance of ***p*** from VP (x_1) may be indicated as P_1. At ***q***' distance of ***q*** from VP (x_2) may be marked as Q_1. P_1 and Q_1 may be joined which is the true length of PQ. The angle between P_1Q_1 and ***p'q'*** is the inclination of the line to VP. While extending P_1Q_1 and ***p'q'***, they will meet at VT only.

On the perpendicular to ***pq*** at ***p***, the distance of ***p'*** from HP (y_1) may be marked as P_2. On the perpendicular to ***pq*** at ***q*** distance of ***q'*** from HP (y_2) may be marked as Q_2. $P_2Q_2 = P_1Q_1$, the true length of PQ. The angle between P_2Q_2 and ***pq*** is the inclination of the line to HP. The point of intersection of P_2Q_2 and ***pq*** will be HT.

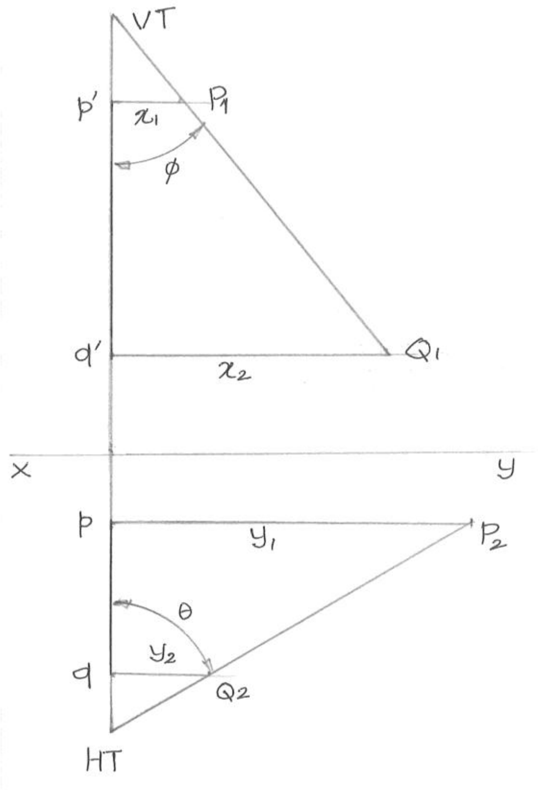


Figure 4.31 Principle of trapezoidal method

Problem 4.15

ab and ***a'b'*** are the top view and front view, respectively of a line AB. The traces can be determined by following the conventional method (refer Figure 4.32).

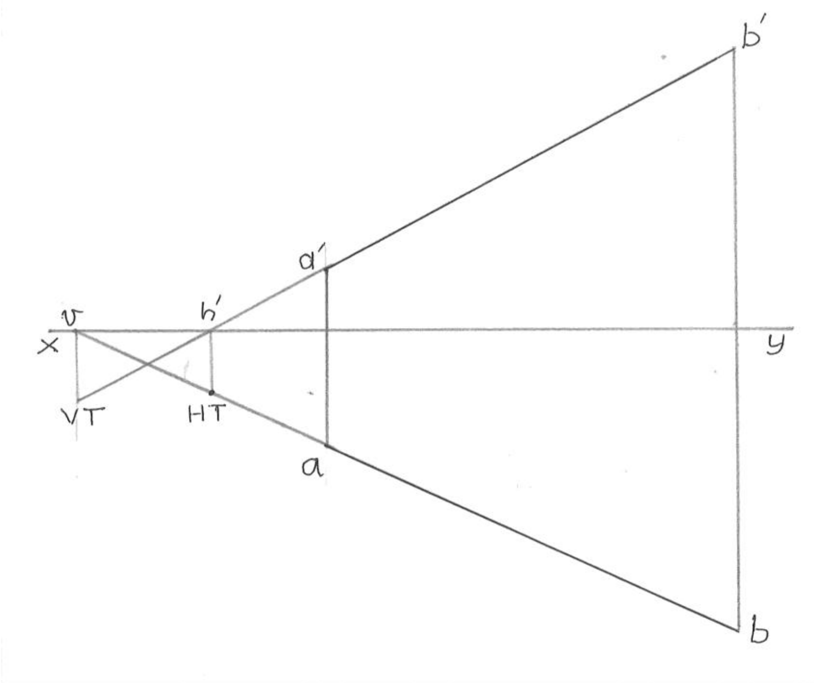


Figure 4.32 Determination of traces of line if top and front views are given

Solution

While extending the front view ***a'b'*** it meets x-y at ***h'***. The point of intersection of the perpendicular drawn from ***h'*** and the top view ***ab*** is HT.

The top view ***ab*** may be extended to meet x-y at ***v***. The point of intersection of the perpendicular drawn from ***v*** and front view ***a'b'*** is VT.

Problem 4.16

The front view ***a'b'*** and top view ***ab*** of a line AB are given. Determine the true length of line and its inclination with HP and VP. Also locate traces.

Solution

The above problem is solved using trpezoidal method. The end B is in the first quadrant and A, in the third quadrant. The ends A and B are in opposite quadrants. While using trapezoidal method the perpendicuars to the ends of projection may be drawn in opposie directions.

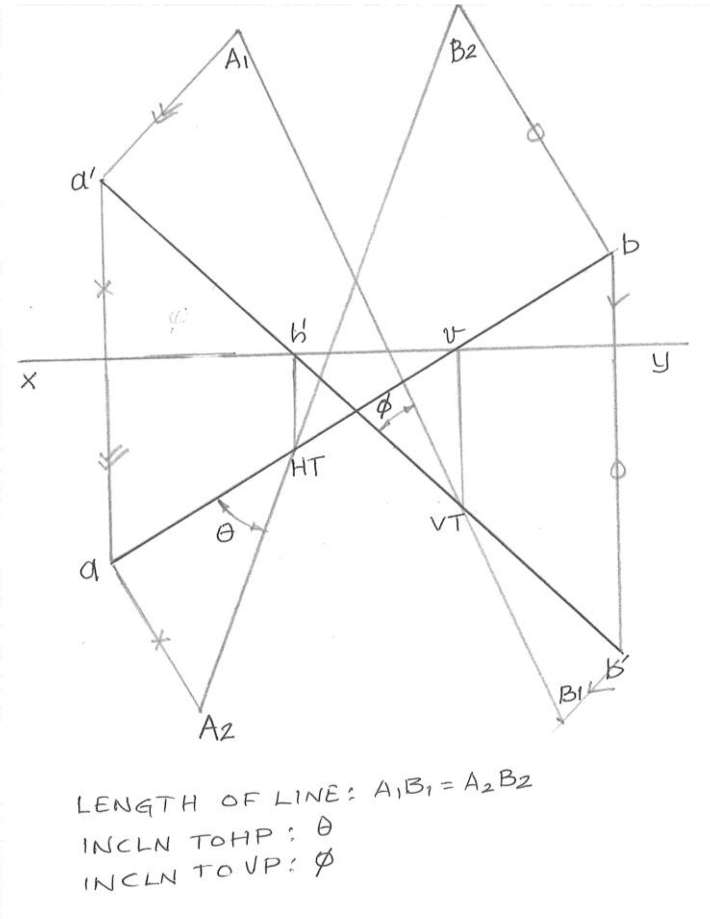


Figure 4.33 Use of alternative method (trapezoidal method) when the ends of line are in opposite quadrants

At ***a′*** a perpendicular to ***a′b′*** is drawn. The distance of ***a*** from x-y is taken and marked as A_1on the perpendicular. After drawing another perpendicular from ***b′***, in the opposite dirction that of ***a′***, mark on it the distance of ***b*** from x-y, as B_1. A_1 and B_1 are joined. A_1B_1 gives the true length of AB. Inclination between A_1B_1 and ***a′b′*** is the true inclination of the line with VP. The point of intersection between ***a′b′*** and A_1B_1 is VT of the line.

At ***a*** a perpendicular to ***ab*** may be drawn. The distance of ***a′*** from x-y is taken and marked as A_2 on the perpendicular. After drawing another perpendicular from ***b***, opposite to that of ***a***, mark on it the distance of ***b′*** from x-y, B_2. A_2 and B_2 are joined. A_2B_2 equals to A_1B_1, the true length of AB. Inclination between A_2B_2 and ***ab*** is the true inclination of the line with HP. The point of intersection between ***ab*** and A_2B_2 is HT of the line.

Problem 4.17

The front view ***p′q′*** and top view ***pq*** of the line PQ are perpendicular to x-y as given in Figure 4.34. Determine the true length of AB, true inclinations with HP and VP, and traces.

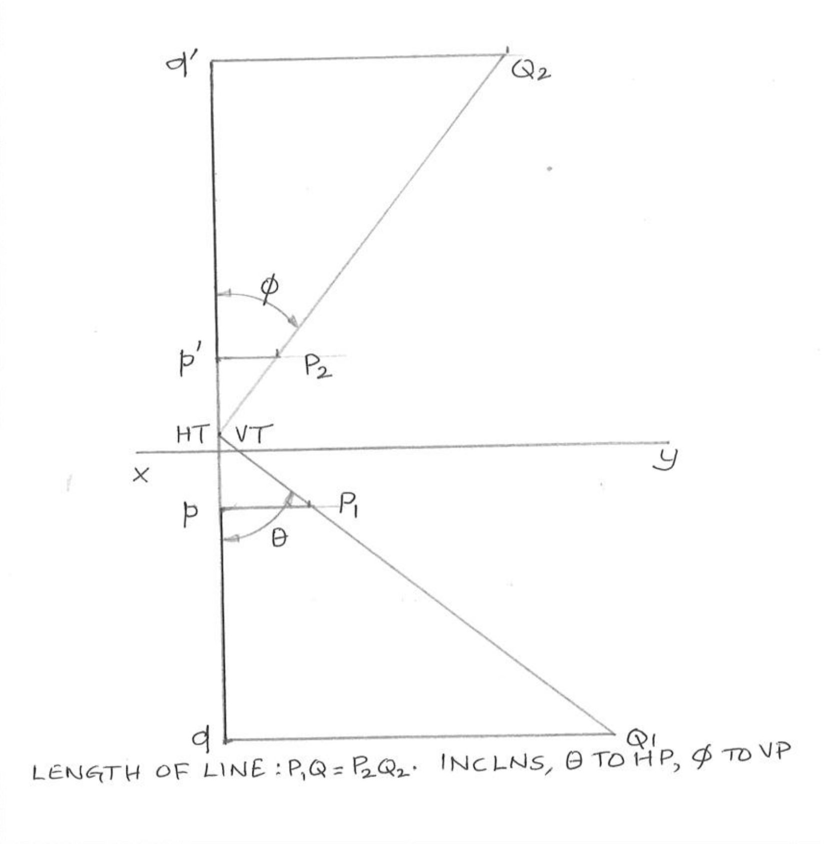


Figure 4.34 Determination of traces, true length and inclinations of line when it lies on a profile plane

As the both projections of the line ***pq*** and ***p′q′*** appear perpendicular to x-y, the line is lying on a profile plane. Using trapezoidal method the problem can be solved.

The distance of ***p′*** from x-y may be marked as P_1 on the perpendicular drawn through ***p*** to ***pq***. The distance of ***q′*** from x-y may be marked as Q_1 on the perpendicular drawn (on the same side, as each projection is on one side of x-y) from ***q*** to ***pq***. P_1 and Q_1 are joined. P_1Q_1 is the true length of PQ. Inclination between ***pq*** and P_1Q_1 is the true inclination of the line with HP. The point of intersection of ***pq*** and P_1Q_1 is HT.

The distance of ***p*** from x-y may be marked as P_2 on the perpendicular drawn through ***p′*** to ***p′q′***. The distance of ***q*** from x-y may be marked as Q_2 on the perpendicular drawn (on the same side, as each projection is on one side of x-y) from ***q′***. P_2 and Q_2 are joined. $P_2Q_2 = P_1Q_1$, the true length of PQ. Inclination between ***p′q′*** and P_2Q_2 is the true inclination of the line with VP. The point of intersection of ***p′q′*** and P_2Q_2 is VT.

Problem 4.18

A straight line AB 80 mm long is inclined 30^o to HP and lying on auxiliary vertical plane (AVP) which is inclined 45^o to VP. Draw the projections. The end A is 10 mm above HP and 20 mm in front of VP. Also find the inclination of the line with VP.

An AVP inclined 45^o to VP is visible in the top view as a line inclined 45^o to x-y. As the line AB is lying on this AVP, the top view of the line AB appears to be inclined at 45^o to x-y.

a′ and ***a*** are marked on the x-y, 10 mm above and 20 mm below, respectively. ***a′b₁′*** is drawn at 30^o inclined to x-y. The corresponding top view, $\boldsymbol{ab_1}$ may be obtained. At ***a*** a line is drawn inclined at $45^o$to x-y; the top view of the line coincides with this line. With ***a*** as centre, $\boldsymbol{ab_1}$ as radius an arc is drawn to meet at ***b*** with the line through ***a***, inclined 45^o to x-y. ***b′*** may be obtained on the line parallel to x-y through $\boldsymbol{b_1'}$ by drawing a projector from ***b***.

$\boldsymbol{a'b_2'}$ may be obtained after rotating ***a′b′*** parallel to x-y. $\boldsymbol{b_2}$ may be obtained by projecting $\boldsymbol{b_2'}$ to the locus of ***b***. Inclination of $\boldsymbol{ab_2}$ with x-y is the true inclination of AB with VP. The traces, HT and VT may be obtained as per the construction detailed earlier.

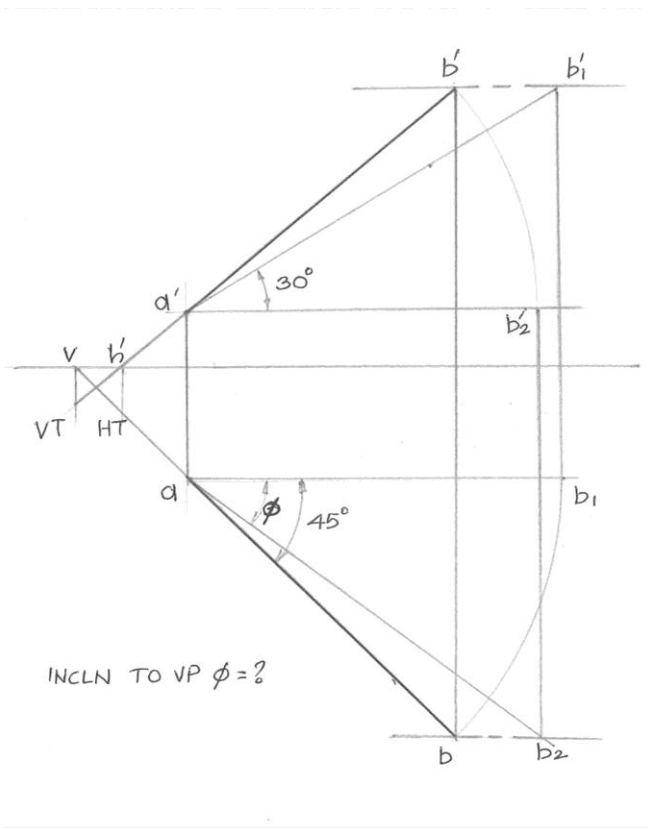


Figure 4.35 Projections of the line lying on an AVP

Problem 4.19

A straight line AB is inclined 30° to VP while its 60 mm long front view is inclined 45° to x-y. The line is in the first quadrant with end A of the line is 10 mm above HP. The VT of the line is 20 mm below HP. Draw the projections of the line. Find out its true length and true inclination with HP. Also locate HT.

Solution

The line AB is inclined to both HP and VP. True inclination to VP and apparent inclination to HP are given. Position of no end is given clearly, rather end A is given as 10 mm above HP. But, the distance of A from VP is unknown.

The problem may be solved considering another point on the straight line, for which both top view and front view can be known. Recollecting, the construction of VT, this solution can be completed.

VT is the point at which the front view intersects the projector from the point of intersection of top view meets with x-y. VT and ***v*** are front view and top view of a point on the line, say extended line. Now the line is extended and hence the front view and top view are also extended.

a' may be marked on the projector 10 mm above x-y. ***a'b'*** = 60 mm may be constructed as inclined at 45^o to x-y. Draw a parallel at a distance of 20 mm below x-y. VT may be marked on this line after extending the front view ***a'b'***.

VT-b₂' may be drawn taking ***VT-b'*** parallel to x-y. From ***b₂'*** a projector is dropped so that it meets with the line drawn from ***v*** at an angle of 30^o to x-y. Locus of ***b*** may be drawn parallel to x-y, through ***b₂***. ***b*** may be marked on the locus by dropping a projector from ***b'***. ***v*** and ***b*** may be joined. Dropping a projector from ***a'*** to ***vb***, ***a*** is also available. ***ab*** is taken parallel to x-y as ***ab₁*** and ***b₁*** may be projected to the locus of ***b'***, to get ***b₁'***. ***a'b₁'*** gives true length of the line AB and inclination of ***a'b₁'*** to x-y is the true inclination of line AB with HP. HT can be located by drawing a projector from the meeting point of front view with x-y to the top view.

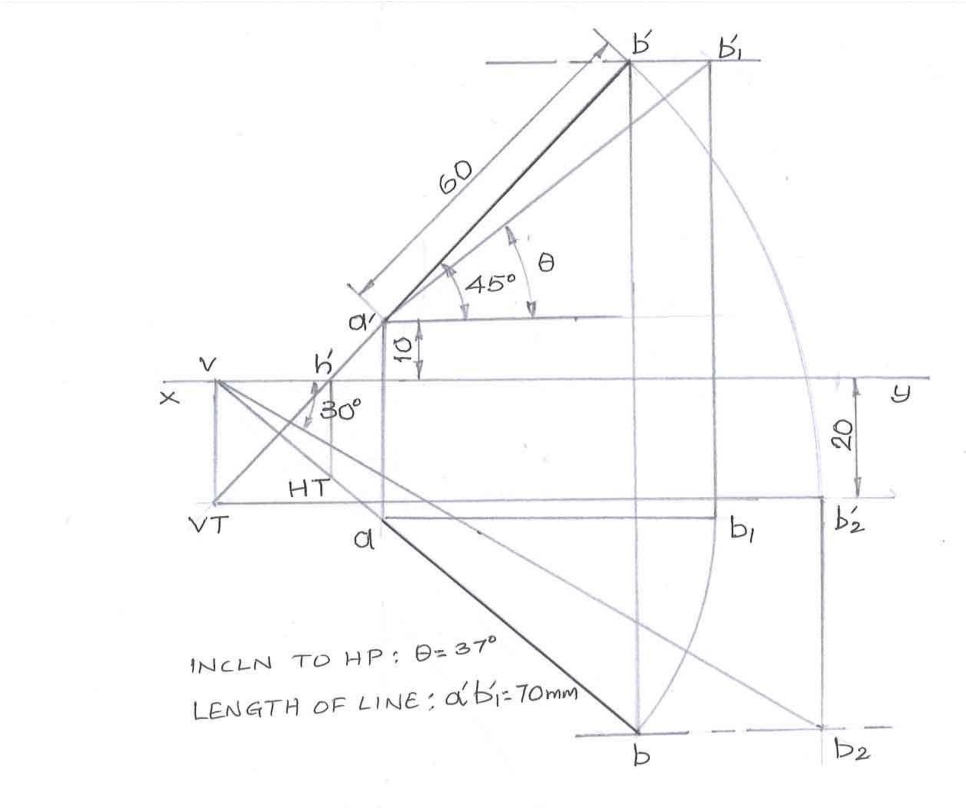


Figure 4.36 Projections of a line when inclination to one plane and a trace are given

Alternative method (Trapezoidal method)

a' may be marked on the projector 10 mm above x-y. ***a'b'***=60 mm may be constructed inclined 45^o to x-y. VT may be marked 20 mm below x-y after extending the front view ***a'b'***.

At VT an angle of 30^o may be constructed for ***a'b'***. From ***a'*** and ***b'*** perpendiculars may be drawn to intersect with the line drawn fromVT at A_1 and B_1, resepctively. A_1B_1 is the true length of the line AB. Projectors may be drawn through ***a'*** and ***b'***.

The distance ***a'A₁*** may be marked from x-y on the projector drawn from ***a'*** to get ***a***. The distance ***b'B₁*** may be marked from x-y on the projector drawn from ***b'*,** for ***b***. ***a*** and ***b*** may be joined. ***ab*** is the required top view of line AB.

Perpendiculars may be drawn from ***a*** and ***b***. On the perpendicular from **a** mark A_2 taking the distance of ***a'*** from x-y. Similarly, mark B_2 on the perpendicular drawn from ***b*** taking a distance ***b'*** from x-y. $A_2B_2 = A_1B_1$, which is true length of the line. Inclination between A_2B_2 and ***ab*** is the inclination of the line with HP and their point of intersection will give HT.

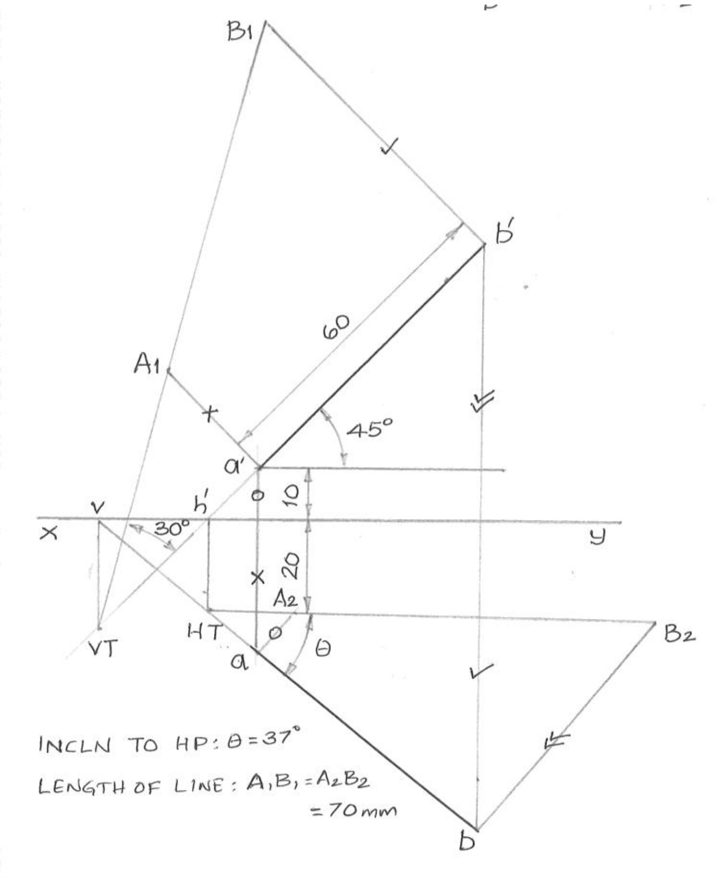


Figure 4.37 Alternative method (Trapezoidal method) for solving Problem 4. 19

Problem 4.20

A straight line AB, having end A 10 mm above HP, is inclined 30° HP. Its front view is 60 mm and inclined 45° inclined to x-y. The VT of the line is 20 mm below HP. Draw the projections and find out its true length, true inclination with VP and location of HT.

Solution

a' may be marked on the projector 10 mm above x-y. ***a'b'***=60 mm may be constructed inclined 45° to x-y. VT may be marked 20 mm below x-y after extending the front view ***a'b'***.

As the distance of the point A from VP is not given, the top view of A, ***a*** can not be fixed, VT and ***v*** may be taken as the projections of a point on the line. At VT a line may be drawn at an angle of 30^o inclined to x-y. This is intersecting the line drawn through **b'** parallel to x-y. Either ***VT-b$_1$'*** or ***VT-a$_1$'*** may be used to complete the solution. ***VT-a'*** may be made parallel to x-y, ***VT-a$_2$'***.

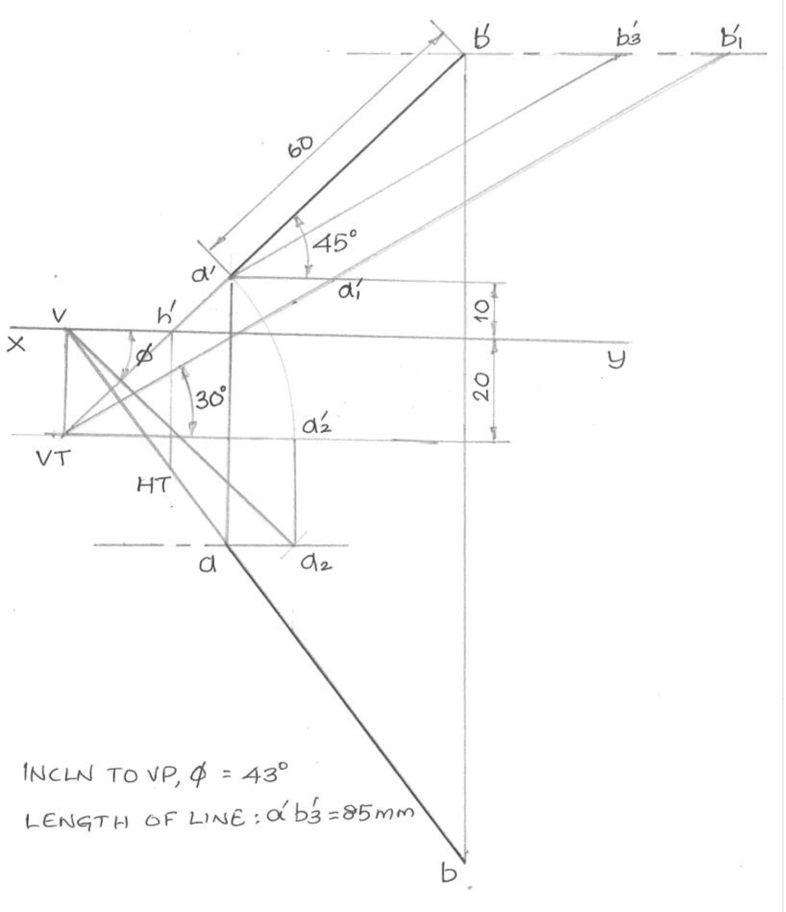


Figure 4.38 Projections of line-given inclination to HP and front view

From ***a$_2$'*** a projector is drawn. An arc is cut to this projector with a radius of ***VT-a$_1$'*** and the point of intersection is ***a$_2$***. ***VT-a$_1$'*** is the true length of the line VA; VT is the front view and v is the top view of the point V. A projector is drawn from ***a'*** to the line drawn through ***a$_2$***, parallel to x-y and ***a*** is marked. v and ***a*** may be joined. ***va*** may be extended to which a projector is drawn from ***b'*** and intesrected at ***b***. ***ab*** and ***a'b'*** are the projections of AB. A line my be drawn from ***a'*** at 30^o inclined to x-y which will give true length (***a'b$_3$')*** of the line. The inclination of ***a'b$_3$'*** to x-y is inclination of the line VP.

Chapter 5

Projections of Planes

Chapter 5

Projections of Planes

5.1 Planes

A plane is a two dimensional object (no height or no thickness, only length and breadth). Any two dimensional object like thin sheet of any shape or lamina is treated as a plane.

Planes can be classified according to their shape.

(i) Triangle (equilateral triangle, isosceles triangle, right angled triangle etc.

(ii) Quadrilateral (rectangle, square, parallelogram, rhombus, trapezium etc.)

(iii) Pentagon, hexagon, heptagon etc.

(iv) Circle, semi-circle, quarter circle

(v) Elliptical plane

5.2 Classifications with respect to position

Irrespective to the shape, a plane can situate at any of the positions given below:

(i) Planes perpendicular to one principal plane and parallel to the other

(ii) Planes perpendicular to both HP and VP.

(iii) Planes perpendicular to one plane and inclined to the other

(iv) Planes inclined to both HP and VP.

5.3 Plane parallel to one principal plane and perpendicular to the other

If a plane is parallel to a principal plane, then it is perpendicular to the other plane. But, if a plane is perpendicular to a plane, then it can be either parallel or perpendicular or inclined to the other plane.

Problem 5.1

A rectangular lamina, length 70 mm and breadth 40 mm, is placed in the first quadrant. The nearest longer side which is parallel to VP is at 15 mm from VP and the plane is parallel to and 20 mm away from HP. Draw the projections.

Analysis

The plane is a rectangle, say ABCD. All sides are parallel to HP. The sides can be treated as independent straight lines. AB, BC, CD, and DA. As all these lines are parallel to HP, projections of all lines will give true length in the top view. Or, the top view of the plane will give the true shape.

As all points on the plane are equi-distant to HP, and the distance from HP to the points will be visible in the front view, the front view of the plane will be a line parallel to x-y. As longer sides are parallel to VP, the front view gives true length of the longer side and as shorter sides are perpendicular to VP, the projections of these sides will appear as points in the front view.

- ***If a plane is parallel to a principal plane, the projection on that plane will give true shape.***
- ***If a plane is perpendicular to a principal plane the projection on that plane will be a straight line.***

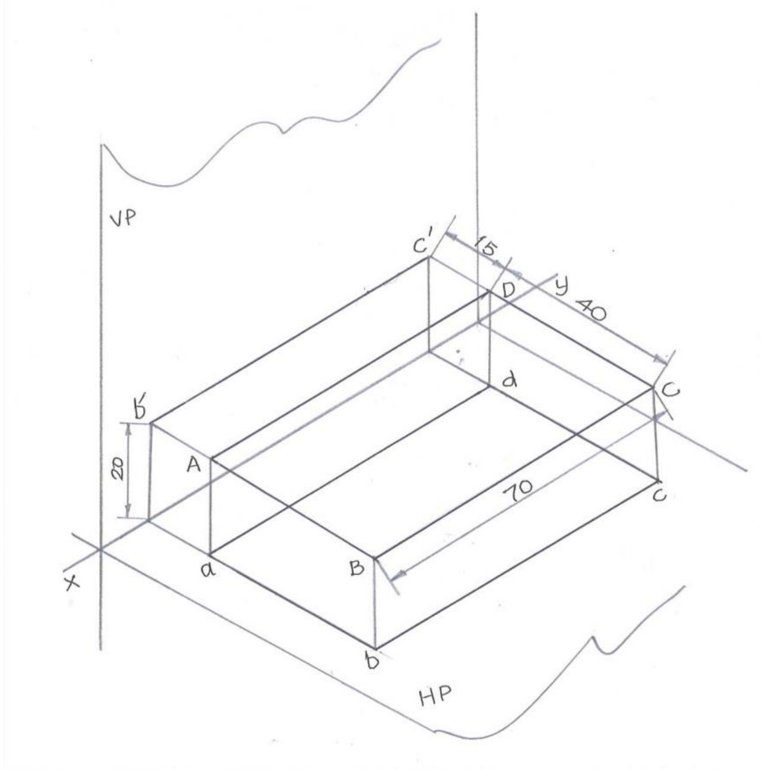


Figure 5.1 Rectangular lamina in the first quadrant

Solution

Figure 5.1 gives the position of the rectangle in three dimensions. After drawing x-y, two projectors may be drawn 70 mm apart. On one projector, below x-y, ***a*** and ***b*** may be marked respectively at 15 mm and 55 mm from x-y. As AB is perpendicular to VP, its projections on VP will be a point. As ***b'*** and ***a'*** are coinciding in the front view, the point may be marked at 20 mm above x-y. Similarly ***d*** and ***c*** at 15 mm and 55 mm below x-y, and ***d'*** and ***c***' may be marked at 20 mm above x-y. ***a, b, c,*** and ***d*** may be joined and finished to get the top view. ***b'*** and ***c′*** may be joined and finished for front view (refer Figure 5.2).

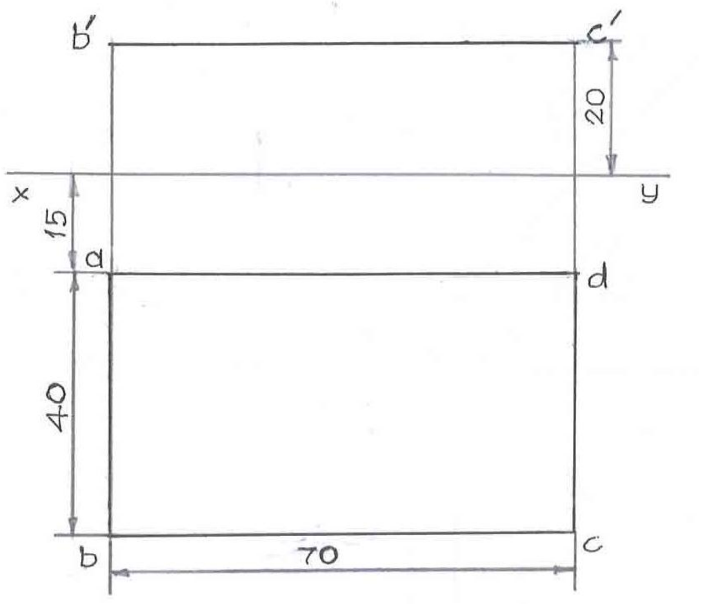


Figure 5.2 Projections of the rectangle, plane parallel to HP

Problem 5.2

An equilateral triangle of sides 60 mm long is parallel to VP and perpendicular to HP. Draw the projections if one side is inclined at 45^{o} HP.

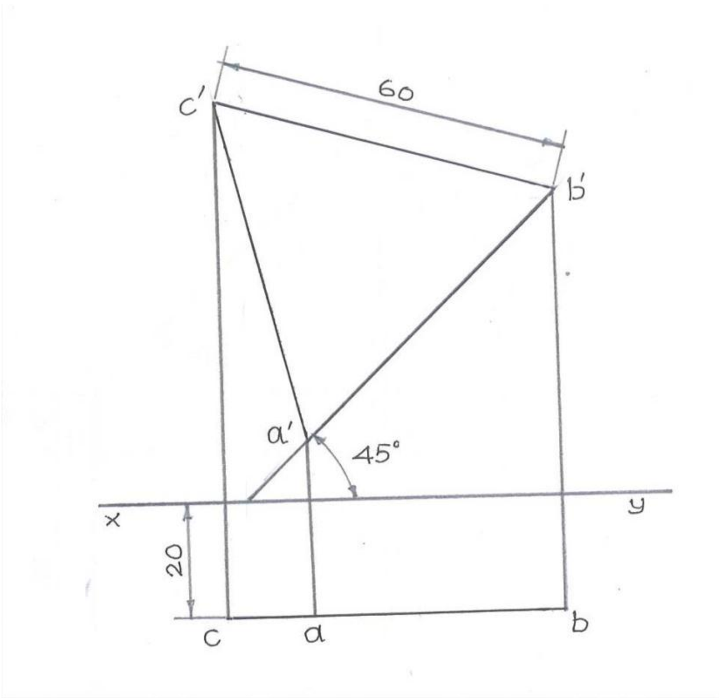


Figure 5.3 Projections of equilateral triangle parallel to VP

Solution

Since the plane is parallel to VP, front view will give the true shape. One side of the triangle is inclined at 45^o to HP. This line is parallel to VP and inclined 45^o to HP. In the front view this line is visible as not only true length but its inclination to x-y will be true inclination to HP also.

A line of 60 mm long is drawn at 45 ^{o} to x-y. Construct the equilateral triangle as the 60 mm long line as one side. This is the required front view ***a'b'c'*** of the triangle. A line at 20 mm below x-y may be drawn. The corner points A, B and C may be projected to the line. The top view ***abc*** is also available now (refer Figure 5.3).

5.4 Plane perpendicular to both HP and VP

Problem 5.3

An equilateral triangular lamina of sides 50 mm is placed on HP in such a way that its plane is perpendicular to both HP and VP. Draw the projections.

Analysis

Since the triangular plane is perpendicular to both HP and VP (or lying on profile plane), both top view and front view will be straight lines perpendicular to x-y.

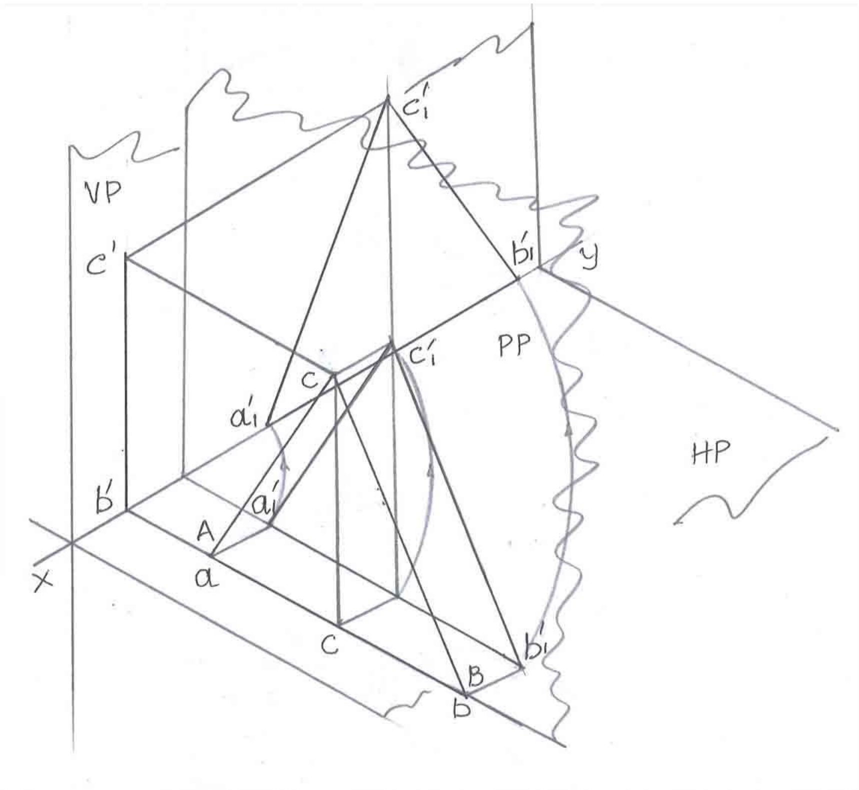


Figure 5.4 Equilateral triangle on HP with plane perpendicular to HP and VP

When the plane is perpendicular to HP with one side on HP, the top view can be drawn. The front view can be drawn easily after drawing the side view. The triangular plane may be projected to the profile plane to get the true shape. The profile plane may be rotated about the line of intersection between VP and PP, to make the front view and side view on the same plane. The distance of the vertex from the base (length of ***b'c'***) may be obtained here from the side view.

Solution

After drawing x-y, equilateral triangle $a_1'b_1'c_1'$ may be drawn as one side on x-y. This is the side view (from left) of the triangle in its position. The length of the top view will be $a_1'b_1'$ which can be projected either by cutting arcs or drawing lines 45^o to x-y. ***acb*** is the required top view. The front view ***b'c'*** may be projected as per the construction given in Figure 5.5.

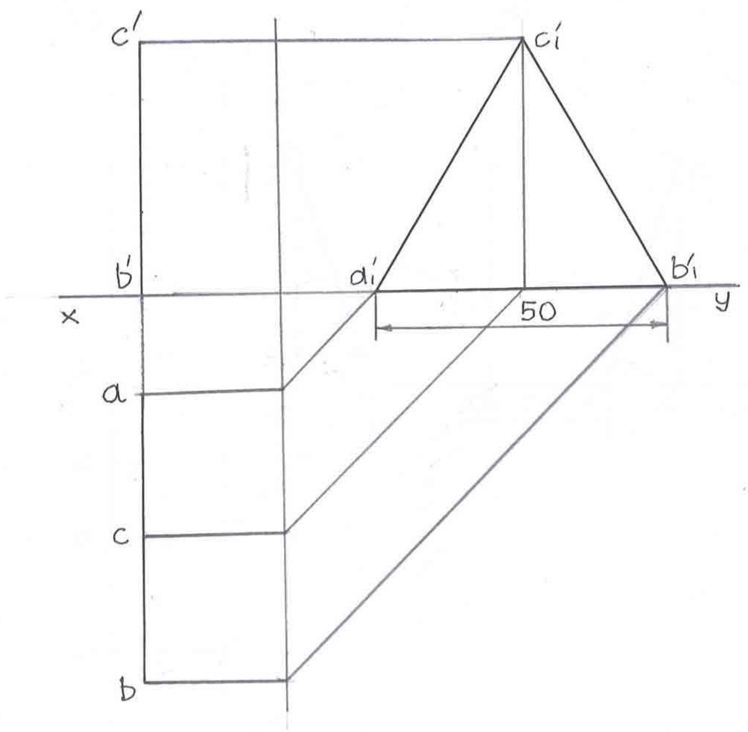


Figure 5.5 Projections of equilateral triangle perpendicular to HP and VP

Problem 5.4

A pentagonal lamina of sides 40 mm is resting on HP with the plane perpendicular to both HP and VP. Draw the projections.

Solution

A regular pentagon $a_1'b_1'c_1'd_1'e_1'$ may be drawn with one side, $a_1'b_1'$ on x-y. This will give the vertical distance of D from AB (distance of d_1' from $a_1'b_1'$) and horizontal distance between C and E (horizontal distance between c_1' and e_1').

The front view ***b'c'd'*** and top view ***eadbc*** may be projected (refer Figure 5.6) and finished.

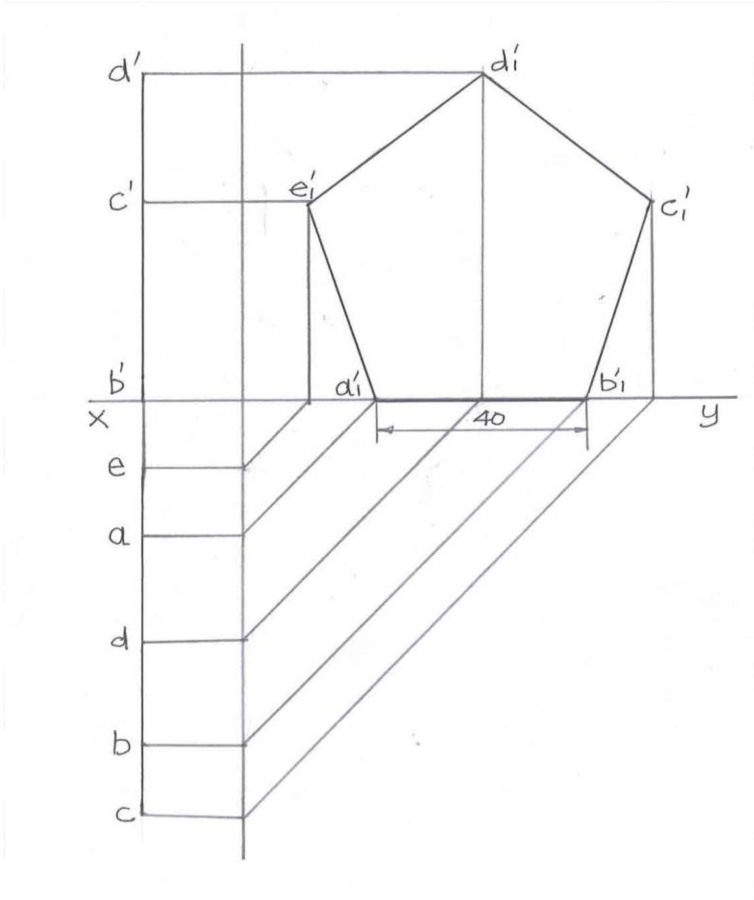


Figure 5.6 Pentagon perpendicular to both HP and VP

- ***When a plane is perpendicular to a principal plane, no edge or a specific line need not be perpendicular to the principal plane. But, when an edge or a specific line is perpendicular to a principal plane, then the plane will be perpendicular to that principal plane.***

5.5 Plane perpendicular to one principal plane and inclined to the other

Problem 5.5

A rectangular lamina, length 70 mm and breadth 40 mm, is resting on the HP with a shorter side. Draw the projections if the plane makes 45^o to HP.

Analysis

The rectangular lamina is inclined 45^o to HP and perpendicular to VP. This position can be achieved by two steps.

Step 1: The plane may be assumed to be parallel to HP, with the shorter side (the side on the HP in the final position) on the HP and perpendicular to VP. Here the plane will be lying on HP. The front view will be a straight line having length equal to longer side. The top view can be drawn first as true shape, and front view can be projected.

Step 2: When the plane is inclined to HP, keeping the plane perpendicular to VP, the front view will be a straight line and its length will be same as in Step 1, but appear 45^o inclined to x-y. When the plane is inclined to HP keeping a shorter side on HP, the other shorter side is parallel to HP and the longer sides are inclined to HP. The shorter sides will be visible as true and the longer sides will appear as reduced length in the top view. The final top view can be projected if the front view is drawn.

Solution

Step 1: After assuming the plane parallel to HP, with one shorter side on HP and perpendicular to VP, the top view may be drawn. It will be a rectangle ($a_1b_1c_1d_1$) having true dimensions, length 70 mm and width 40 mm, with a shorter side perpendicular to x-y. The front view may be projected; it is a line lying on x-y ($b_1'c_1'$).

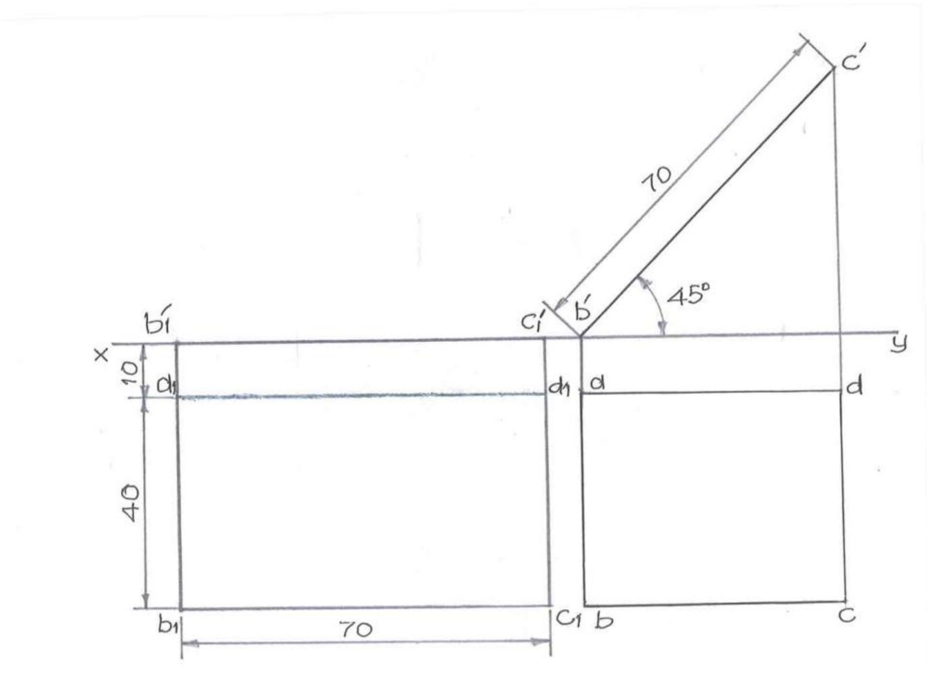


Figure 5.7 Rectangular lamina inclined to HP and perpendicular to VP

Step 2: The length of the final front view is equal to the length of front view in the first step. $b'c'$ =70 mm may be drawn inclined 45^o to x-y. The final top view may be projected from the front view. From a_1 and d_1, a and d, and from b_1 and

c_1*, *b and ***c*** can be obtained.

abcd and ***b'c'*** may be finished to get the final projections (refer Figure 5.7).

- ***For keeping an edge of a plane on a principal plane, the edge must be taken as perpendicular to the other plane.***

Alternative solution

Step 1: The plane may be assumed to be parallel to HP, with one shorter side on HP and perpendicular to VP. The top view will be a rectangle (***abc_1d_1***) having true dimensions, length 70 mm and width 40 mm, with a shorter side perpendicular to x-y. The front view may be projected; it is a line lying on x-y (***$b'c_1'$***).

Step 2: The length of the final front view is equal to the length of front view (***$b'c_1'$***) in the first step. ***b'c'*** =70 mm may be drawn inclined 45^o to x-y. The final top view may be projected from the front view. From ***c', c*** and ***d*** can be obtained.

abcd and ***b'c'*** may be finished to get the final projections (refer Figure 5.8).

This solution is no way different from the previous one; except in the less space and time consumed.

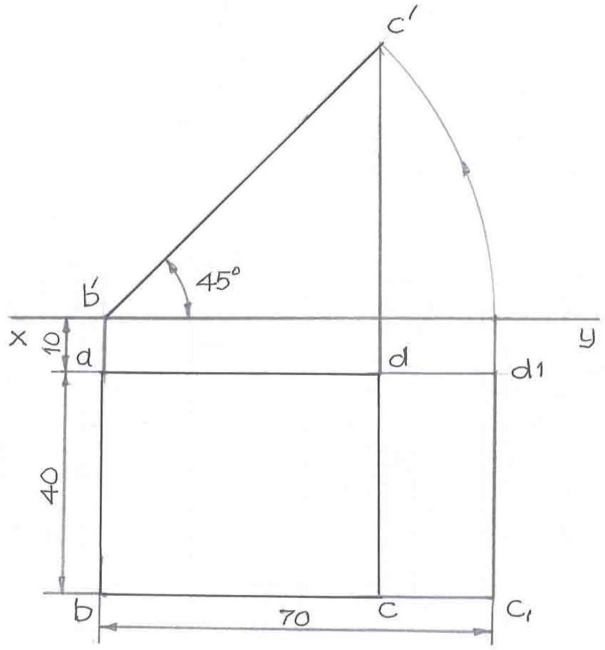


Figure 5.8 Rectangular lamina inclined to HP and perpendicular to VP

Problem 5.6

A regular pentagon of sides 40 mm is resting on VP with a side. Draw the projections if the plane is inclined 35^o to VP and perpendicular to HP.

Analysis

The plane is inclined to VP and perpendicular to HP. In the first step, the plane must be assumed to be parallel to VP, with one side on VP and perpendicular to HP. Or, the plane is lying on VP, with one side perpendicular to HP. The front view will have true shape and top view will be a line. The top view may be made inclined 35^o to x-y. The front view may be projected. For saving time and space, the projections can be drawn as shown in Figure 5.9.

Solution

Draw a pentagon of sides 40 mm, above x-y with one side perpendicular to x-y. ***a'b'c₁'d₁'e₁'*** is the front view of the plane in the first step. ***ae₁d₁*** is the corresponding top view.

ad₁ is the length of the final top view. At ***a*** an angle of 35^o may be constructed. ***A*** as centre ***ae₁*** and ***ad₁*** as radii, arcs are cut to the line and the final top view ***aed*** may be drawn. ***aed*** may be projected to get the final front view ***a'b'c'd'e'***.

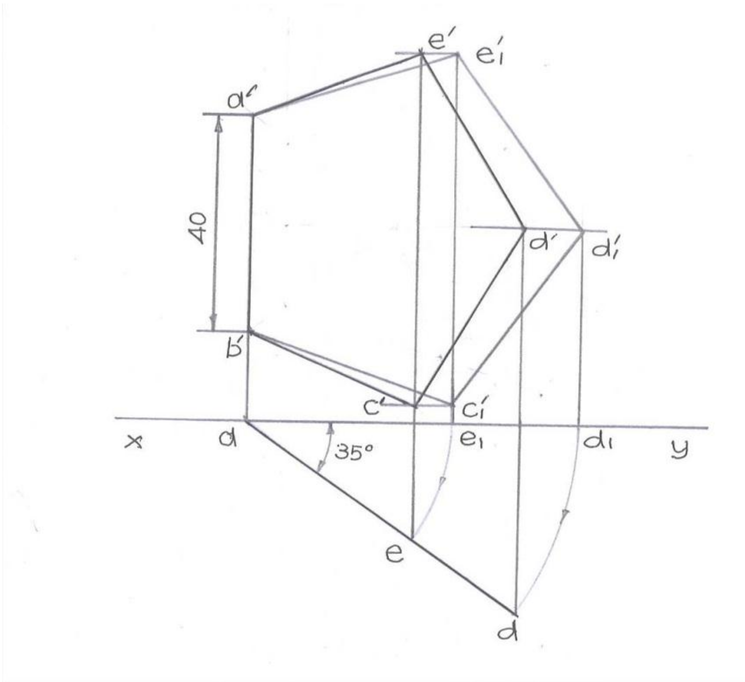


Figure 5.9 Pentagon inclined to VP and perpendicular to HP

Problem 5.7

A circle of diameter 80 mm rests on HP with a point on its circumference. Draw the projections if the plane is inclined 45^o to HP.

Solution

The plane is assumed to be lying on HP. Draw a circle of diameter 80 mm below x-y. It may be projected to x-y. The circle may be divided in to 12 equal parts. They may be numbered sequentially as ***1, 2, 3, ……12***. Please note that if ***1*** must be on the HP in the final position, the diameter ***1-7*** must be parallel to VP or ***1-7*** may be parallel to x-y in the top view. All divisions may be marked in the front view also. ***123….12*** is the top view while ***1'2'3'4'5'6'7'*** is the front view (refer Figure 5.10).

At ***1*** an angle of 45^o may be drawn and ***2', 3', 4', 5', 6', 7'*** may be marked on that line. ***1'-7'*** is the required front view of the circle. The corresponding top view may be projected from the front view and previous top view as shown in Figure 5.10. Since the circular plane is inclined to HP, the top view appears to be an ellipse.

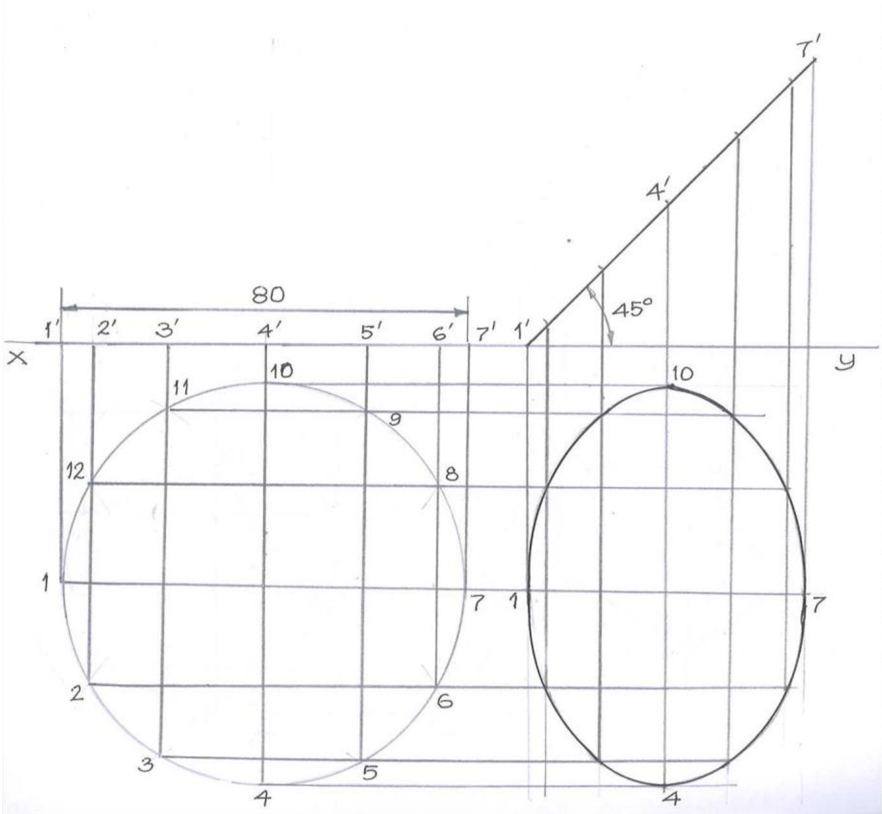


Figure 5.10 Circular plane inclined to HP and perpendicular to VP

- ***For keeping a point on the periphery of a plane on a principal plane, a line passing through the point must be taken as parallel to the other plane.***

Problem 5.8

An isosceles triangle, base 40 mm and altitude 75 mm, appears to be an equilateral triangle of sides 40 mm in the front view. Draw the projections of the triangle if the plane is perpendicular to HP. Also find its inclination to VP.

Solution

The isosceles triangle having base 40 mm appears to be an equilateral triangle of side 40 mm. Naturally the base is parallel to VP, but the altitude is shortened by the inclination of the plane with VP.

Assuming the isosceles triangle parallel to VP, with the base perpendicular to HP, the front view ***a'b'c₁'*** as the true shape and top view ***ac₁*** as a line may be drawn. With the base ***a'b'*** as common side equilateral triangle ***a'b'c'*** may be constructed. This is the front view of the plane. A projector may be dropped from ***c'***. ***a*** as centre and ***ac₁*** as radius an arc is drawn to the projector from ***c'*** and the meeting point on the projector is marked as ***c***. ***a*** and ***c*** may be joined to get the final top view. ***a'b'c'*** and ***ac*** may be finished (Figure 5.11).

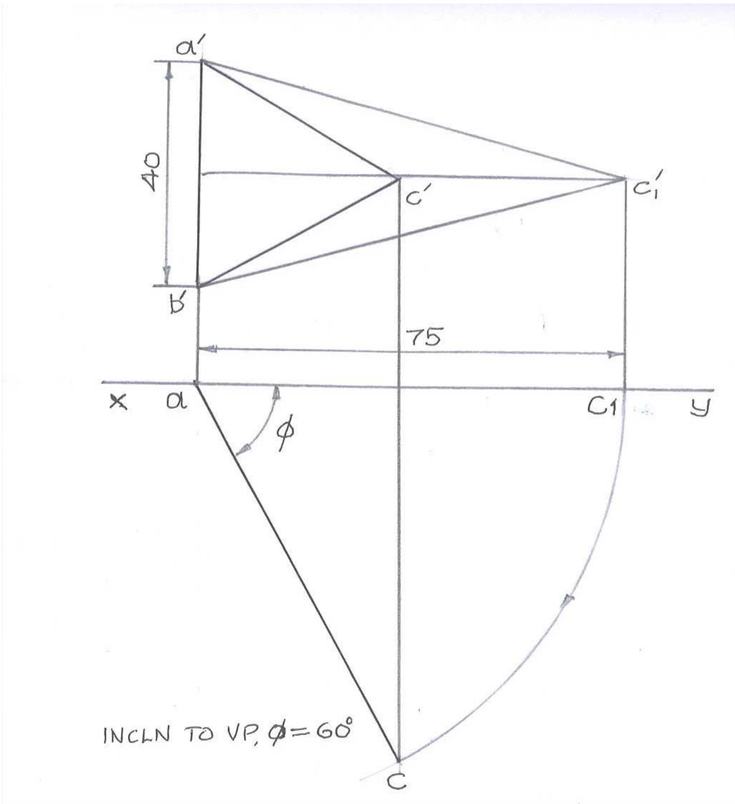


Figure 5.11 Isosceles triangle appears to be an equilateral triangle in the front view

5.6 Planes inclined to both HP and VP

Planes inclined to both the principal planes are called as oblique planes. The projections of planes inclined to both HP and VP can be solved in three steps. Such problems of planes are typically stated in two ways:

(i) The inclination of the plane to a principal plane will be given directly;

along with a line (edge, diagonal, diameter etc.) on the plane parallel to that plane, and inclination of the referred line to the other principal plane will also be given.

(ii) The inclinations of the plane to both the HP and VP will be given. Here in place of plane, sometimes line may also be referred, but as good as plane only.

Problem 5.9

A pentagonal lamina, 30 mm sides, is resting on HP with one of its sides. The plane of the pentagon is inclined at 45^o to HP. The side with which it is resting on HP, is inclined at 30^o to VP. Draw the projections of the lamina.

Analysis

The plane of the pentagon is inclined 45^o to HP with one side on HP. It is also given that the edge which is on HP is inclined 30^o to VP.

Step 1: This problem is falling in the first category. The pentagon may be assumed to be parallel to HP, with one side on HP and perpendicular to VP. The projections may be drawn. The top view will be of true shape and front view will be a straight line.

Step 2: The length of front view in the first step and second step will be same, but 45^o inclined to x-y in the second step. The front view, which is a line, may be made inclined to x-y. The top view may be drawn.

Step 3: As the plane keeps 45^o to HP, irrespective to the position of the plane with VP, top view remains same. The side which is on HP will be always equal to true length in the top view and it will be inclined at 30^o to x-y. From this position the final front view can be obtained.

It may be noted that while making the plane inclined to VP, keeping the inclination to HP same, the corners will be moving along a straight line parallel to x-y in the front view. Or, the distance of every corner of the plane from HP will respectively be equal when the plane is inclined to VP and perpendicular to VP.

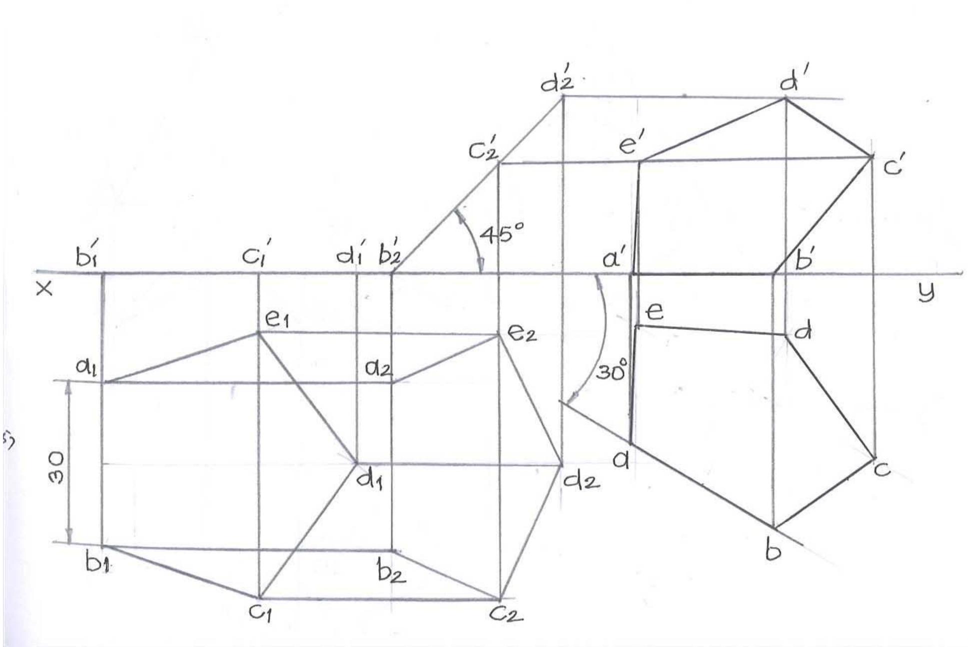


Figure 5.12 Pentagon inclined resting on HP, plane inclined 45° to HP, and the side on HP inclined at 30° to VP

Solution

Step 1: The pentagon may be assumed to be parallel to HP, with one side on HP, and perpendicular to VP. A pentagon $a_1b_1c_1d_1e_1$, having side 30 mm, may be drawn as one side a_1b_1 perpendicular to x-y, below x-y. This is the top view of the pentagon. The corresponding front view $b_1'c_1'd_1'$ may be drawn, which is a straight line coinciding to x-y.

Step 2: The length of front view in the first step and second step will be same, but 45° inclined in the second step. The line, $b_2'c_2'd_2'=b_1'c_1'd_1'$ may be drawn inclined 45° to x-y. The corresponding top view $a_2b_2c_2d_2e_2$ may be obtained.

Step 3: The plane keeps 45° to HP, irrespective to the position of the plane with VP, top view remains same and the side which is on HP will be always equal to true length in the top view. Since the edge AB is inclined 30° to VP, ***ab*** will be inclined at 30° to x-y. The top view $a_2b_2c_2d_2e_2$ may be re-constructed as ***abcde*** with ***ab*** 30° inclined to x-y. The final front view ***a'b'c'd'e'*** may be obtained projecting from the top view and drawing parallel lines from the front view in Step 2 (refer Figure 5.12).

Problem 5.10

A pentagonal lamina, 30 mm sides, is resting on HP with one of its sides. The plane of the pentagon is inclined at 45° to HP. The perpendicular bisector of the side with which it is resting on HP, is inclined at 30^o to VP. Draw the projections of the lamina.

Analysis

The pentagon is resting on HP with a side. The plane makes 45^o to HP and the perpendicular bisector of the side with which it is resting on HP is inclined 30^o to VP.

While carefully analyzing, it can be observed that the perpendicular bisector of the side which is on HP is inclined 45^o to HP and 30^o to VP. In such cases, in the first step, the line which is to be made inclined to both HP and VP, (here perpendicular bisector of the side on HP) is assumed to be parallel to both HP and VP.

Step 1: This problem is falling in the second category. The pentagon may be assumed to be parallel to HP, with one side on HP, and the perpendicular bisector of the side parallel to VP (the side will be perpendicular to VP). The projections are drawn. The top view will be of true shape and front view will be a straight line.

Step 2: The length of front view in the first step and second step will be same, but 45^o inclined to x-y in the second step. The front view may be drawn inclined to 45^o to x-y. The top view may be projected.

Step 3: As the plane keeps 45^o inclined to HP (the perpendicular bisector of the side makes 45^o to HP), irrespective to the position of the plane with VP, top view remains same. The side which is on HP will be always equal to true length in the top view. But, the perpendicular bisector is inclined at 30^o to VP. After obtaining the apparent inclination of the perpendicular bisector, the final position of top view may be drawn and the final front view may be obtained.

Solution

Step 1: The pentagon may be assumed to be parallel to HP, with one side on HP, and perpendicular to VP. A pentagon $a_1b_1c_1d_1e_1$, having side 30 mm, may be drawn below x-y as one side a_1b_1 perpendicular to x-y. This is the top view of the pentagon. The perpendicular bisector of a_1b_1, $1d_1$ is parallel to VP also. The corresponding front view $b_1'c_1'd_1'$ may be drawn, which is a straight line coinciding to x-y.

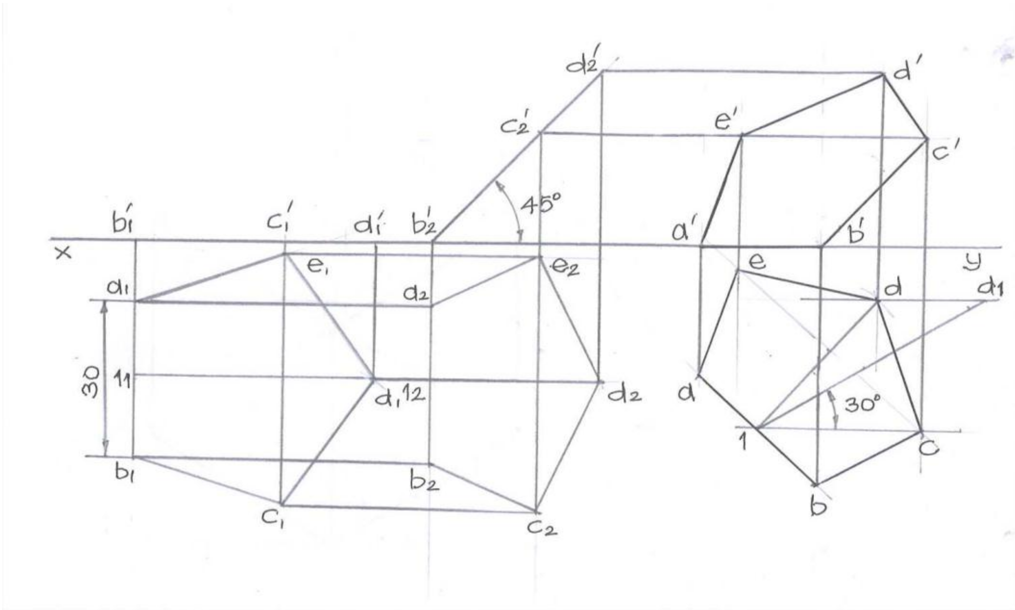


Figure 5.13 Pentagon inclined resting on HP, plane inclined 45^o to HP, and perpendicular bisector of the side on HP inclined 30^o to VP

Step 2: The length of front view in the first step and second step will be same, but 45^o inclined to x-y in the second step. The line, $b_2'c_2'd_2' = b_1'c_1'd_1'$ may be drawn inclined 45^o to x-y. By this construction, the perpendicular bisector $1d_2$ is made inclined 45^o to HP. The corresponding top view $a_2b_2c_2d_2e_2$ may be projected.

Step 3: The plane keeps 45^o to HP. Irrespective to the position of the plane with VP, top view remains same and the side which is on HP will be always equal to true length. Since the perpendicular bisector is inclined 30^o to VP, its apparent inclination to VP may be found out. $1d_1$, the true length of perpendicular bisector, may be drawn at 30^o to x-y. The locus of ***d*** may be drawn through d_1 parallel to x-y. With ***1*** as centre and $1d_2$ (length of top view) as radius an arc is drawn to the locus of ***d***. The top view in Step-2 may be reconstructed (***abcde***) with ***1d*** as the perpendicular bisector. The final front view ***a'b'c'd'e'*** may be obtained projecting from the top view and drawing parallel lines from the front view in Step 2 (refer Figure 5.13).

Problem 5.11

A regular hexagon, sides 40 mm long, is resting on HP with a side. Draw the projections if the plane is inclined 60^o to HP and 30 ^{o} to VP. The side opposite to the side on HP is on VP.

Solution

Step 1: The hexagon ABCDE may be assumed to be parallel to HP, with one side

AB on HP, and perpendicular to VP. A hexagon $a_1b_1c_1d_1e_1f_1$, having side 40 mm, may be drawn below x-y as one side a_1b_1 perpendicular to x-y. This is the top view of the hexagon. The perpendicular bisector of a_1b_1 (if drawn) is parallel to VP also. The corresponding front view $b_1'c_1'd_1'$ may be drawn, which is a straight line coinciding with x-y.

Step 2: The length of front view in the first step and second step will be same, but 60^o inclined to x-y in the second step. The line, $b_2'c_2'd_2'=b_1'c_1'd_1'$ may be drawn inclined 60^o to x-y. By this construction, the perpendicular bisector of the side on HP is made inclined 60^o to HP. The corresponding top view $a_2b_2c_2d_2e_2f_2$ may be obtained.

Step 3: The plane keeps 60^o to HP. Irrespective to the position of the plane with VP, top view remains same and the side which is on HP will be always equal to true length. Since the perpendicular bisector is inclined 30^o to VP, its apparent inclination to VP may be found out.

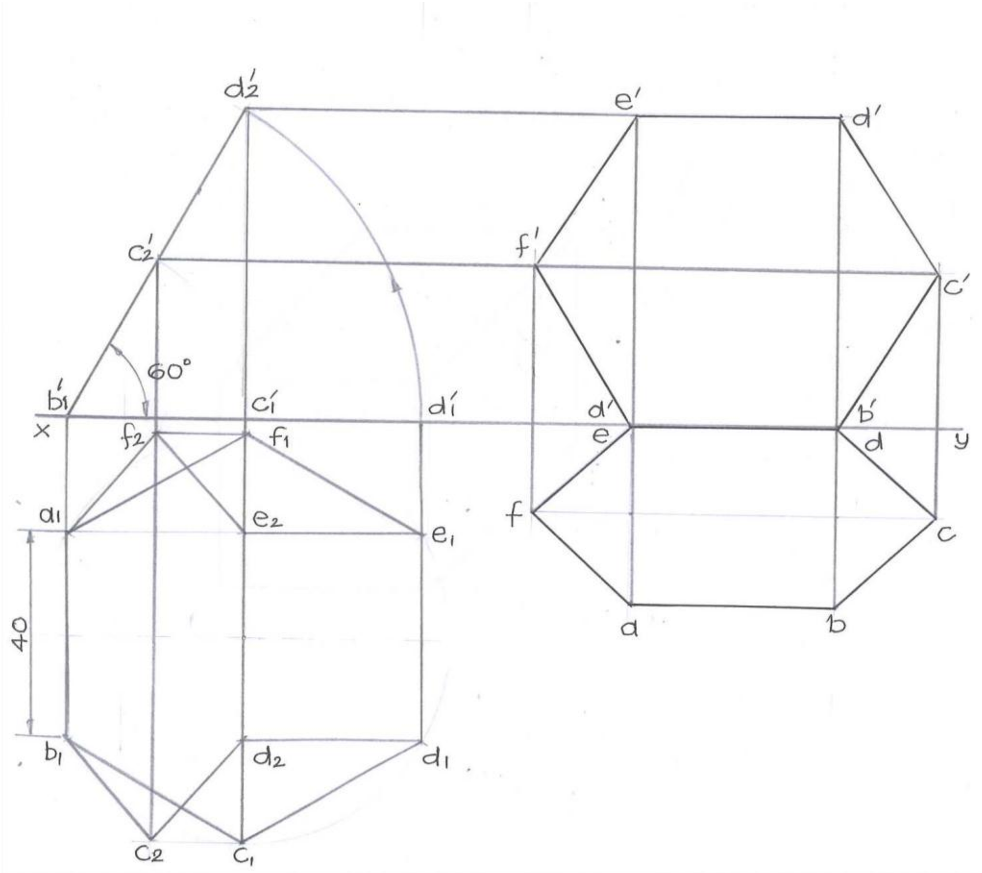


Figure 5.13 Hexagon inclined 60^o to to HP and 30^o to VP

Being inclinations to HP and VP are complimentary, the perpendicular bisector (if drawn) will be lying on a profile plane and its projections will be perpendicular to x-y. The solution can be simplified here. As the side opposite (DE) to one on HP (AB) is on VP, its top view (***de***) will be on x-y. The front view (***a′b′***) of the side on HP will be on x-y.

The top view $a_2b_2c_2d_2e_2f_2$ may be re-constructed (***abcde***) in such a way that ***de*** is on x-y. Now the perpendicular bisector of AB (if drawn) will be perpendicular to x-y The final front view ***a'b'c'd'e'*** may be obtained projecting from the top view and drawing parallel lines from the front view in Step 2 (refer Figure 5.14).

Chapter 6

Projections on Auxiliary Planes

Chapter 6

Projections on Auxiliary Planes

6.1 Auxiliary Planes

Profile plane (PP), auxiliary vertical plane (AVP), and auxiliary inclined plane (AIP) are the auxiliary planes employed in orthographic projections. Depending up on the complexity of the object, when two views (top view and front view) of the object are not sufficient, projections on auxiliary planes are also used. The projections on auxiliary planes are called as auxiliary projections.

In addition to the above, projections of objects (irrespective to the dimensions, line, plane or solid) especially in oblique positions can be obtained conveniently by auxiliary projection method. The time required for drawing and space consumed will be less compared to conventional method when auxiliary projection method is employed. Auxiliary projections can also be used to determine (i) the true length of straight lines and true inclinations to the principal planes if projections are given and (ii) true shape of planes and true inclinations to principal planes.

The principle of auxiliary projections is illustrated here using a few examples.

6.2 Projection on Profile Plane

Problem 6.1

A point A is 40 mm above HP and 30 mm in front of VP. Draw its side view from left.

Analysis

The point A is assumed to be in the first quadrant. A may be projected to HP, VP and a profile plane (PP) which is on right side (refer Figure 6.1). The top view ***a***, front view ***a***' and projection on PP, $\boldsymbol{a_1'}$ are respectively available. Referring the figure, $\boldsymbol{Aao_1a_1'}$ can be shown as a rectangle. Or, the distance of $\boldsymbol{Aa = a_1'o_1}$.

- ***The distance of side view of a point from x-y equals to the distance of the point from HP (same as the distance of front view from x-y).***

All the three projections may be made on a single plane. After detaching HP from PP, viewing along x-y, HP may be rotated in clock-wise direction to make it coinciding with VP. Now top view and front view are on the same plane.

Viewing from the front, the PP may be rotated about the line of intersection of VP and PP, and PP may be made coinciding with VP. The top view, front and side view will be available on a single plane (VP).

The side view from left is presented on the right side of the front view as per Figure 6.2.

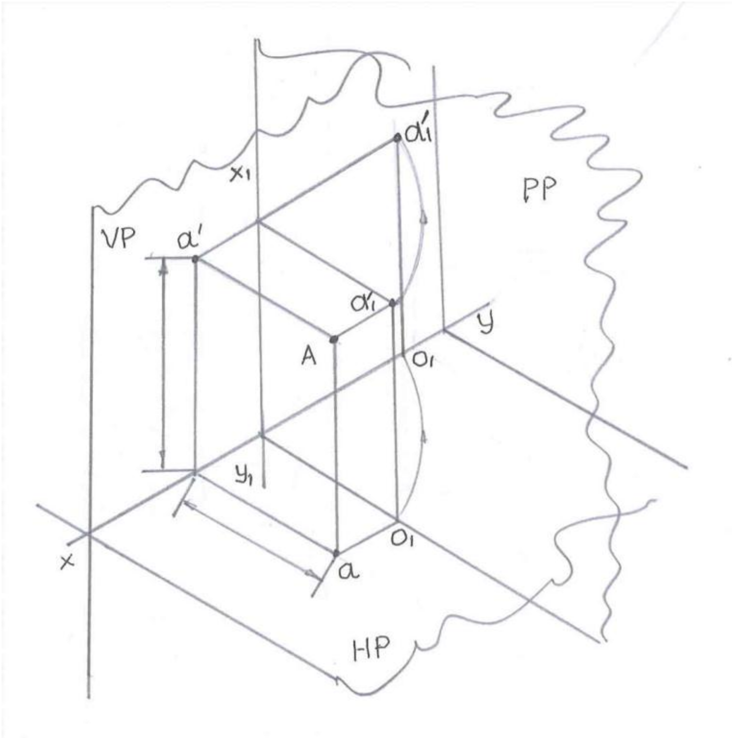


Figure 6.1 Projections of A on HP, VP and PP

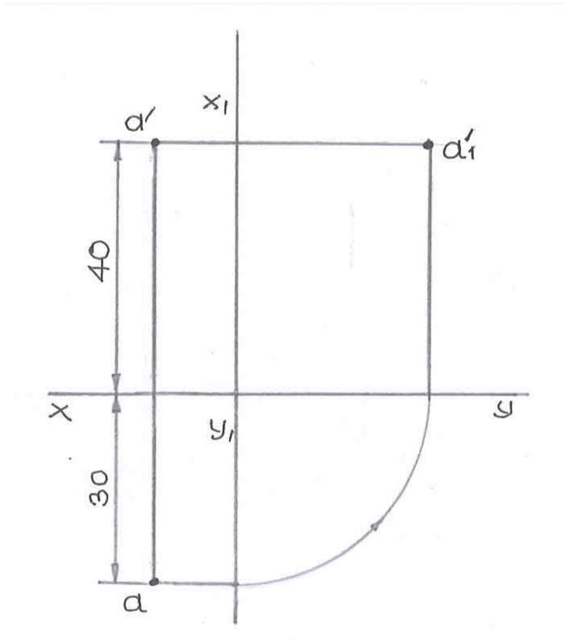


Figure 6.2 Side view from left for a point

Solution

After drawing a projector, ***a′*** and ***a*** may be marked taking distances 40 mm above x-y and 30 mm below x-y on the projector. The line of intersection of VP and PP, x_1-y_1 may be shown conveniently. Parallel lines to x-y may be drawn from the top view ***a*** and front view ***a***' to meet x_1-y_1. From the point of intersection of the line from ***a*** and extended x_1-y_1 an arc is cut with the point of intersection of x_1-y_1 and x-y as centre. From the cutting point of arc with x-y, a projector is drawn to intersect with the line from ***a***' parallel to x-y. The meeting point is $\boldsymbol{a_1}$', the required side view from left. The distance of $\boldsymbol{a_1'}$ from x-y is equal to the distance from x-y to ***a'*** (refer Figure 6.2).

6.3 Projection on auxiliary vertical plane

Problem 6.2

A point B is 40 mm above HP and 30 mm in front of VP. Draw its projection on an AVP which is inclined 45° to VP.

Analysis

The point B is in the first quadrant. B may be projected to HP, VP and to auxiliary vertical plane (AVP) (refer Figure 6.3). The top view ***b***, front view ***b***' and projection on AVP, $\boldsymbol{a_1'}$ are respectively obtained. Referring the figure, $\boldsymbol{Bbo_1b_1'}$ is a rectangle and hence the distance of $\boldsymbol{Bb = b_1'o_1}$=distance of ***b'*** from x-y.

- ***The distance of auxiliary front view of a point from auxiliary reference line equals to the distance of the point from HP (same as the distance of front view from x-y).***

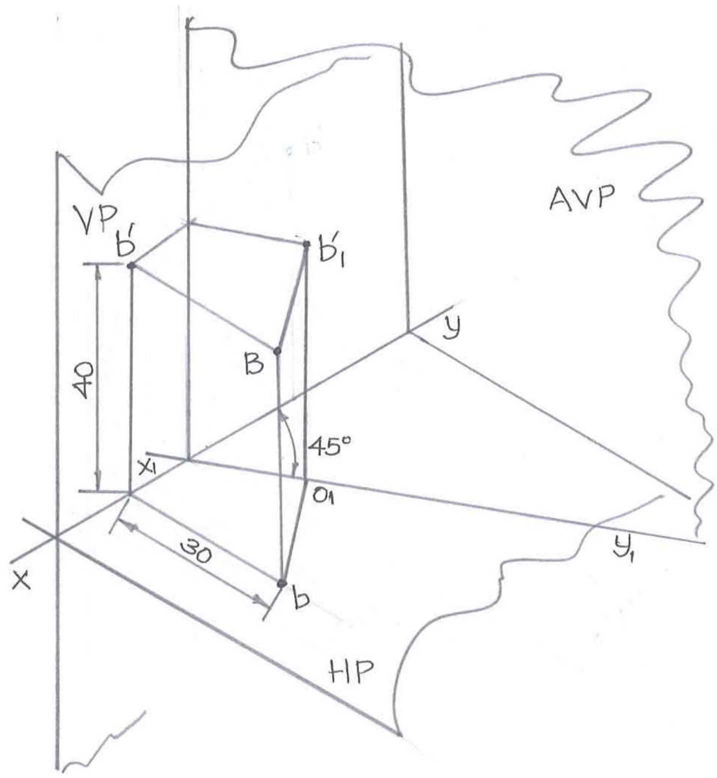


Figure 6.3 Point is projected to auxilairy vertical plane

After detaching AVP from VP, viewing along x-y, HP may be rotated in the clock-wise direction to make it coinciding with VP. Now top view and front view are on the same plane.

AVP may be rotated about the line of intersection of HP and AVP, and AVP may be made coinciding with VP (already HP and VP are coincided). The top view, front view and auxiliary front view will be on a single plane (VP) (refer Figure 6.4).

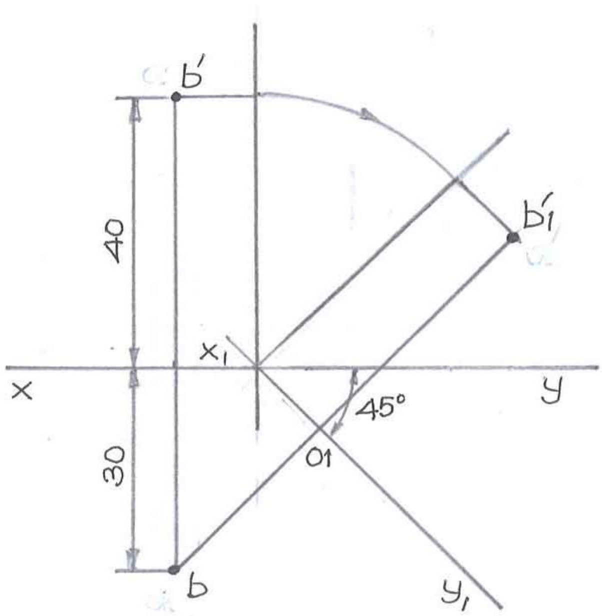


Figure 6.4 (a) Projection of a point on auxiliary vertical plane

Solution

x-y may be drawn conveniently. ***b'*** and ***b*** may be marked on a projector taking distances 40 mm above x-y and 30 mm below x-y. The line of intersection of HP and AVP, x_1-y_1 may be shown at an angle of 45^o to x-y. Parallel line to x-y may be drawn from front view ***b***' to meet the vertical line to x-y drawn from the point of intersection of x-y and x_1-y_1. From ***b*** a projector is drawn to x_1-y_1. An arc is cut from the meeting point of vertical line to x-y with the point of intersection of x_1-y_1 and x-y as centre. From the cutting point of arc on the line parallel to the projector to x_1-y_1, a parallel line is drawn to x_1-y_1. This parallel line will meet the projector for x_1-y_1 at ***b_1***', the required auxiliary front view. The distance of ***b_1'*** from x_1-y_1 is equal to the distance of ***b'*** from x-y (refer Figure 6.4a).

Alternative Solution

x-y may be drawn conveniently. ***b'*** and ***b*** may be marked on a projector taking distances 40 mm above x-y and 30 mm below x-y. The line of intersection of HP and AVP, x_1-y_1 may be shown at an angle of 45^o to x-y.

From ***b*** a projector is drawn to x_1-y_1. The distance of ***b'*** from x-y may be taken and ***b₁'*** may be marked on the projector drawn from ***b*** to x_1-y_1. ***b₁'*** is the required auxiliary front view of point B (Figure 6.4b).

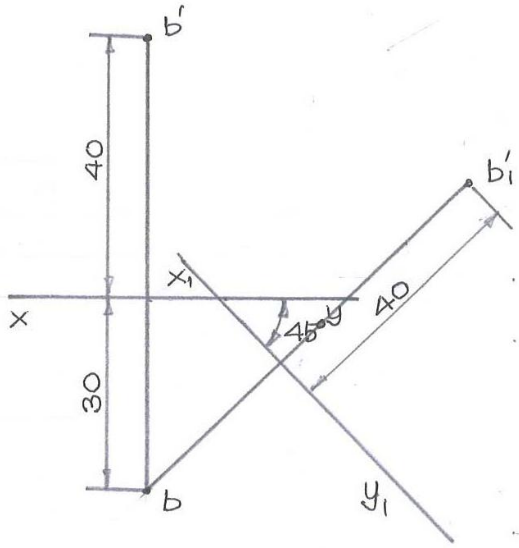


Figure 6.4 (b) Projection of a point on auxiliary vertical plane

6.4 Projection on auxiliary inclined plane

Problem 6.3

A point B is 40 mm above HP and 30 mm in front of VP. Draw its projection on an auxiliary inclined plane (AIP) 50^o inclined to HP.

Analysis

The point B is in the first quadrant. B may be projected to HP, VP and to auxiliary inclined plane (AIP) (refer Figure 6.5). The top view ***b***, front view ***b***' and projection on AIP, $\boldsymbol{b_1}$ are respectively obtained. Referring the figure, $\boldsymbol{Bb'o_1b_1}$ is a rectangle and hence the distance of $\boldsymbol{Bb' = b_1o_1}$=distance of ***b*** from x-y.

- ***The distance of auxiliary top view of a point from auxiliary reference line equals to the distance of the point from VP (same as the distance of top view from x-y).***

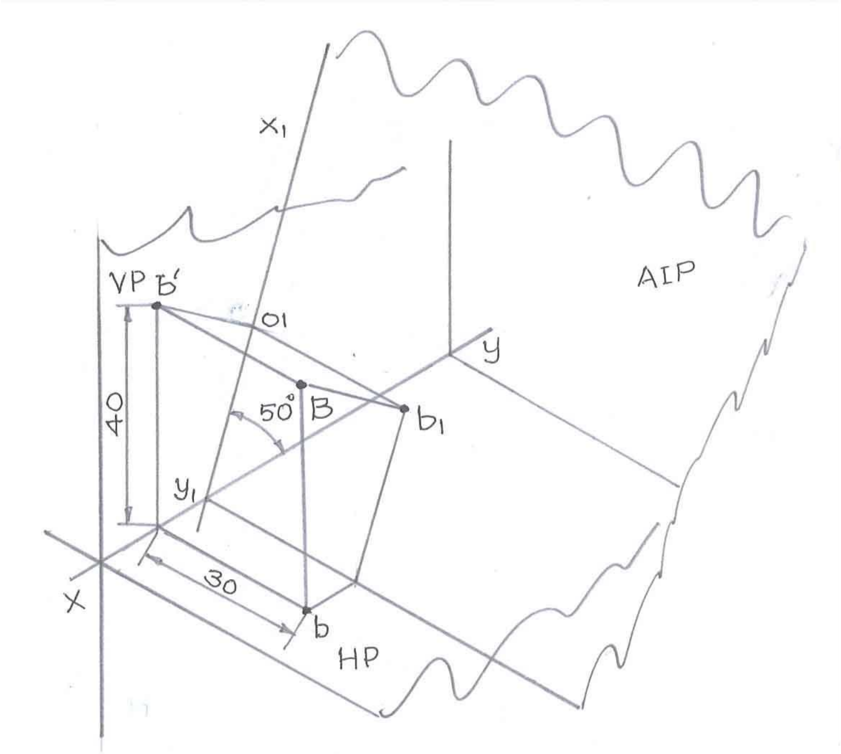


Figure 6.5 Projection of B on auxiliary inclined plane

After detaching AIP from HP, viewing along x-y, HP may be rotated in the clock-wise direction to make it coinciding with VP. Now top view and front view are on the same plane.

AIP may be rotated about the line of intersection of VP and AIP, and AIP may be made coinciding with VP. The top view, front view and auxiliary top view will be on a single plane (VP) (refer Figure 6.6).

Solution

b' and ***b*** may be marked on a projector taking distances 40 mm above x-y and 30 mm below x-y. The line of intersection of VP and AIP, x_1-y_1 may be shown at an angle of 50^o to x-y. Parallel line to x-y may be drawn from top view ***b*** to meet the vertical line to x-y drawn from the point of intersection of x-y and x_1-y_1. From ***b'*** a projector is drawn to x_1-y_1. An arc is cut from the meeting point of vertical line to x-y with the point of intersection of x_1-y_1 and x-y as centre. From the cutting point of arc on the line parallel to the projector to x_1-y_1, a parallel line is drawn to x_1-y_1. This parallel will meet the projector from ***b'*** to x_1-y_1 at $\boldsymbol{b_1}$, the required auxiliary top view. The distance of $\boldsymbol{b_1}$ from x_1-y_1 is equal to the distance from x-y to ***b*** (refer Figure 6.6a).

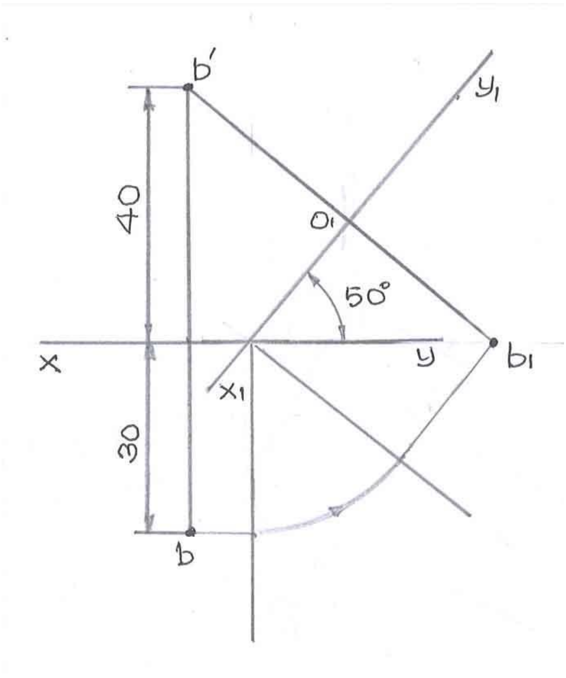


Figure 6.6 (a) Projection of a point on auxiliary inclined plane

Alternative Solution

x-y may be drawn conveniently. ***b′*** and ***b*** may be marked on a projector taking distances 40 mm above x-y and 30 mm below x-y. The line of intersection of VP and AIP, x_1-y_1 may be shown at an angle of 50^o to x-y.

From ***b′*** a projector is drawn to x_1-y_1. The distance of ***b*** from x-y may be taken and ***b₁*** may be marked on the projector of ***b′*** to x_1-y_1. ***b₁*** is the required auxiliary top view of point B (Figure 6.6b).

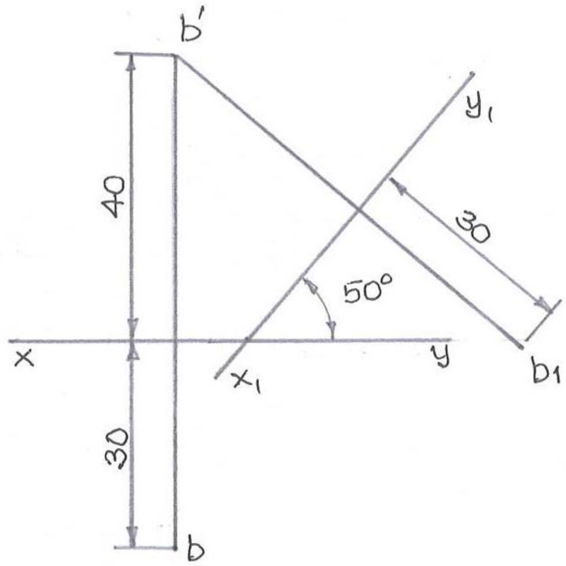


Figure 6.6 (b) Projection of a point on auxiliary inclined plane

Problem 6.4

The ends A and B of straight line AB are respectively 40 mm above HP and 20 mm in front of VP, and 10 mm above HP and 50 mm in front of VP. The distance between projectors drawn through end points is 60 mm. Draw the projection of the line AB. Determine the true length and inclinations of the line. Use auxiliary projection method.

Analysis

The true length and inclinations have been obtained earlier by assuming the straight line inclined to one plane and parallel to the other.

Instead of assuming the line inclined to one plane and parallel to the other, say inclined to HP and parallel to VP, an auxiliary vertical plane is assumed which is parallel to the line. Naturally the line is inclined to only one plane, to HP.

Similarly in place of assuming the line inclined to VP and parallel to HP, an auxiliary inclined plane is assumed to be parallel to the line. In both cases, projections are obtained separately to get the required answers.

Solution

After drawing x-y, ***a***' and ***a*** may be marked at 40 mm above and 20 mm below x-y. Another projector is drawn 60 mm away from the projector of A. ***b'*** and ***b*** may be marked on the projector at 10 mm above and 50 mm below x-y. While joining ***a*** and ***b***, and ***a'*** and ***b'*** respectively the top view and front view of AB are available.

Assuming the straight line is parallel to VP and inclined to HP, top view will be parallel to x-y and corresponding front view will give true length and its inclination to the reference line will be true inclination to HP (refer Figure 6.7).

From ***a*** and ***b*** perpendiculars may be drawn to ***ab***. Draw a parallel line, x_1-y_1, at 20 mm away from ***ab***. The straight line may be projected to the AVP. a_1' may be marked on the projector from ***a*** at 40 mm away from x_1-y_1; b_1' may be marked on the projector from ***b*** at 10 mm away from x_1-y_1. $a_1'b_1'$ is the required true length of the line. The inclination of $a_1'b_1'$ with x_1-y_1 gives true inclination (θ) of the line with HP.

Similarly, from ***a'*** and ***b'*** perpendiculars may be drawn to ***a'b'***. Draw a parallel line, x_2-y_2, at 10 mm away from ***a'b'***. The straight line may be projected to the AIP. a_2 may be marked on the projector from ***a'*** at 20 mm away from x_2-y_2; b_2 may be marked on the projector from ***b'*** at 50 mm away from x_2-y_2. $a_2b_2 = a_1'b_1'$, the true length of the line. The inclination of a_2b_2 with x_2-y_2 gives true inclination (ϕ) of the line with VP.

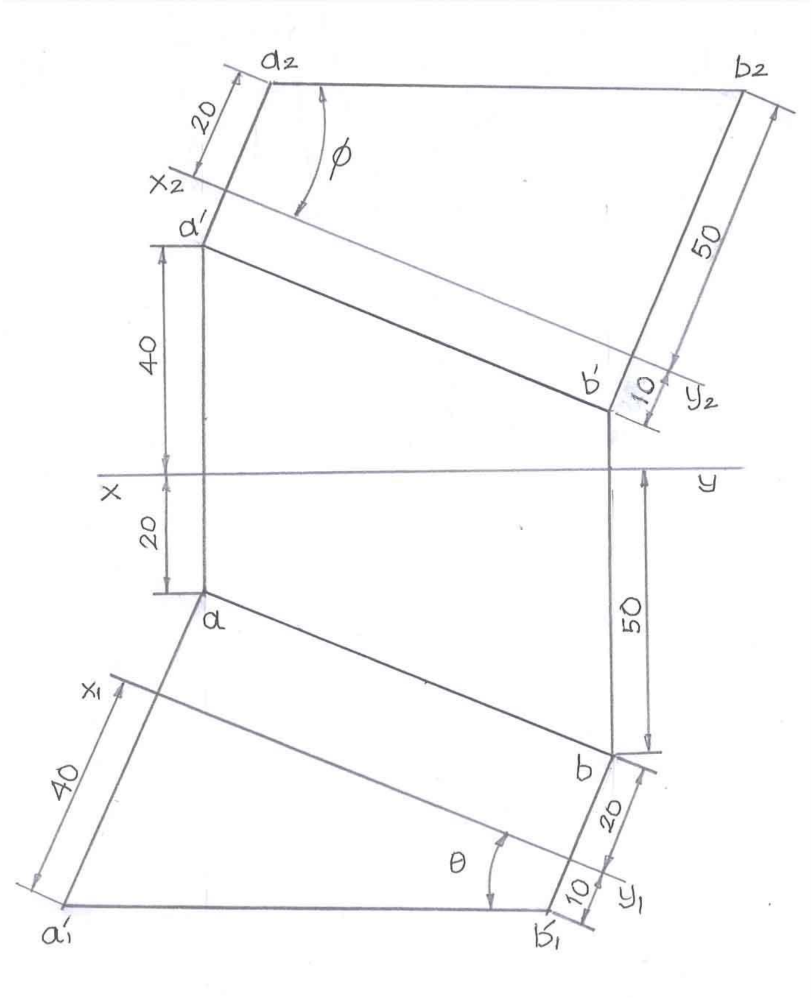


Figure 6.7 Determination of true length and inclinations of a line

Problem 6.5

A straight line AB, 80 mm long, is inclined 45^o to HP and 30^o to VP. Draw the projections. The end A is 20 mm away from HP and 10 mm away from VP. Use auxiliary projection method.

Solution

a_1' and ***a_1*** may be marked on a projector at 20 mm above and 10 mm below x-y. The line is assumed to be parallel to both HP and VP. Both top view (***a_1b_1)*** and front view (***a_1'b_1')*** are parallel to x-y and both give true length (refer Figure 6.8).

Instead of assuming the straight line is parallel to HP and inclined to VP, an AVP is assumed which is inclined 30^o to the line. A line is drawn through ***a_1*** to ***a_1b_1*** at an angle of 30^o. ***a_1*** and ***b_1***.are projected perpendicular to this line passing through ***a_1***. A parallel line (x_1-y_1) is drawn for the line through ***a_1*** at 10 mm away. Draw a line ***a'b'*** parallel to x_1-y_1 at 20 mm away. ***a_1b_1*** and ***a'b'*** are the top view and front view respectively when the line is parallel to HP and inclined to VP.

a'b' is the length of the final front view. Since the straight line is inclined to HP also, apparent angles must be obtained. A line ***a'b₂'***=80 mm may be drawn through ***a'*** at an angle of 45^o to ***a'b'***. A line may be drawn through ***b₂'*** parallel to ***a'b'***. This is the locus of ***b'***. To minimize the space and time, ***a'b'*** as radius ***a'*** as cntre an arc is drawn to the locus of ***b'***, say the arc meets at ***b₃'***. x_2-y_2 may be drawn parallel to ***a'b₃'*** and 20 mm away. ***a'*** and ***b'*** may be projected perpendicular to x_2-y_2. The distances of ***a₁*** and ***b₁*** from x_1-y_1 respectively may be marked on the projectors of ***a'*** and ***b'***. While finishing ***a'b'*** and ***ab*** the required projections are available.

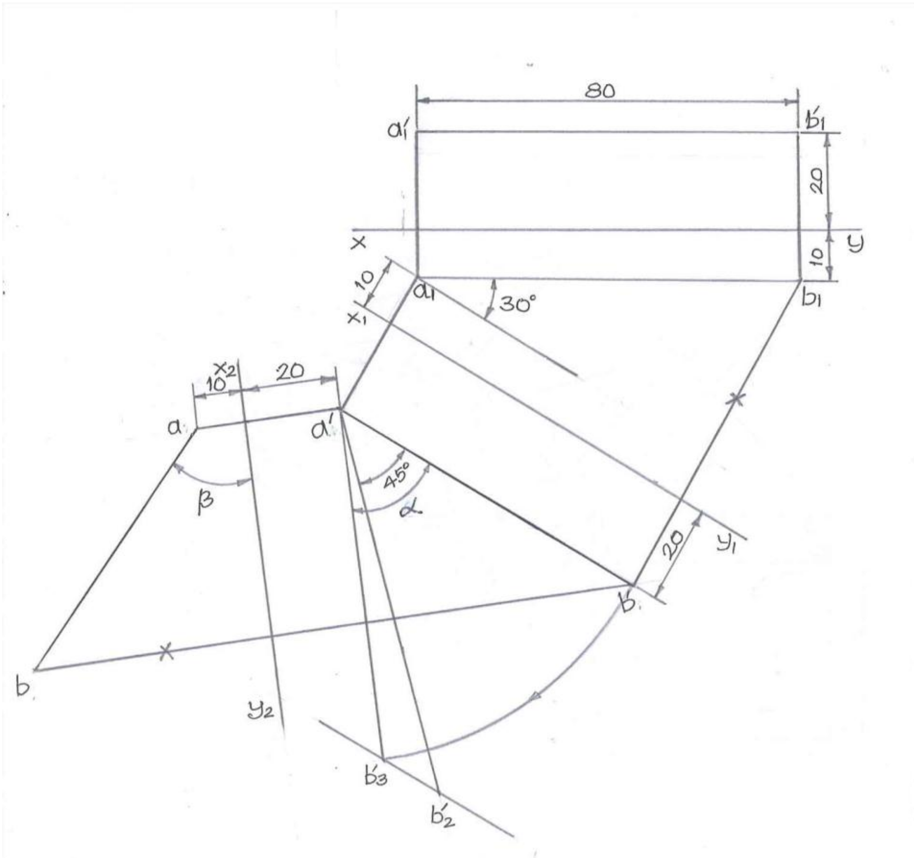


Figure 6.8 Projctions of lines inclined to both HP and VP

Problem 6.6

A rectangular lamina, length 50 mm and breadth 30 mm, is resting on HP with a shorter side. The plane makes 45^o to HP while the side on HP makes 30^o to VP. Draw the projections. Also find the true inclination of the plane with VP.

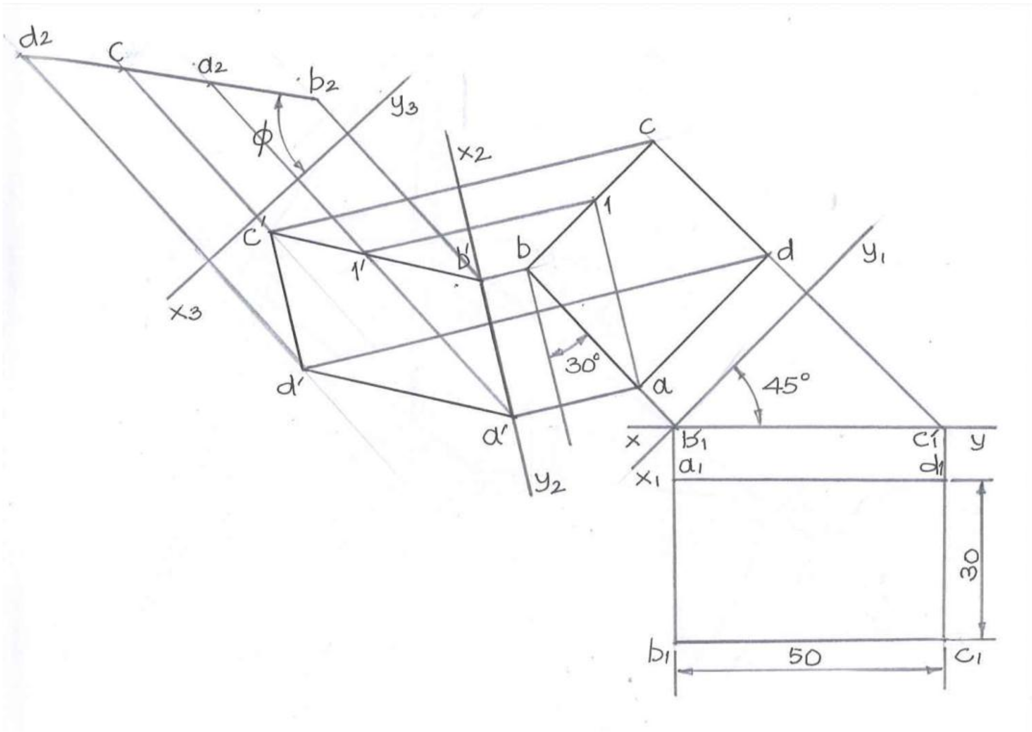


Figure 6.9 Projections of a rectangle inclined to HP and VP

Solution

The projection of the first step may be drawn by assuming the plane parallel to HP, with one shorter side perpendicular to VP and on the HP. Or the plane is lying on HP with one shorter side perpendicular to VP.

An AIP is assumed which is inclined at 45^o to the plane, (the line of intersection of VP and AIP, x_1-y_1, in Figure 6.9) passing through the shorter edge which is to be made on HP. Draw the projections of the plane on this AIP. After projecting ***b₁'*** and ***c₁'*** to x_1-y_1, distances of $\boldsymbol{a_1}$, $\boldsymbol{b_1}$, $\boldsymbol{c_1}$, and $\boldsymbol{d_1}$ from x-y may be marked to get respectively ***a***, ***b***, ***c*** and ***d***. x_2-y_2 is drawn at 30^o inclined to ***ab***. ***a***, ***b***, ***c*** and ***d*** may be projected to x_2-y_2. (In place of making the shorter side inclined at 30^o to VP, an AVP is assumed at 30^o inclined to the shorter side.)

The distances of $\boldsymbol{a_1'}$, $\boldsymbol{b_1'}$, $\boldsymbol{c_1'}$, and $\boldsymbol{d_1'}$ from x_1-y_1 may be marked respectively on the projectors to get ***a'***, ***b'***, ***c'***, and ***d'***. While finishing ***abcd*** and ***a'b'c'd'*** the top and front views of the rectangle are available.

The line ***a1*** (parallel to x-y) is drawn in the top view such that A1 is visible as true length in the front view (***a'1'***). When the projections of the plane is obtained assuming the plane perpendicular to HP inclination to VP can be found out. x_3-y_3 may be drawn perpendicular to ***a'1'*** (an AIP is assumed perpendicular to A1) and the plane may be projected to the new reference line. Marking the distances of ***a***, ***b***, ***c***, and ***d*** from x_2-y_2, the plane will appear as a straight line and its inclination with x_3-y_3 will give true inclination of the plane with VP (refer Figure 6.9).

Problem 6.7

An isosceles triangle, base 35 mm and altitude 55 mm, appears to be an equilateral triangle in the top view. The base is on the HP, inclined 40^o to VP and the vertex is on VP. Draw the projections. Also find out the inclination of the plane with HP.

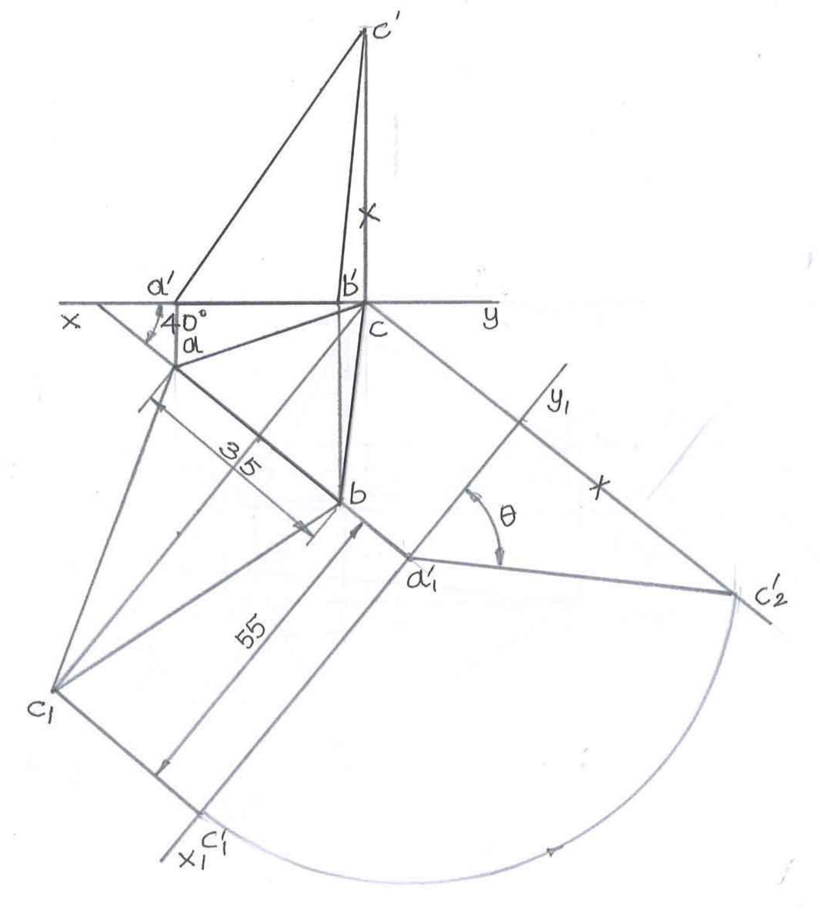


Figure 6.10 Projections of isosceles triangle resting on HP with vertex on VP

Solution

After drawing x-y conveniently, a line may be drawn at 40^o inclined to and below x-y. The equilateral triangle ***abc*** may be constructed with ***c*** on x-y. A line may be drawn from ***c*** perpendicular to ***ab***, then extended to c_1, where abc_1 gives the true shape of the isosceles triangle. Assuming the plane perpendicular to VP and parallel to HP (lying on HP with the base perpendicular to VP), $a_1'c_1'$ gives the corresponding front view. a_1' as centre $a_1'c_1'$ as radius an arc is cut to the projector drawn from ***c*** to x_1-y_1. Inclination of $a_1'c_2'$ with x_1-y_1 gives the inclination of the triangle with HP (refer

Figure 6.10).

The distance of ***c_2'*** from x_1-y_1 may be marked on the projector drawn from ***c*** to x-y, to get ***c'***. ***ab***c and ***a'b'c'*** may be finished to get the projections of the triangle ABC.

Problem 6.8

A triangle ABC has its corners A, on the HP and 10 mm in front of VP, B, 25 mm above HP and 30 mm in front of VP and C, 10 mm above HP and 15 mm in front of VP. The distance between projectors of A and C are 40 mm. The projector of B falls between the projectors of A and C, at 10 mm away from that of A. Draw the projections of the triangle ABC. Determine its inclinations with HP and VP. Also find out the true shape of the triangle.

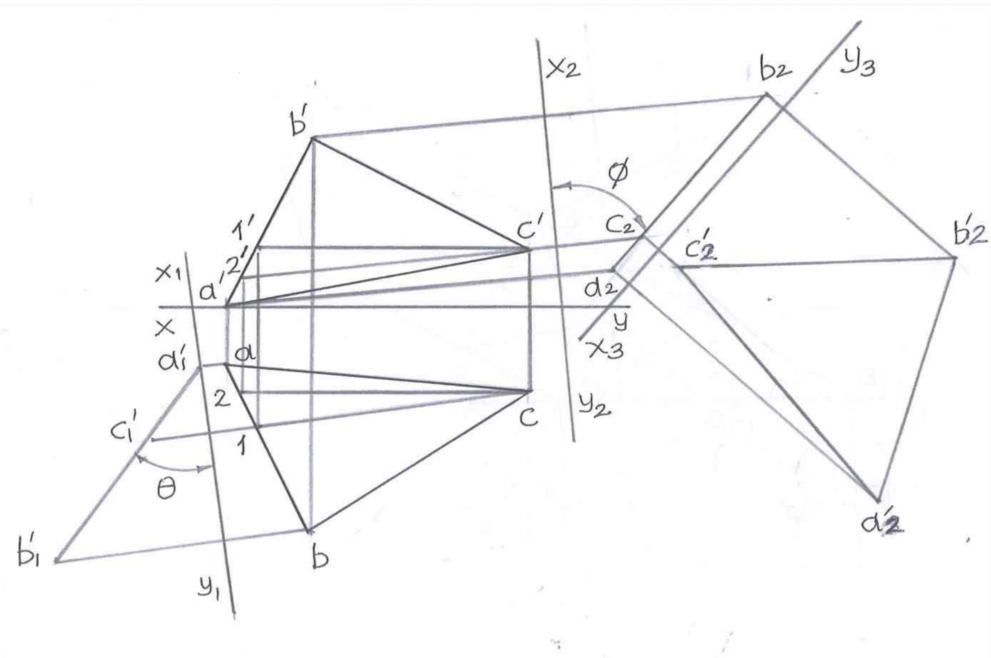


Figure 6.11 Determination of true inclinations and true shape of triangle

Solution

a' and ***a*** may be marked on a projector, ***a'*** on x-y, ***a*** 10 mm below x-y. Another projector is drawn 10 mm away from that of A (projector of B) and ***b'*** and ***b*** may be marked 25 mm above x-y and 30 mm below x-y. On the same side, another projector (projector of C) may be drawn at 40 mm from that of A, and ***c'*** and ***c*** may be marked on it at 10 mm above x-y and 15 mm below x-y. While joining and finishing ***abc*** and ***a'b'c'*** the required top and front views of the plane are available (Figure 6.11).

c'1' is drawn parallel to x-y in the front view. The corresponding top view ***c1*** will give its true length. If the projections of the plane are obtained assuming perpendicular to VP, the inclination to HP can be found out.

x_1-y_1 may be drawn perpendicular to ***c1***. ***a***, ***b***, and ***c*** may be projected to x_1-y_1. The distances of ***a'***, ***b'***, and ***c'*** from x-y respectively may be marked on the projectors of ***a***, ***b***, and ***c***. $\boldsymbol{a_1'c_1'b_1'}$ will be a straight line and its inclination with x_1-y_1 will give true inclination of the plane with HP.

c2 is drawn parallel to x-y in the top view. The corresponding front view ***c'2'*** will give its true length. If the projections of the plane are obtained assuming perpendicular to HP, the inclination to VP can be found out.

x_2-y_2 may be drawn perpendicular to ***c'2'***. ***a'***, ***b'***, and ***c'*** may be projected to x_2-y_2. The distances of ***a***, ***b***, and ***c*** from x-y respectively may be marked on the projectors of ***a'***, ***b'***, and ***c'***. $\boldsymbol{a_2c_2b_2}$ will be a straight line and its inclination with x_2-y_2 will give true inclination of the plane with VP.

While projecting the plane parallel to a plane, the true shape is available. x_3-y_3 may be drawn parallel to $\boldsymbol{a_2c_2b_2}$. The corners ***a***, ***b***, and ***c*** may be projected with respect to x_3-y_3. The distances of ***a'***, ***b'*** and ***c'*** from x-y may be marked on the respective projectors from x_3-y_3. $\boldsymbol{a_2'b_2'c_2'}$ will be the true shape of ABC.

Problem 6.9

A hexagonal lamina, 40 mm side, is resting on HP with a side. The plane makes 60^o to HP and 30^o to VP. Draw the projections of the lamina, using auxiliary projections. Note that the side opposite to that on HP is on VP.

Solution

The hexagonal lamina may be assumed to be parallel to HP, with one side on HP and perpendicular to VP (the hexagon is lying on HP with one side perpendicular to VP).

After drawing x-y, the hexagon with side 40 mm may be drawn ($\boldsymbol{a_1b_1c_1d_1e_1f_1}$) taking one side perpendicular to and below x-y. This is the top view of the plane in the first step. The corresponding front view will be a straight line, $\boldsymbol{b_1'c_1'd_1'}$.

Instead of making the plane inclined 60^o to HP, an AIP is assumed inclined 60^o to the plane. x_1-y_1 may be drawn through $\boldsymbol{b_1'}$ 60^o inclined to the front view, $\boldsymbol{b_1'c_1'd_1'}$. The corresponding top view may be obtained. Perpendiculars may be drawn from $\boldsymbol{b_1', c_1',}$ and $\boldsymbol{d_1'}$ to x_1-y_1. The distances of $\boldsymbol{a_1, b_1, c_1, d_1, e_1}$ and $\boldsymbol{f_1}$ from x-y may be marked on the respective projectors from x_1-y_1. ***abcdef*** is the top view of the hexagon if it is inclined 60^o to HP.

The plane makes 30^o to VP also. The side AB is on HP and the opposite side, DE is on VP. When AB is on HP, ***a'b'*** will be on x-y and when DE is on VP, ***de*** will be on x-y.

Instead of making the plane inclined to VP, an AVP is assumed inclined 30^o to the plane. x_2-y_2 may be drawn along ***de*** and all points in the top view may be projected perpendicular to x_2-y_2. The distances of a_1', b_1', c_1', d_1', e_1' and f_1' may be taken from x_1-y_1 and marked to get ***a'b'c'd'e'f',*** the final front view (Figure 6.12).

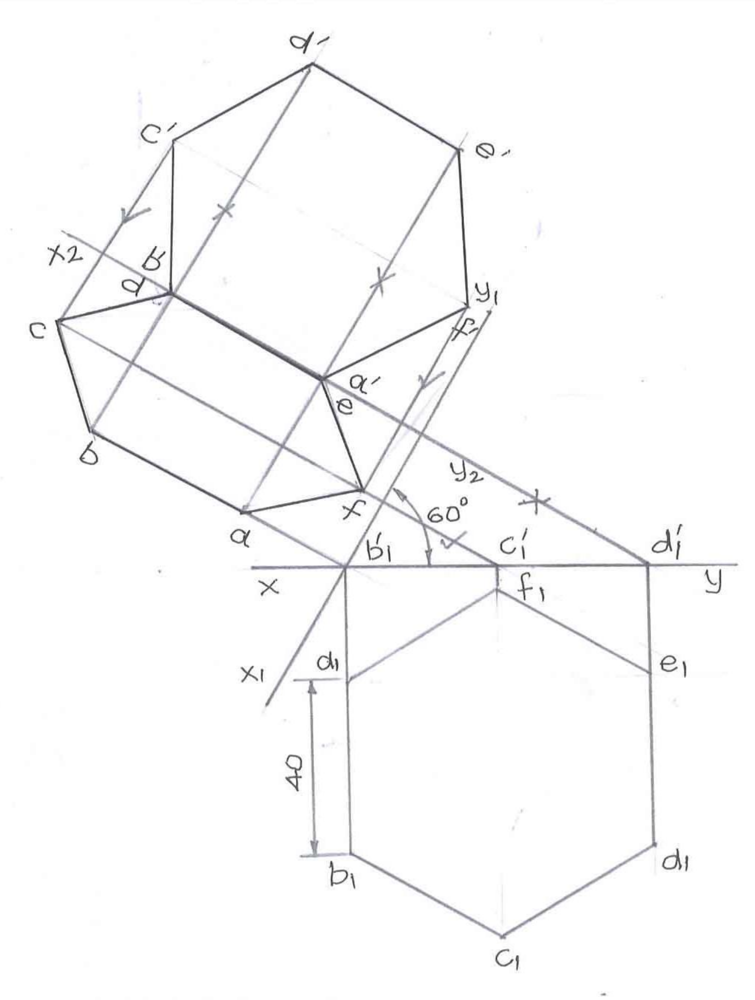


Figure 6.12 Projections of a hexagon inclined 60^o to HP and 30^o to VP

Chapter 7

Projections of Solids

Chapter 7

Projections of Solids

7.1 Solids

Solids are three dimensional objects, (having length, breadth and height). Solids are classified in to two:

(i) polyhedra

(ii) solids of revolution

7.1.1 Polyhedra

A polyhedron is a solid made up of faces (or planes) and having straight edges. Polyhedra are again classified in to:

(i) regular polyhedra

(ii) prisms

(iii) pyramids

Regular Polyhedra

A regular polyhedron has equal faces having equal edges and equal angles. Examples are:

(i) Tetrahedron: A Tetrahedron has ***four*** equal equilateral triangular faces (Figure 7.1).

(ii) Cube: A Cube has ***six*** equal square faces.

(iii) Octahedron: An Octahedron has ***eight*** equal equilateral triangular faces (Figure 7.2).

(iv) Dodecahedron: A Dodecahedron has ***twelve*** equal regular pentagonal faces.

(v) Icosahedron: An Icosahedron has ***twent***y equal equilateral triangular faces.

Prism

A prism can be either right prism or oblique prism. In right prism, the axis will be perpendicular to the base (end face), while in oblique prism the axis will be inclined to the base.

This chapter will discuss only right solids.

A prism is named based on the shape of its end faces (base and its opposite face). If

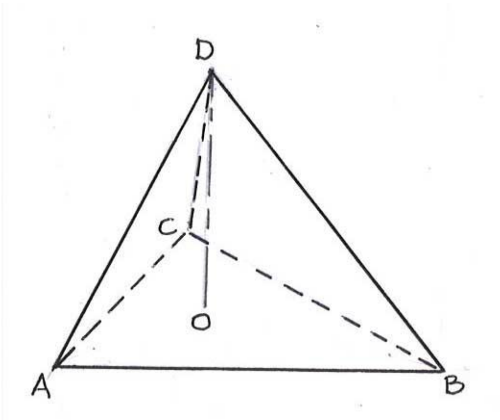


Figure 7.1 Tetrahedron

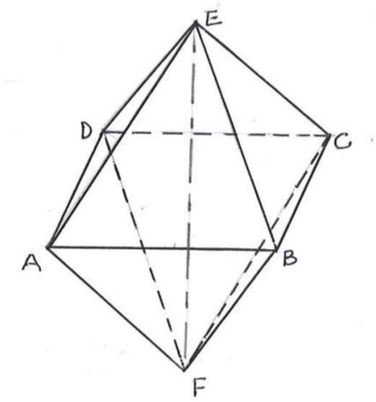


Figure 7.2 Octahedron

the base is triangle, triangular prism; for square, square prism and so on. The imaginary line joining the centres of end faces is the axis. The corners of the end faces are respectively connected by straight edges and hence prisms have rectangular faces equal in number of the sides of the end faces (Refer 7.3 for triangular prism).

Pyramid

In right pyramids, the axis will be perpendicular to the base of the pyramid, while it is inclined to the base in oblique pyramids.

Pyramids are also named based on the shape of the base, triangular pyramid, square pyramid etc. In pyramids, the corners of the base are joined with the vertex of the solid by straight edges; these edges are the slant edges. Two slant edges and a side of the base, will make a slant face. All slant faces are equal isosceles triangles (in right pyramids). The imaginary line joining the vertex with the centre of the base is the axis (Refer Figure 7.4 for square pyramid).

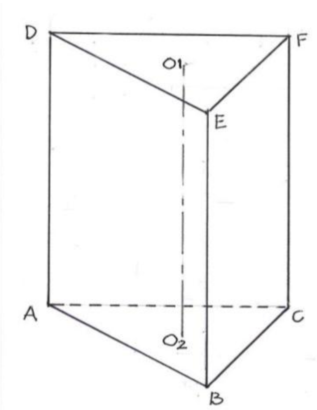


Figure 7.3 Triangular prism

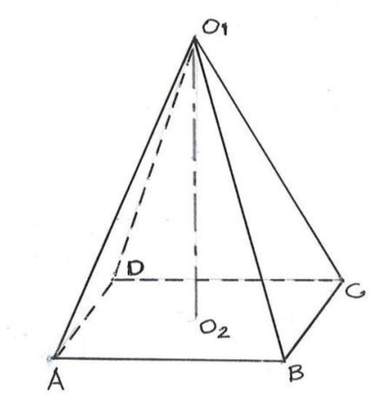


Figure 7.4 Square pyramid

7.1.2 Solids of revolution

Examples for solids of revolution are:

(i) Cylinder

(ii) Cone

(iii) Sphere

Cylinder

When a rectangle (or a square) is revolved about a side the cylinder is generated. The side which will generate the cylindrical surface, at its various positions, is the generator or the line joining a point on the circumference of the base circle with the corresponding point on the opposite face is the generator. The side or line about which the rectangle is revolved will become the axis of the solid or axis is the imaginary line joining the centres of the end faces.

Cone

When a right angled triangle is revolved about one of the sides that make right angle, a cone is generated. Any line joining a point on the circumference of the base circle and the apex is called as a generator (Figure 7.5).

Sphere

When a semi-circle is revolved about its straight edge, a sphere is generated.

Frustums

In addition to the above full solids, there can be sectioned objects. If a pyramid or cone is sectioned such that the sectioning is parallel to the base, then the portion containing the base is the frustum of that solid (eg: frustum of cone, frustum of square pyramid etc.-Refer Figure 7.6 for the frustum of square pyramid).

If the sectioning is inclined to the base, the object is called as a truncated solid.

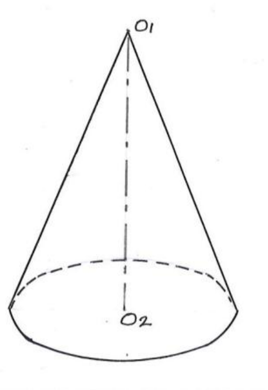


Figure 7.5 Cone

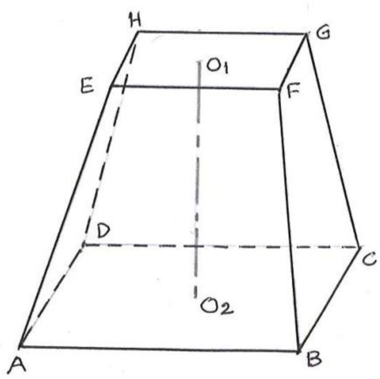


Figure 7.6 Frustum of square pyramid

7.2 Classification of solids with respected to their position

(i) Axis of the solid is perpendicular to one principal plane and parallel to the other

(ii) Axis of the solid is parallel to both HP and VP

(iii) Axis of the solid is inclined to one principal plane and parallel to the other

(iv) Axis of the solid is inclined to both HP and VP

7.2.1 Projections of solids with axis perpendicular to one principal plane

Problem 7.1

A square prism, side of base 30 mm and height 50 mm, is resting on HP with its base. Draw the projections if the axis is perpendicular to HP and two rectangular surfaces are parallel to VP.

Analysis

The square prism is resting on HP with its base with the axis perpendicular to HP. The axis is parallel to VP.

The base (on the HP) and the top face are parallel to HP and perpendicular to VP. As the end faces are parallel to HP, their top view will give true shape. As they are perpendicular to VP, their front views will be straight lines. The base in front view will be on x-y line.

All longer edges are perpendicular to HP. The longer edges will give true length in the front view and top views will be points only.

All rectangular faces are perpendicular to HP. Being a square prism, as two rectangular surfaces are parallel to VP, the other two are perpendicular to VP. The rectangular faces which are parallel to VP will give true shape in the front view while the other two will appear as straight lines.

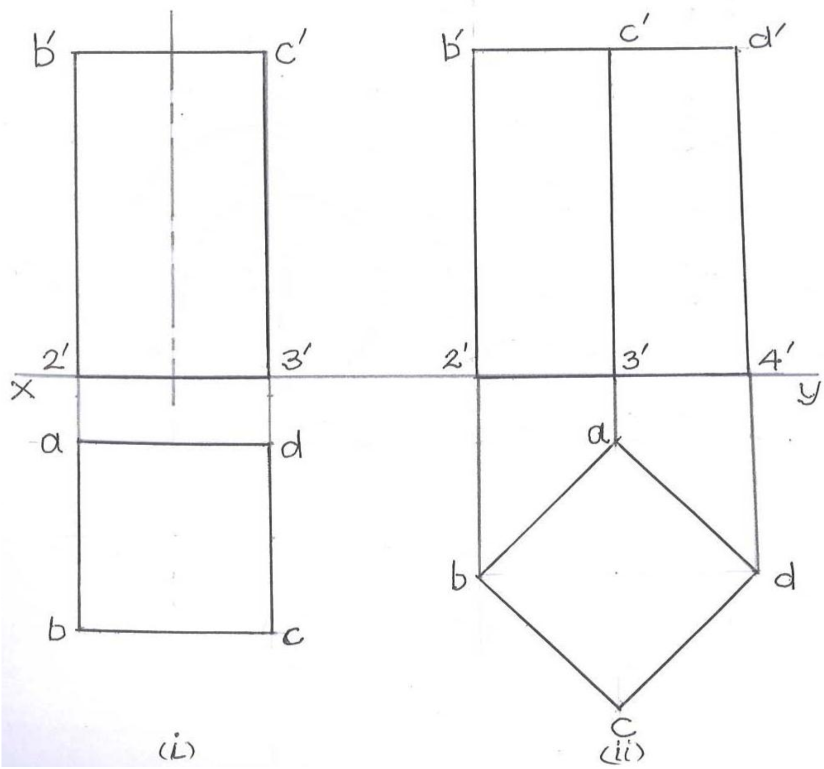


Figure 7.7 Square prism with axis perpendicular to HP (i) Two rectangular surfaces are parallel to VP (ii) rectangular surfaces are equally inclined to VP

Solution

Draw a square of sides 30 mm below x-y with two sides parallel to x-y. This is the top view of the prism in its position. All the corners may be named say the base as ***1234*** and top face ***abcd***. The geometric centre of the plane may be marked. The axis of the solid (thin chain line) may be indicated in the front view. After taking a height of 50 mm, width equals the side of the square from the top view, the front view may be completed as a rectangle. The corresponding names may be given to the corners. The square ***abcd*** (top view) and the rectangle ***2'3'c'b'*** (front view) may be finished - refer Figure 7.7(i).

Problem 7.2

A square prism, side of base 30 mm and height 50 mm, is resting on HP with its base. Draw the projections if the axis is perpendicular to HP and all rectangular surfaces are equally inclined to VP.

Solution

In this position also, the top view appears as a square, but all sides will be equally inclined to x-y (The sides of the square in the top view indicate the rectangular surfaces). As these are equally inclined to VP, the sides of the square will appear equally inclined - 45 ^{o} - to x-y.

The square of sides 30 mm may be drawn below x-y, with all sides equally inclined - 45 ^{o} - to x-y. The corners may be named - ***abcd***. All the points may be projected to get the front view. The front view will appear as a rectangle ***2'4'd'b'***, with longer edge ***3'c'*** visible centrally – Figure 7.7(ii).

Problem 7.3

A pentagonal pyramid, side of base 25 mm and height 50 mm, is resting on VP with its base, axis perpendicular to VP. Draw the projections if one side of the base is perpendicular to HP.

Solution

The base of the pentagonal pyramid is on the VP, having one side perpendicular to HP. The base is visible as true shape in the front view, with one side perpendicular to x-y.

A regular pentagon of sides 25 mm may be drawn above x-y, with one side perpendicular to x-y - ***a'b'c'd'e'***. The geometric centre of the pentagon may be located as ***o'***. ***a', b', c', d', e'*** may be joined with ***o'***. This is the front view of the solid.

The axis may be drawn in the top view projecting from the geometric centre. The vertex ***o*** may be marked taking a distance of 50 mm from x-y. All the corners (***a, b, c, d, e***) of the base may be projected to x-y (base will be on x-y in the top view). ***a,*** (coincides with ***b***) ***c,*** (coincides with ***d***) and ***e*** may be joined with ***o***. This is the top view of the pyramid. The required top view and front view may be finished – refer Figure 7.8 (i).

Problem 7.4

A tetrahedron, sides 50 mm long, is resting on VP with a face. Draw the projections if one side of the face on VP is parallel to HP.

Analysis

A tetrahedron has four equal faces and all are equilateral triangles. When it rests on VP with a face, the front view will have true shape of the equilateral triangle. As one side of the face on VP is parallel to HP, it will be parallel to x-y in the front view. The geometric centre will give the position of the fourth corner.

The top view of the face on the VP can be projected easily as its length is true side and it is on x-y. The height of the fourth corner from the base (face on VP) may be found by assuming one side parallel to HP.

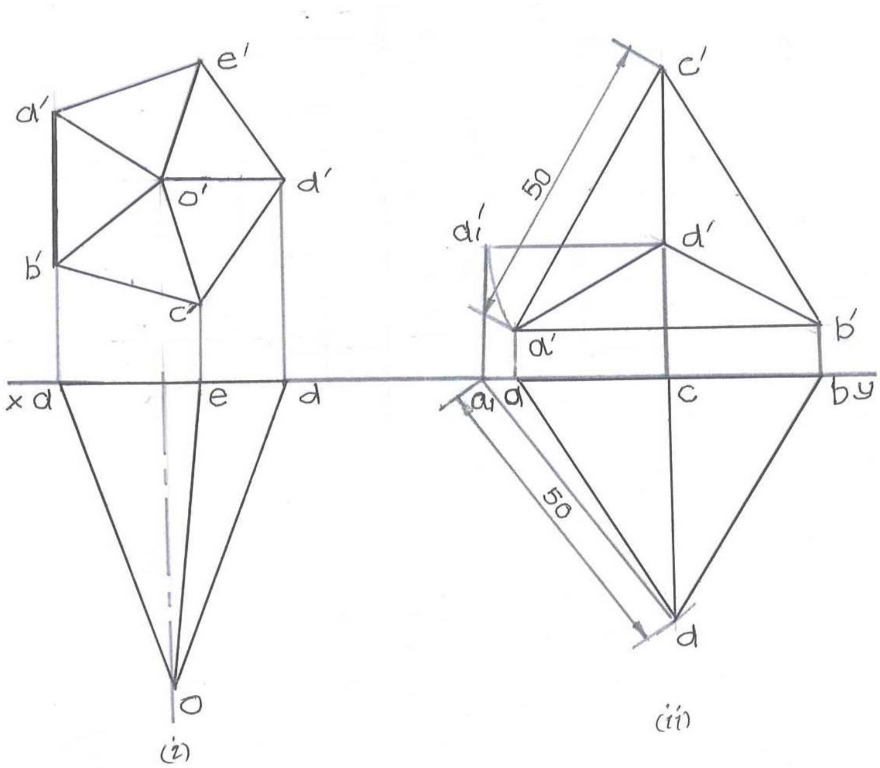


Figure 7.8 (i) Pentagonal pyramid resting on VP, (ii) Tetrahedron resting on VP

Solution

An equilateral triangle of side 50 mm ($a'b'c'$) may be drawn above x-y, keeping one side parallel to x-y. After locating its geometric centre, d', the corners a', b', c' may be joined with d'. This is the front view of the tetrahedron.

The top view of the face on VP will be a straight line and that will coincide with x-y (abc). A vertical line may be drawn from c.

One side, say AD, may be made parallel to HP. Now $d'a'$ will be parallel to x-y ($d'a'_1$) and its top view (da_1) will give true length.

a_1 as centre and a_1d=50 mm as radius an arc is drawn to meet with the perpendicular already drawn from c. Th point of interesection is d. d may be joined with a, b, c. After finsihing $a'b'c'd'$ and $abcd$ the required front view and top view are ready.

7.2.2 Projections of solids with axis parallel to both principal planes

Problem 7.5

A triangular prism, side of base 50 mm and length 75 mm, is lying on HP with a rectangular face. Draw the projections if the axis is parallel to VP.

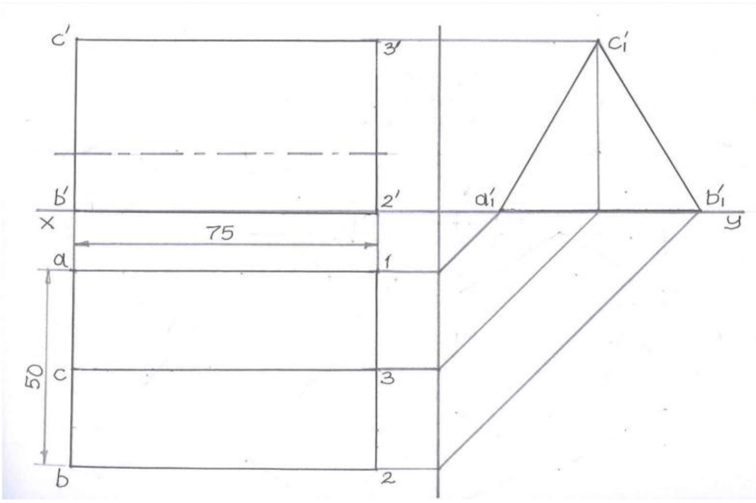


Figure 7.9 Triangular prism with axis parallel to both HP and VP

Analysis

As the triangular prism is resting on HP with a rectangular face, the axis is parallel to HP. The rectangular face which is on HP will be visible as true shape in the top view.

The axis of the prism is parallel to VP. As the longer edges are parallel to both HP and VP, all longer edges will be appearing as ture length in both the views.

The edge opposite to the face on HP, will be visible centrally in the rectangular top view. While, in the front view it is visible at a height equal to the altitude of the triangulular base, which may be found out from the side view. In the side view, the prism appears to be an equilateral triangle, with one side on x-y.

Solution

An equilateral triangle of side 50 mm (***a_1'b_1'c_1'***) may be drawn with one side (***a_1'b_1'***) on x-y. This is the side view (from left) of the prism.

Using the side view, the top view ***acb-123*** may be projected as per Figure 7.9. ***a12b*** is the rectangle which indicates true shape of one face of the solid. The longer edge, ***c3*** appears as the central line of the rectangle.

The corresponding front view may be projected. The geometric centre of the side view may be located and then the axis may be indicated using thin chain line. The vertical distance of ***c'3'*** from x-y may be found out from the side view by direct projection from ***c_1'***. The front view and top view may be finished.

7.2.3 Projections of solids with axis inclined to one principal plane and parallel to the other

Problem 7.6

A square pyramid, side of base 30 mm and height 50 mm, is resting on HP with a side of its base. Draw the projections if the axis is inclined to 45^{o} to HP and parallel to VP.

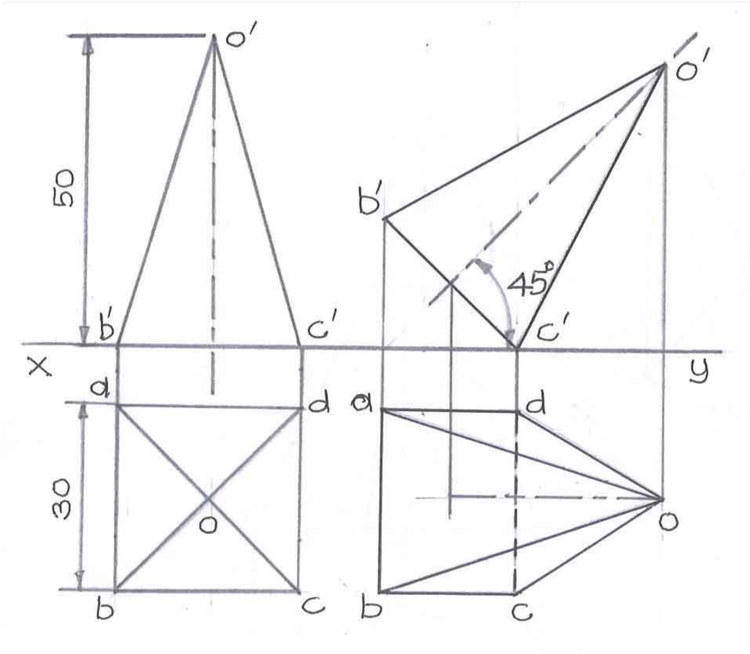


Figure 7.10 Square pyamid resting on HP with a side of base and axis inclined 45^{o} to HP

Analysis

The pyramid is resting on HP with a side of the base, axis is inclined to HP and parallel to VP. This can be solved in two steps.

Step-1:

The solid may be assumed to be resting on HP with its base and axis is perpendicular to HP. The side which is on HP must be perpendicular to VP. The top view will be a square of side 30 mm. The geometric centre may be joined with the corners to get the top view. The front view will be an isosceles triangle with base on x-y and altitude, the height of the pyramid.

Step-2:

The axis of the solid is a line parallel to VP and inclined to HP. The axis is visible in the front view as true with true inclination to x-y. The isosceles triangle in the front view, in the first step and second step will be the same without any change in the dimensions, appears to be in such a way that the altitude of the triangle will be inclined at true inclination to x-y. The final top view can be projected.

Solution

Step-1:

The square pyramid may be assumed as resting on HP with its base, the axis is perpendicular to HP. One side of the base (the side which is on HP in the final position) may be perpendicular to VP.

A square (***abcd***) of side 30 mm may be drawn below x-y, keeping one side (***cd***) perpendicular to x-y. The geometric centre (***o***) may be marked and all the corners (***a, b, c, d***) may be joined with ***o***. This is the top view of the pyramid in the first step.

The front view may be projected. ***a, b, c, d*** may be projected to x-y. As the base is on HP, the front view of the base will be a line and it (***b'c'***) is coinciding with x-y. ***o*** may be projected to x-y and a perpendicular may be drawn from the meeting point on x-y.

A length of 50 mm (from x-y to ***o'***) may be marked on this perpendicular. ***b'*** and ***c'*** may be joined with ***o'***; this is the front view of the pyramid in the first step.

<u>Step-2:</u>

It has been proved that front view in the first step and final front view are similar, but the axis is inclined 45^{o} to x-y.

The isosceles triangle ***b'c'o'*** may be re-constructed in such a way that ***c'*** (representing side CD) is on x-y, and the axis (altitude of the triangle) is inclined 45^{o} to x-y. ***b'***, ***c'***, and ***o'*** may be projected and intersected by the respective horizontal lines drawn from ***a***, ***b***, ***c***, ***d*** and ***o*** to obtain the final top view. The final (second step) front view and top view may be finished (Figure 7.10).

The pyramid is resting on HP with a side of base (CD). In this position, the side which is on HP is the bottom most edge and above that the solid rests. Hence, the edge which is on HP will be hidden (see ***cd***) in the top view and all other edges will be visible.

Alternative Solution (Auxiliary Projection method)

<u>Step-1:</u>

The square pyramid may be assumed as resting on HP with its base, the axis is perpendicular to HP. One side of the base (the side which is on HP in the final position) may be perpendicular to VP.

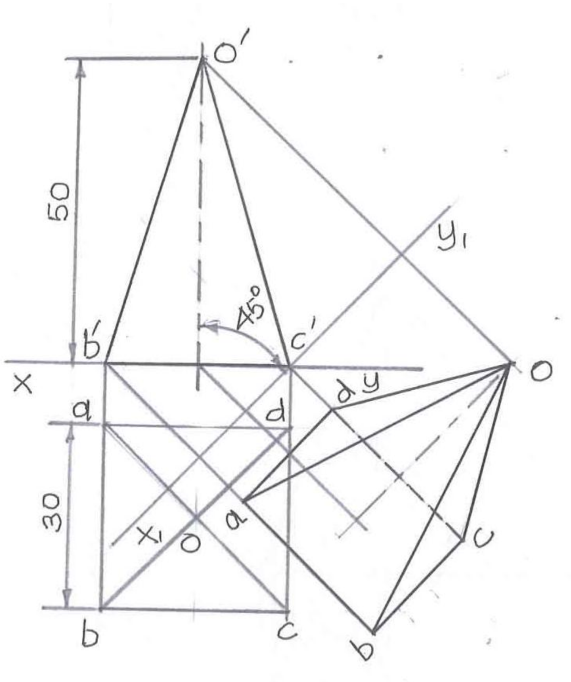


Figure 7.11 Square pyamid resting on HP with a side of base and axis inclined 45° to HP

A square (***abcd***) of side 30 mm may be drawn below x-y, keeping one side (***cd***) perpendicular to x-y. The geometric centre (***o***) may be marked and all the corners (***a, b, c, d***) may be joined with ***o***. This is the top view of the pyramid in the first step.

The front view may be projected. ***a, b, c, d*** may be projected to x-y. As the base is on HP, the front view of the base will be a line and it is coinciding with x-y (***b'c'***). ***o*** may be projected to x-y and a perpendicular may be drawn from the meeting point on x-y.

A length of 50 mm (from x-y to ***o'***) may be marked on this perpendicular. ***b'*** and ***c'*** may be joined with ***o'***; this is the front view of the pyramid in the first step.

<u>Step-2:</u>

Instead of making the axis of the solid inclined to HP, an auxiliary inclined plane is assumed, inclined 45° to the axis of the solid. x_1-y_1 may be drawn through ***c'*** which makes 45° to the axis in the front view. ***o'***, ***b'*** and ***c'*** may be projected perpendicular to x_1-y_1. Distances of ***a***, ***b***, ***c***, ***d*** and ***o*** from x-y may be taken and marked on respective projectors from x_1-y_1. After joining ***a***, ***b***, ***c***, ***d*** and then every point with ***o***, the top view is available. Front view and top view may be finished as shown in Figure 7.11. As ***cd*** is hidden it may be marked as dotted line.

Problem 7.7

A pentagonal pyramid, side of base 30 mm and height 50 mm, is hung from a corner of its base. Draw the projections if its axis is parallel to VP.

Analysis

Since the pyramid is hanging freely from a corner of its base, the axis is inclined to HP and the axis is parallel to VP.

<u>Step-1:</u>

As the axis of the solid is inclined to HP and parallel to VP, in the first step the axis must be assumed to be perpendicular to HP. The side opposite to the corner from which the solid is hung, must be perpendicular to VP. The top view can be drawn, which is the true shape of the pentagonal base, corners joined with the geometric centre. The front view may be obtained by projecting from the top view.

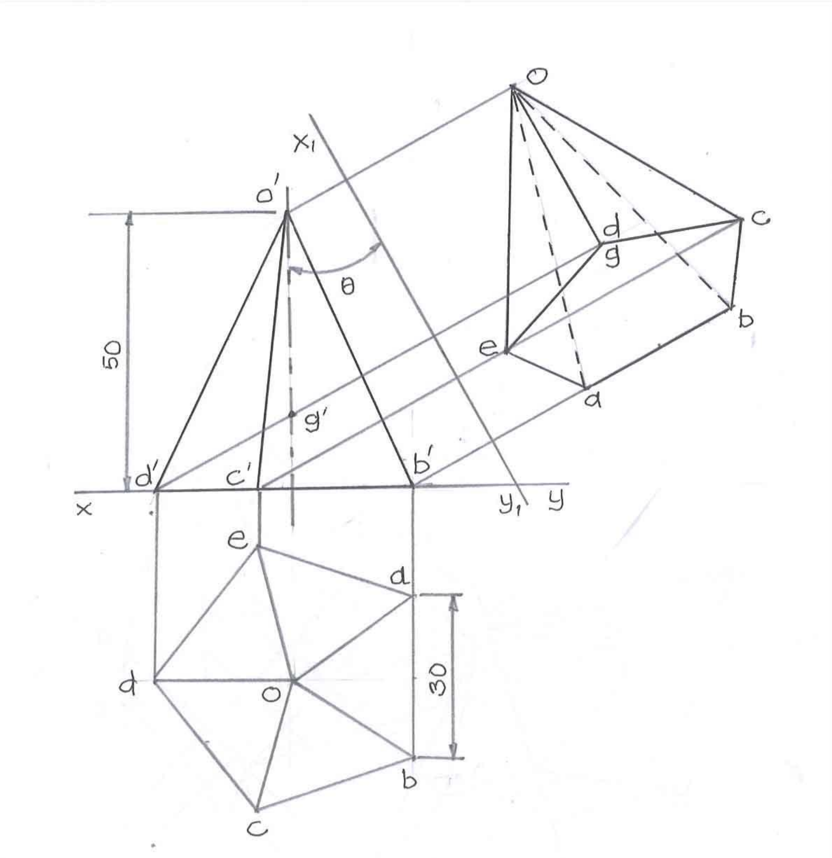


Figure 7.12 Pentagonal pyramid hanging from a corner of base, axis parallel to VP

<u>Step-2:</u>

When the solid is hanging from the corner of the base, the axis is inclined to HP. The corner from which the solid is hung and the centre of gravity of the solid will be lying on a vertical line (or a line perpendicular to x-y).

An auxiliary inclined plane may be assumed to be perpendicular to the line joining the specified corner and centre of gravity of the solid. The shape of the front view is retained. The top view may be projected from the front view.

Solution

Step-1:

A regular pentagon of sides 30 mm (***abcde***) may be drawn below x-y with one side (***ab***) perpendicular to x-y. The geometric centre (***o***) may be marked. While joining ***a***, ***b***, ***c***, ***d***, ***e*** with ***o***, the top view is available. The corresponding front view can be projected. ***o*** may be projected to x-y. A perpendicular of 50 mm long (height of the solid) may be drawn to x-y, which represents the axis of the solid. Front views of corners of base, ***d'***, ***c'*** and ***b'***, may be marked on x-y. Joining ***d'***, ***c'*** and ***b'*** with ***o'***, the front view is obtained.

Step-2:

The centre of gravity ***g'*** may be marked on the axis (one fourth of the height from the base) in the front view. x_1-y_1 may be drawn in such a way that it is perpendicular to ***c'g'***. ***b'***, ***c'***, ***d'*** and ***o'*** may be projected perpendicular to x_1-y_1. The distances of ***a***, ***b***, ***c***, ***d***, ***e*** and ***o*** in the first step top view from x-y may be taken and marked on the respective projectors from x_1-y_1. While joining ***a-b***, ***b-c***, ***c-d***, ***d-e***, ***e-a*** and ***a***, ***b***, ***c***, ***d***, and ***e*** to ***o*** (as per Figure 7.12) and finishing properly both top view and front view, the projections of the pyramid are completed.

It may be noted that the triangular face ABO is below the solid, that face will be hidden in the top view. The hidden longer edges AO and BO in the top view (***ao*** and ***bo***) may be drawn as dotted line.

Problem 7.8

A cone, of base diameter 50 mm and height 60 mm, is lying on VP with a generator. Draw the projections if the axis is parallel to HP.

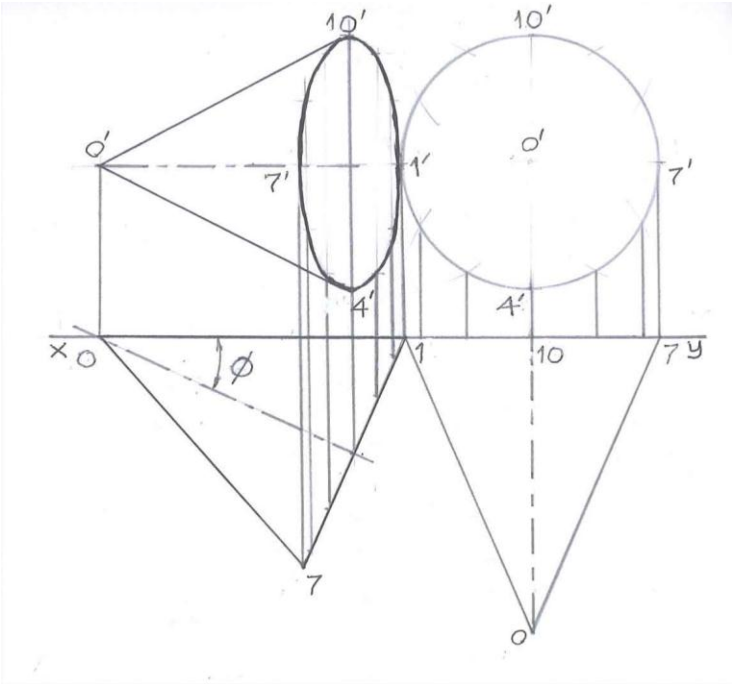


Figure 7.13 Cone resting on VP with a generator, axis parallel to HP

Solution

<u>Step-1:</u>

When the cone is lying with a generator on VP, the axis is inclined to VP. Hence in the first the solid must be assumed to be perpendicular to VP. As a generator (line joining a point on the circumference and the apex) is on VP, then the diameter through that point on the circumference must be parallel to HP.

A circle of diameter 50 mm may be drawn above x-y. The circle may be divided into minimum 12 equal parts. The diameter ***1'7'*** may be taken parallel to HP. It is assumed that the generator O1 will be on VP in the final position.

All the points on the circumference of the base may be projected to x-y and the corresponding top view may be drawn after taking a height of 60 mm.

<u>Step-2:</u>

The top view may be re-constructed in such a way that ***o1*** is on x-y (generator O1 is on VP). All the points from the top view of the first step may be marked in the re-constructed top view. Points ***1*** to ***12*** and ***o*** may be projected and the front view may be obtained. While observing from front, the base is fully visible, but appears to be an ellipse (refer Figure 7.13).

7.2.4 Projections of Solids with axis inclined to both HP and VP

Projections of solids with axis inclined to both HP and VP can be obtained from the knowledge of projections of solids with axis inclined to one plane.

While closely observing the description of solids with the axis inclined to both the principal planes, there are generally two types of problems.

(i) The position of the solid will be described, giving inclination of the axis to one principal plane and inclination of an edge (which is parallel to the plane to which inclination of the axis is given) to the other principal plane.

(ii) In the second type, inclination of the axis to both the principal planes will be given.

By identifying and analyzing, using three steps (generally) both the types of problems can be solved easily.

Problem 7.9

A square pyramid, side of base 30 mm and height 50 mm, is resting on HP with a side of its base. The axis of the solid is inclined 45^o to HP while the edge on HP is inclined 30^o to VP. Draw the projections.

Analysis

This is a typical problem from the first category of problems. The square pyramid is resting on HP with a side of base, axis inclined at 45^o to HP and the side which is on HP is inclined 30^o to VP.

Step-1:

As the axis is inclined to HP, in the first step, axis must be treated as perpendicular to HP. The side of the base, which is on HP and inclined to VP, must be on HP and perpendicular to VP in the first step. Ultimately the solid is resting on HP with its base, with the axis perpendicular to HP and one side of base perpendicular to VP.

Step-2:

Keeping the side which is to be made inclined to VP on HP, the axis may be made inclined to HP. The corresponding projections may be drawn.

Step-3:

Keeping the inclination of the axis to HP same, the side which is perpendicular to VP initially, may be made inclined to VP. Since the inclination of the axis to HP is retained, the top view remains same. The corresponding front view may be obtained.

Solution

Step-1:

As the axis is inclined 45^o to HP, in the first step, axis must be treated as perpendicular to HP. The side of the base, which is on HP and inclined to 30^o VP, must be on HP and perpendicular to VP. Now, the solid is resting on HP, with the axis perpendicular to HP and one side of base perpendicular to VP.

A square ***abcd*** of side 30 mm may be drawn below x-y, taking ***ab*** (and ***cd***) perpendicular to x-y. After locating the geometric centre ***o***; the corners ***a***, ***b***, ***c*** and ***d*** may be joined with ***o***. This is the top view of the pyramid.

The axis may be indicated in the front view taking 50 mm length. From top view ***a***, ***b***, ***c*** and ***d*** may be projected to x-y. Joining ***b′*** and ***c′*** with ***o′***, the front view is completed.

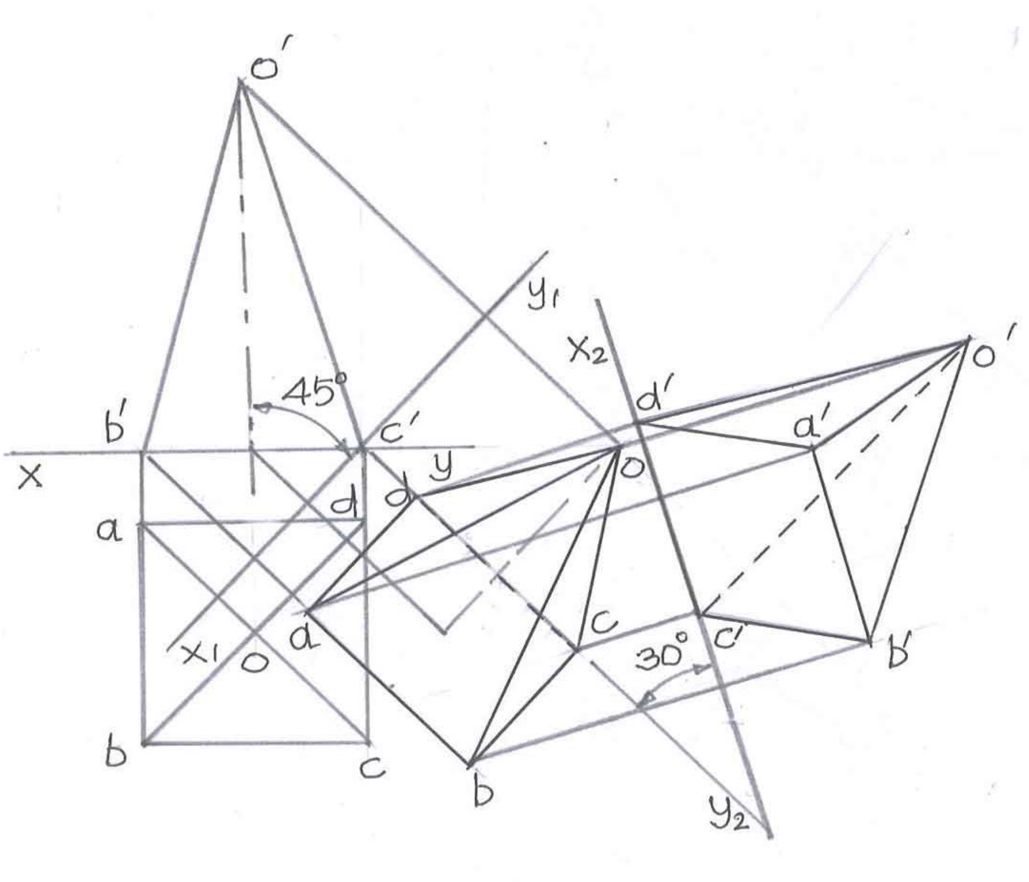


Figure 7.14 Projections of pyramid-axis inclined 45^o to HP and one side of base inclined 30^o to VP

Step-2:

Keeping the side ***cd*** (which is to be made inclined 30^o to VP) on HP, the axis may be made inclined to HP.

x_1-y_1 may be drawn through ***c′*** at 45^o to the axis. The corresponding top view may be drawn by projecting every point in the front view with respect to x_1-y_1 and marking the distances from previous top view, from x-y.

By viewing along the front view perpendicular to x_1-y_1, it can be understood that ***cd*** is hidden in the top view (as ***c'*** or ***c'd'***, comes as the bottom most point in the front view or the solid actually rests with the side CD as the bottom most).

Step-3:

Retaining the inclination of the axis to HP, the side CD which is perpendicular to VP initially, may be made inclined 30^o to VP. Since the inclination of the axis to HP is retained, the top view remains same.

x_2-y_2 may be drawn at 30^o inclined to ***cd***. All the points, ***a***, ***b***, ***c***, ***d*** and ***o*** may be projected with respect to x_2-y_2. The corresponding front view may be obtained by marking the distances of the previous front view from x_1-y_1.

After finishing top view ***abcd-o*** and front view ***a'b'c'd'-o'***, the projections are available (Figure 7.14).

By observing the solid in its position, it is understood that the base is visible in the front view. It also indicates that the sides of base AB, BC, CD and DA, and slant edges AO, BO and DO are visible while CO is hidden.

Problem 7.9

A square pyramid, side of base 30 mm and height 50 mm, is resting on HP with a side of its base. The axis of the solid is inclined 45^o to HP and 30^o to VP. Draw the projections.

Analysis

This is a typical problem from the second category of problems. The square pyramid is resting on HP with a side of base, axis inclined at 45^o to HP and 30^o to VP.

Step-1:

As the inclinations of the axis to HP and VP are given, with a side on the HP, axis must be treated as perpendicular to HP in the first step. The side of the base, which is on HP must be on HP and perpendicular to VP. In effect the solid is resting on HP, with the axis perpendicular to HP and one side of base perpendicular to VP. The top view may be drawn first and front view may be projected then.

Step-2:

Keeping the side on HP, the axis may be made inclined to HP. The corresponding projections may be drawn. The front view may be drawn and top view may be obtained.

Step-3:

Keeping the inclination of the axis to HP same, it may be made inclined to VP. Since the inclination of the axis to HP is retained, the top view remains same. The axis can be considered as a line, which is inclined 45^o to HP and 30^o to VP. After obtaining the apparent inclination the solution can be completed.

Solution

<u>Step-1:</u>

As the axis is inclined 45^o to HP, in the first step, axis must be treated as perpendicular to HP. The side of the base, which is on HP must be on HP and perpendicular to VP. Now the solid is resting on HP, with the axis perpendicular to HP and one side of base perpendicular to VP.

A square ***abcd*** of side 30 mm may be drawn below x-y, taking ***ab*** (and ***cd***) perpendicular to x-y. After locating the geometric centre ***o***; ***a***, ***b***, ***c*** and ***d*** may be joined with ***o***. This is the top view of the pyramid.

The axis may be indicated ($o_1'o'$) in the front view taking 50 mm length. From top view ***a***, ***b***, ***c*** and ***d*** may be projected to x-y. Front view is obtained by joining ***b′*** and ***c′*** with ***o′***.

<u>Step-2:</u>

Keeping the side ***cd*** on HP, the axis may be made inclined to HP.

The front view from the first step may be re-constructed in such a way that ***c′*** on x-y and the axis is inclined 45^o to x-y. The corresponding top view may be drawn by projecting every point in the front view and drawing parallel lines from the previous top view.

By viewing along the front view perpendicular to x-y, it can be understood that ***cd*** is hidden in the top view (as ***c′*** or ***c′d′***, comes as the bottom most point in the front view or the solid actually rests with the side CD as the bottom most).

<u>Step-3:</u>

As the inclination of the axis to HP is 45^o, whatever being the inclination to V, the top view will be unchanged.

A line o_1o_2=50 mm (length of axis) may be drawn at 30^o inclined to x-y.A line parallel to x-y may be drawn through o (locus of O in the top view). o_1 as centre and the top view of the axis from second step as radius, an arc is drawn to cut the locus of o. o_1o as axis, the top view may be re-constructed to get the final top view. The corresponding front view may be obtained by drawing parallel lines to x-y from the previous front view.

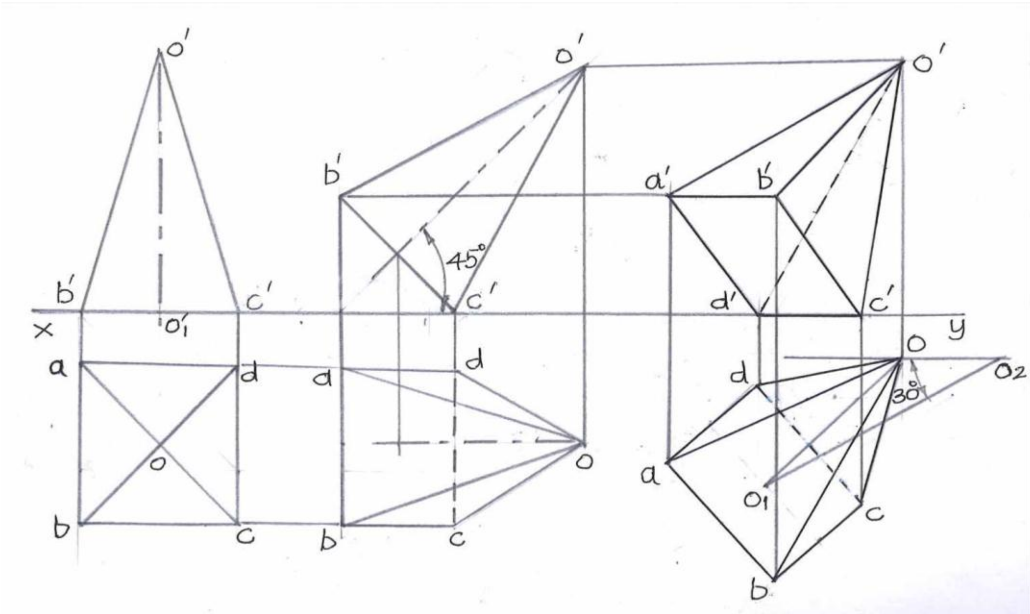


Figure 7.15 Projections of pyramid-axis inclined 45° to HP and 30° to VP

After finishing top view ***abcd-o*** and front view ***a'b'c'd'-o'***, the projections are completed (Figure 7.15).

By observing the solid in its position, it is understood that the base is visible in the front view. It also indicates that the sides of base AB, BC, CD and DA, and slant edges AO, BO and CO are visible while DO is hidden.

Problem 7.10

A square pyramid, side of base 30 mm and hieght 50 mm, is resting on HP with a triangular face. Draw the projections if the shorter side of the face on HP is inclined 45° to VP.

Solution

The pyramid is resting on HP with a triangular face and the shorter side of the face is inclined 45° to VP. When the pyramid is resting on HP with a slant face, the axis is inclined to HP. It is also given that one shorter side on this slant face is inclined to VP. This is a problem from the first category.

Step-1:

The pyramid is assumed to be resting on HP with its base, axis perpendicular to HP and one side of the base (The shorter side of the slant face is the side of the base only as every slant face is constituted by two slant edges and one side of the base) is perpendicular to VP.

A square (***abcd***) of side 30 mm may be drawn below x-y, with one side perpendicular to x-y. ***a***, ***b***, ***c*** and ***d*** may be joined with its geometric centre ***o***. This is the top view in the first step.

The axis of the solid may be shown taking a length of 50 mm. ***a***, ***b***, ***c*** and ***d*** may be projected to get the corresponding front view on x-y. ***b'*** and ***c'*** may be joined with ***o'***. The front view is drawn now.

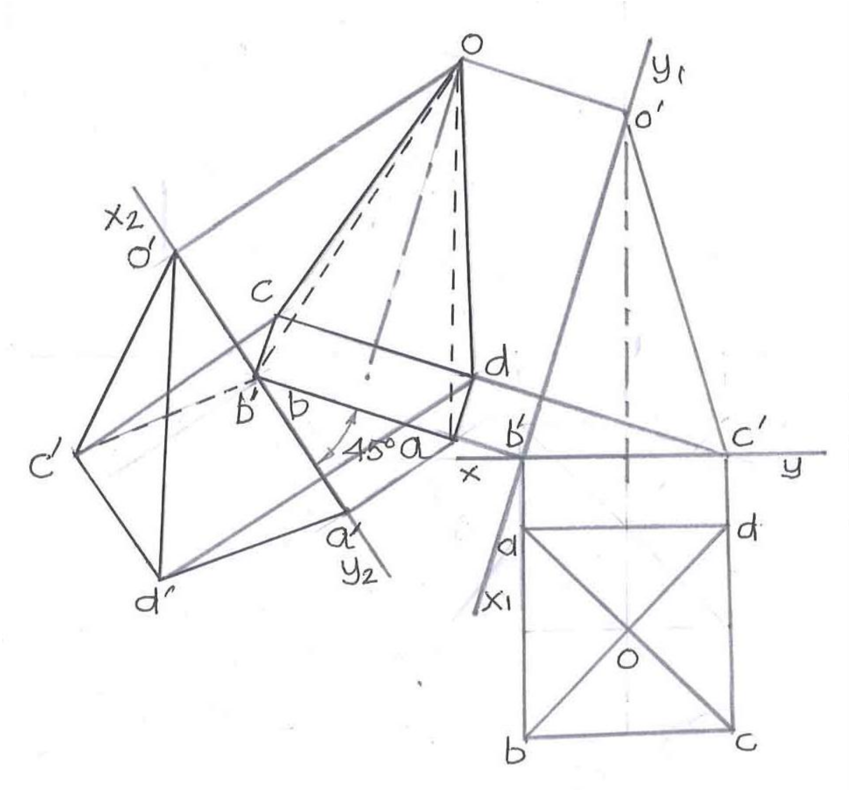


Figure 7.16 Square pyramid resting on HP with a slant face with the shorter side on HP, inclined 45^{o} to VP

Step-2:

For making one slant face on HP, x_1-y_1 may be drawn through ***o'b'***. All the points in the front view may be projected to get the corresponding top view. The distances of ***a***, ***b***, ***c***, ***d*** and ***o*** from x-y may be taken and marked, from x_1-y_1, to obtain the top view.

Step-3:

Keeping the slant face on the HP, the shorter side of the slant face may be made inclined to VP. The top view will be retained and in the top view the shorter side of the slant face may be made inclined to x-y.

x_2-y_2 may be drawn at an angle of 45^{o} with ***ab***. All the points in the top view may be projected with respect to x_2-y_2 and the distances of ***a'***, ***b'***, ***c'***, ***d'*** and ***o'*** may be taken from x_1-y_1 and marked on the projectors, from x_2-y_2.

After finishing ***abcd-o*** and ***a'b'c'd'-o'*** both projections are available (Figure 7.16).

Problem 7.11

A hexagonal prism, side of base 20 mm and height 50 mm, is resting on HP with a side of the base. The axis is inclined 30^o to HP and 60^o to VP. Draw the projections if a side of the other end face is on VP.

Solution

Step-1:

Basically this problem falls in the second category. As one side of the base is on HP, in the first step itself that side must be on HP and perpendicular to VP. To make the axis inclined to HP, the axis must be kept perpendicular to HP in the first step.

A regular hexagon (***abcdef***) may be drawn with side 20 mm, below x-y, keeping one side ***12*** or ***45*** (***ab*** or ***de***) perpendicular to x-y. Taking a length of 50 mm for the axis the front view (***2'3'4'-b'c'd'***) may be completed.

Step-2:

The axis has to be made inclined 30^o to HP keeping one side of base, ***45***, on HP. x_1-y_1 may be drawn inclined 30^o to the axis through ***4'5'***. All points in the front view may be projected to get top view, by taking distances of ***1***, ***2***, ***3***, ***4***, ***5***, ***6***; and ***a***, ***b***, ***c***, ***d***, ***e***, ***f*** from x-y and marking from x_1-y_1.

Step-3:

As per the question, 45 is on HP or its front view ***4'5'*** will be on x-y. The geometrically oposite side DE is on VP or ***de*** will be on x-y. As the sum of the inclinations of the axis to HP and VP is 90^o, the axis appears to be straight lines perpendicular to x-y in both top view and front view.

x_2-y_2 may be drawn through ***de***. All the points from the top view may be projected, and the distances of of all points in the front view from x_1-y_1 may be marked from x_2-y_2.

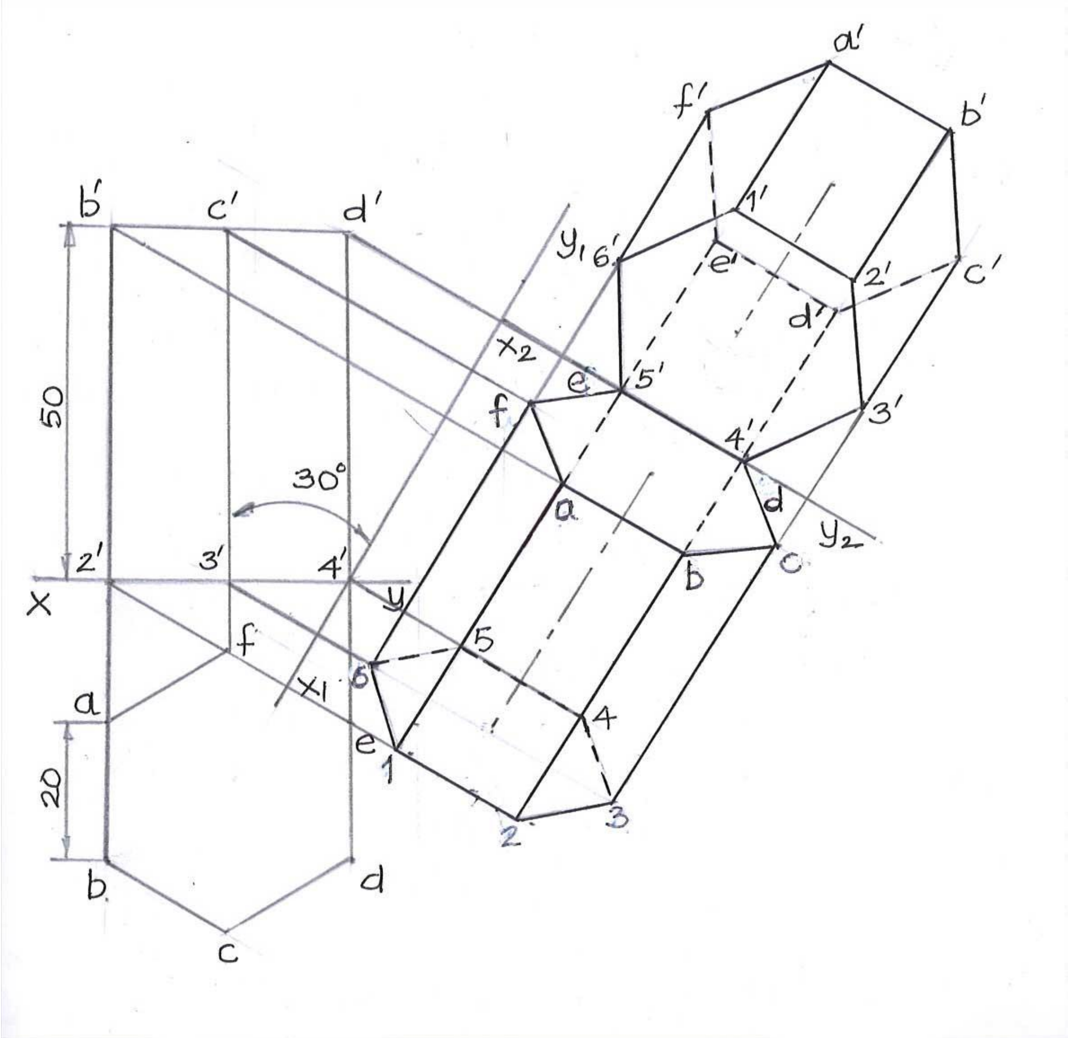


Figure 7.17 Projections of hexagnal prism inclined to both HP and VP, on side of base on HP and geometrically opposite side on VP

While joining the corresponding points the front view is also completed (Figure 7.17). On verification, it can be found that the axis, both in front view and top view, will be perpendicular to x-y. All longer edges are also perpndicular to x-y in both views as all these are parallel to the axis.

7.3 Projections of spheres

Projections of spheres are basically simple as spheres in any direction appear to be circles. But, care must be taken in deciding whether the sphere or part of sphere is visible or hidden by other solids including spheres.

A few illsutrations are given from projctions of spheres having equal diameter and un-equal diameters.

Problem 7.12

A sphere of diameter 48 mm is placed centrally on three equal spheres of diameter

25 mm each, which are resting on the HP, each touching the other two. Assume the line joining the centres of two equal spheres is parallel to VP.

Analysis

Three equal spheres which are resting on the HP carry a fourth sphere. While joining the centres of the spheres which are resting on HP, it will give an equilateral triangle, whose one side is parallel to VP. As the centres are equai-distant from HP, the plane of the equilateral triangle is parallel to HP. The top view of the triangle will be of true shape.

When the fourth sphere is resting on the three sphere combination, a portion of the upper sphere, which is in the gap between the sphere combination, will be hidden in the front view. By observing from top, as the bigger sphere is visble fully, major portions of the smaller spheres will become hidden by the upper bigger one.

Solution

An equilateral triangle ($c_1c_2c_3$) of side 25 mm (sum of radii of two spheres) may be constructed below x-y with one side parallel to x-y. This will indicate the equilateral triangle made by joining the centres of equal spheres which are resting on HP.

c_1, c_2, c_3 as centres circles may be drawn with radius 12.5 mm each. This is the top view of three equal spheres each touching other two (refer Figure 7.18). A circle of diameter 48 mm may be constructed with centre c_4, the geometric centre of the triangle. $c_1c_2c_3$-c_4 represents the equailateral traingular pyramid made by joining the cntres of all spheres. c_1c_4, c_2c_4, and c_3c_4 are the slant edges of the pyramid. These also indicate the lines joining the centres of spheres that are resting on the HP and the centre of the bigger sphere.

While observing from the top, the bigger sphere will be visible fully, while the major portions of the smaller spheres are hidden.

c_1, c_2, c_3 are projected to get the front views of the centres. The line parallel to and 12.5 mm above x-y is drawn to get the front views of centres. c_1', c_2', c_3' as centres cicles may be drawn with radius 12.5 mm, which will give the front view of three sphere combination.

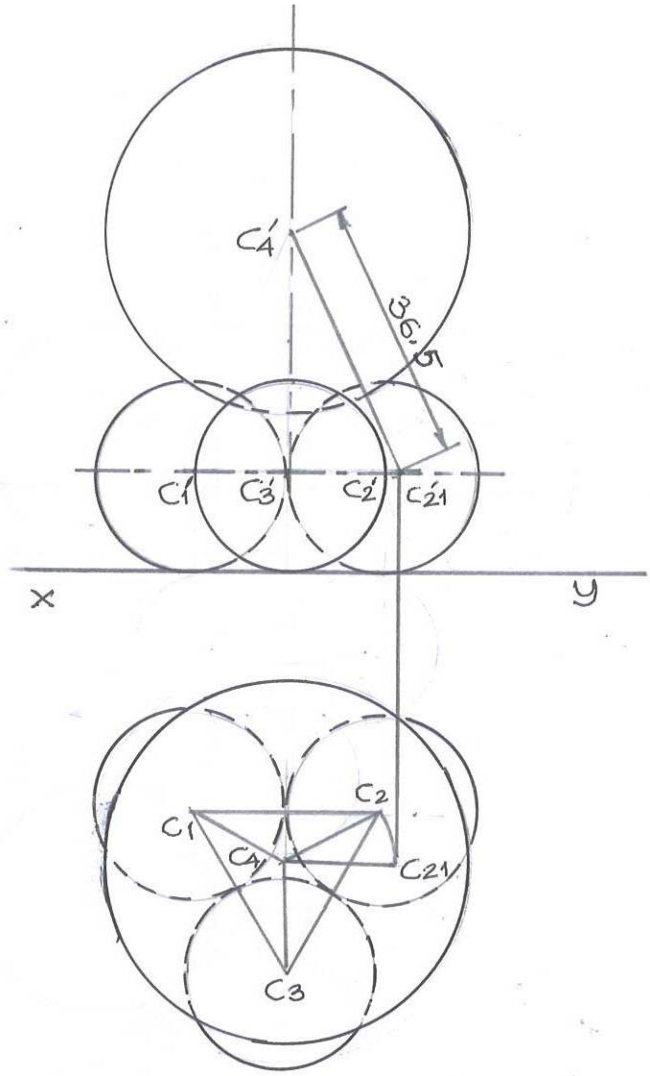


Figure 7.18 A bigger sphere is resting on three equal spheres

The projector from c_4 may be drawn. One of the slant edges of the imaginary triangular pyramid, say c_2c_4, may be made parallel (c_4c_{21}) to x-y or the corresponding front view will give the true length of the line joing the centres of a small sphere and the bigger sphere (12.5 mm + 24 mm= 36.5 mm). c_{21} may be projected to the plane of the centres of the smaller spheres (c_{21}'). c_{21}' as centre 36.5 mm as radius, an arc is drawn to intersect with the projector from c_4. The point of intersection is c_4'. c_4'as centre, with radius 24 mm the circle drawn will indicate the bigger sphere in the front view.

The visible part of spheres (or the hidden part) of spheres can be identified. Considering the position of the centres of spheres with rspect to VP, this can be found out. The farthest from VP or the nearest to the observer is the small sphere having centre C_3. This sphere will be visible fully viewing from front or the circle representing this sphere will be visible fully in the front view. Next distant sphere from VP is that having centre C_4. The portion of the bigger sphere other than hidden by the sphere with centre C_3 will be visible in the front view.

The remaining two spheres are equi-distant from VP. The portions of these spheres other than hidden by the spheres with centres C_3 and C_4, are visible. The corresponding circles may be finished properly after identifying the visible lines and hidden lines.

Problem 7.13

Three spheres of diameters 48 mm, 25 mm and 12 mm resepctively are placed on the HP each touching th other two. Draw the projections.

Solution

C_1, C_2, and C_3 are the centres of the spheres having diameters 48 mm (big), 25 mm (medium), and 12 mm (small), respectivley.

For convenience, the line joining C_1 and C_2 is assumed to be parallel to VP. Now $c_1'c_2'$ will give true length and c_1c_2 will be parallel to x-y. A line is drawn parallel to and below x-y, and c_1 is marked on the line. A circle of diameter 48 mm is drawn c_1 as centre, which is the top view of the big sphere. c_1 is projected and c_1' is marked on the projector taking 24 mm from x-y. c_1' as centre if a circle of diameter 48 mm is drawn, its front view is also available.

A line is drawn parallel to x-y, and 12.5 mm above, on which the centre of medium size sphere lies. c_1' as centre and 36.5 mm as radius an arc is made on the line already drawn parallel to x-y. The point of intersection is c_2'. c_2' as centre 12.5 mm as radius a circle is drawn. This is the front view of the sphere of medium size. c_2' may be projected to the line parallel to x-y through c_1. The meeting point of the projector is c_2. The circle drawn with 25 mm diameter, c_2 as centre will give the top view of the medium size pshere.

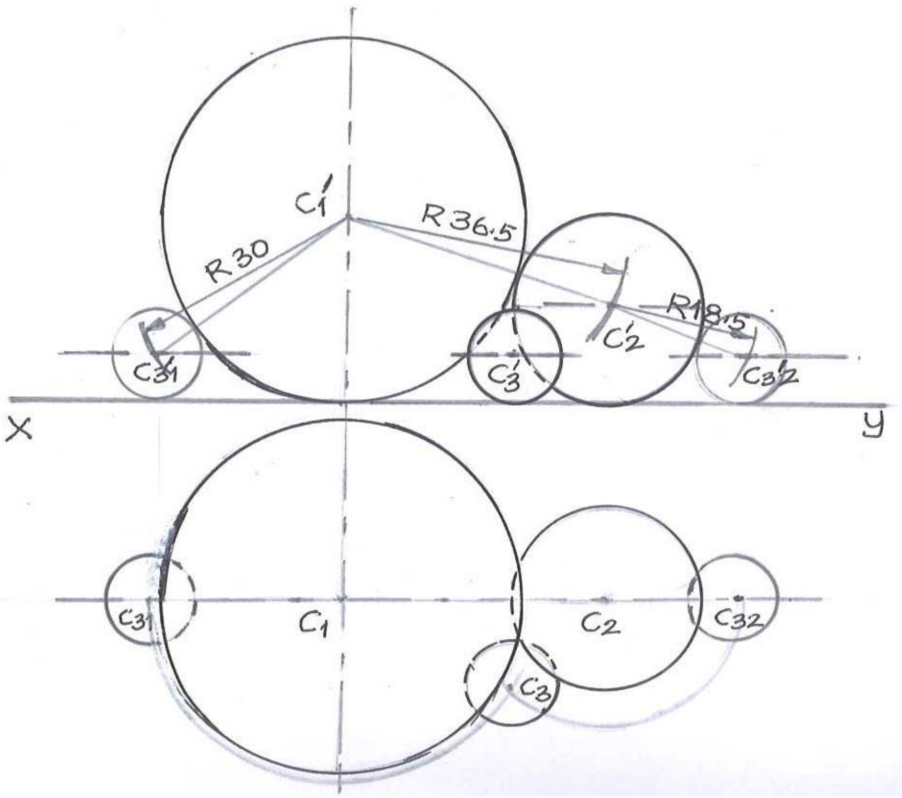


Figure 7.19 Three unequal spheres lie on HP each touching the other two

In the same way, the relative positions of the small sphere with the big sphere and medium sphere can be found out.

A line is drawn parallel to x-y at 6 mm above. c_{31}' may be marked by making an arc of radius 30 mm. The circle of diameter 12 mm shows the position of the small sphere with respect to the big one. The corresponding top view may be drawn after projecting c_{31}' to the line on which c_1 lies. The circle having 12 mm diameter, c_{31} as centre will give the raltive position of small sphere if it is placed nearer to the big one. Observing from top, the distance between centres of small and big spheres is c_1c_{31}, whereever the small sphere is placed touching with the big one.

By making an arc of radius 18.5 mm and centre c_2 to the centre line of small sphere above x-y, the position of the centre (c_{32}') of small sphere is available when it is kept on the HP and adjacent to the medium sphere. c_{32}' as centre, 6 mm radius circle will be the front view of the small sphere in the position. The corresponding top view can be drawn by projecting c_{32}' to the line parallel to x-y passing through c_1. The distance between the centres of medium sphere and small sphere in the top view when they are touhing mutually will be c_2c_{32}.

The position of c_3 in the top view can be found out. c_3 is at a distance of c_1c_{31} from c_1

and c_2c_{32} from c_2. c_1 as centre, c_1c_{31} as radius and c_2 as cntre c_2c_{32} as radius arcs may be drawn. The point of intersection of these arcs will be c_3. c_3 as centre 6 mm as radius a circle may be drawn to get the top viw of small sphere. c_3 may be projected to get its front view, c_3'. The circle with c_3' as centre, 6 mm as radius, will be the small sphere in the front view.

After deciding the visible lines and hidden lines the top view and front view of the combination may be finished (Figure 7.19).

Problem 7.14

Four equal spheres are placed on the HP around a square prism, side of base 15 mm and height, 70 mm, which is resting on the HP with its base, axis is perpendicular to HP and one rectangular face parallel to VP. Each sphere touches one rectangular face of the prism and neighbouring spheres.

Solution

A square of side 15 mm (***abcd***) may be drawn below x-y taking one side (representing one rectangular face) parallel to x-y. The front view (***2'3'-c'b'***) may be constructed taking a height of 70 mm.

Diagonals in the top view, ***ac*** and ***bd*** may be drawn, which may be extended to ***6*** and ***5***, respectively. The bisectors of angle ***5bc*** and ***6cb*** will intersect at ***2***. The perpendicular distance from ***2*** to ***bc*** is the radius of the sphere (say ***R***). The point of intersection of ***ac*** and ***bd*** as centre, its distance from ***2*** as radius a circle may be drawn. The centres of all four spheres will lie on the circle, respectively ***1, 2, 3, 4***. ***1, 2, 3, 4*** as cetres ***R*** as radius circles may be drawn. Now the top view of the combination is available.

Taking the radius from the top view, a line may be drawn parallel to x-y and baove it. This indicates the plane on which the centres of four spheres lie looking from front.

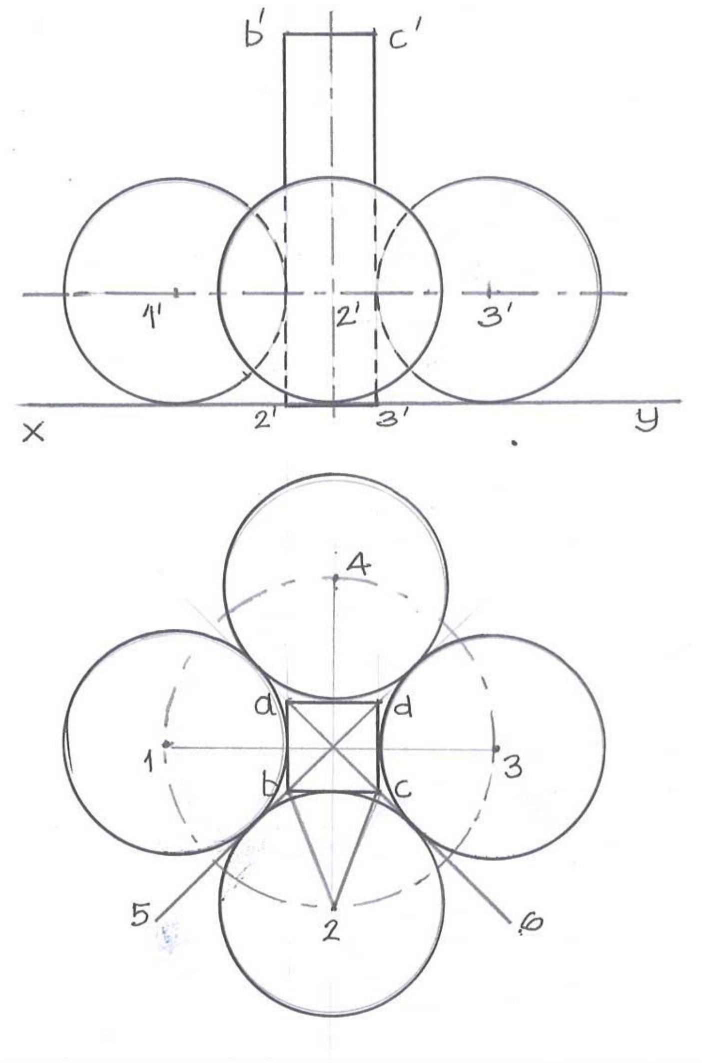


Figure 7.20 Four equal spheres are placed around the square prism

Centres c_1, c_2, c_3, c_4 may be projected to get their front view, c_1', c_2', c_3', c_4'. c_1', c_2', c_3' as centres and radius of sphere R as radius circles may be drawn. These circles will represent the spheres in the front view. While looking from front, c_2' and c_4' coincide and hence these two spheres will appear as one cicle (Figure 7.20).

Problem 7.15

Four equal spheres are placed around a square pyramid, side of base 20 mm and height 60 mm, which is resting on HP with its base, axis is perpendicular to HP, one side of the base perpendicular to VP. Darw the projections if each sphere touches the neighbouring spheres and one slant face of the pyramid.

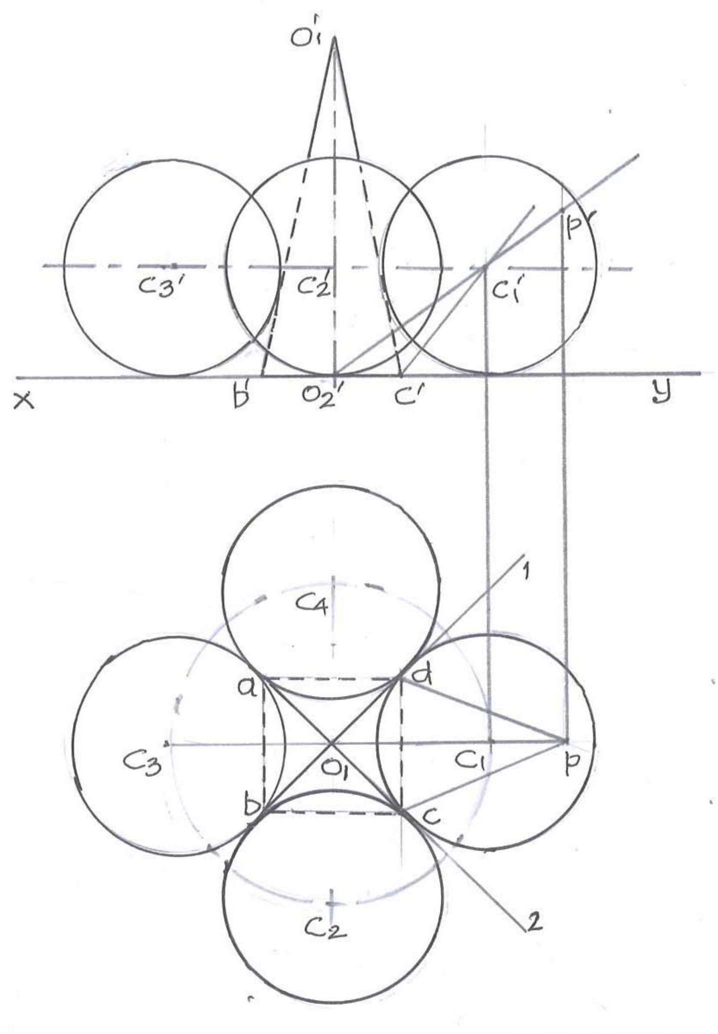


Figure 7.21 Four equal spheres are placed around a pyramid

Solution

A square of side 20 mm (***abcd***) may be drawn below x-y with one side perpendicular to x-y. This is the top view of the square pyramid. The front view (***b'o₁'c'***) of the pyramid may be constructed taking a height of 60 mm.

The diagonals of the square in the top view, ***ac*** and ***bd*** may be drawn, will intersect at o_1, and extended respectively to ***2*** and ***1***. The bisectors of angles ***2cd*** and ***1dc*** will intersect at ***p***. The perpendicular distance from ***p*** to ***cd*** is the radius of the sphere if the solid were a prism.

p may be projected to get its front view ***p'***. ***p'*** may be joined with the geometric centre o_2' of the base in the front view ($p'o_2'$). The bisector of angle made by $o_1'c'$ (angle $o_1'c'y$) with x-y may be constructed. The bisector will intersect $p'o_2'$ at c_1'. The length of the perpendicular dropped from c_1' to x-y will give the radius ***R*** of the required spheres. From c_1', its top view c_1 may be obtained. With respect to c_1 centres of other spheres c_2, c_3, c_4 may be obtained.

c_1, c_2, c_3, c_4 as centres R as radius circles may be drawn which will give the top view of all spheres. c_1, c_2, c_3, c_4 may be projected to get their front views c_1', c_2', c_3', c_4'. The circles drawn with c_1', c_2', c_3', c_4'. As centres and R as radius will be the corresponding front views of the spheres. Here c_2' and c_4' are coinciding and hence circles representing the spheres having these centres also concide (Figure 7.21).

Chapter 8

Projections of Sectioned Objects

Chapter 8
Projections of Sectioned Objects

8.1 Section

Projections of full solids were discussed in Chapter 7. Solids need not be available as full. Those can be sectioned (or cut) also. This chapter deals with the projections of sectioned objects.

Section

Any solid can be cut off or sectioned any portion with the help of any cutting tool like blade, knife, saw etc. depending up on the material used to make the object. If a part of the object is removed by cutting, then the object is called as 'Sectioned object'.

In engineering graphics in place of any cutting tool, its plane only will be referred. Hence it is known as 'cutting plane'. The cutting plane is represented by 'cutting plane line', which has resemblance with centre line, but extreme end long dashes are thick.

After cutting, the portion nearer to the observer is usually removed to view the fresh surface formed during cutting. The new surface formed is called as 'Section'. The section is indicated by using 'Section lines'. Section lines are thin, parallel to one another, uniformly spaced and inclined 45^o to the reference line.

True Section

The true section or true shape of section is visible only while observing perpendicular to the section or perpendicular to the cutting plane. Hence, to obtain the true section a view must be obtained projecting perpendicular to the cutting plane.

8.2 Cutting Planes

The commonly used cutting planes are perpendicular to at least one of the principal planes.

They are:

(i) Cutting planes parallel to one principal plane and perpendicular to the other

(ii) Cutting planes inclined to one principal plane and perpendicular to the other

(iii) Planes perpendicular to both HP and VP

The projections of objects sectioned by different cutting planes are discussed in the following section.

8.3 Illustrated Examples

Problem 8.1

A square prism, side of base 30 mm and height 60 mm, is resting on HP with its base, axis perpendicular to HP. The solid is cut by a horizontal section plane, passing through the mid-point of the axis. Assume that one rectangular face of the object is parallel to VP.

Solution

The projections of the sectioned object are usually drawn after drawing the projections of full solid, the section is represented and the required portion alone will be finished.

Every problem from sections of solids will have two parts, (i) projections of the solid in its position, (ii) showing the section of the object.

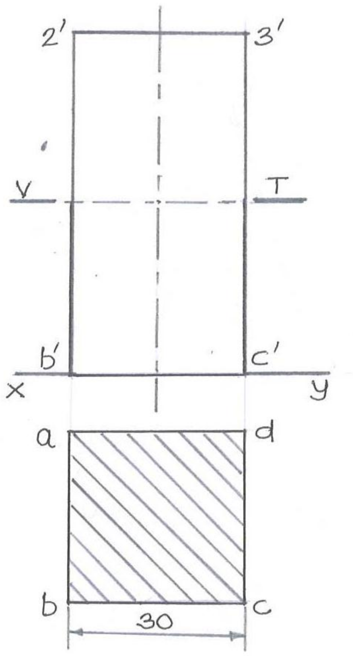


Figure 8.1 Square prism sectioned parallel to the base through the mid-point of the axis

A square ***(abcd)*** of side 30 mm may be drawn below x-y, with one side parallel to x-y. This is the top view of the square prism in the position.

The front view may be projected which is a rectangle of width 30 mm and height

60 mm. All the points in the top view and front view may be indicated. The axis of the solid may also be drawn.

The mid-point of the axis may be indicated. The cutting plane may be drawn (VT of the plane) as given in Figure 8.1.

As the axis is sectioned along the mid-point, all the vertical edges are also cut at their mid-points. It can be understood that the object is cut into two equal halves.

After sectioning, the portion nearer to the observer is removed to see the section directly.

In the front view, the part below V-T may be finished indicating only the bottom half exists (or the upper half is removed).

In the top view, the cutting points on every vertical edge coincide with ***a, b, c,*** and ***d***, respectively. Hence the shape of the section will be square, which may finished and indicated by section lines (refer Figure 8.1).

As the cutting plane is parallel to HP, the section available in the top view has true shape.

Problem 8.2

A square prism, side of base 30 mm and height 60 mm, is resting on HP with its base, axis perpendicular to HP. The solid is cut a by horizontal section plane, passing through the mid-point of the axis. Assume that all rectangular faces of the object are equally inclined to VP.

Solution

All the rectangular faces of the square prism are 45^{o} inclined to VP.

A square ***abcd***, of sides 30 mm may be drawn below x-y, taking every side is inclined 45^{o} to x-y. This is the top view of the prism.

After projecting the front view, the V-T of the cutting plane may be marked along the mid-point of the axis. Since the upper part is removed, the bottom half may be finished.

The plane cuts through the mid-points of the vertical edges and cutting points, in the top view, coincide with the corners of the base. The whole square is the section, which may be indicated by section lines (Figure 8.2).

As the cutting plane is parallel to HP, here also the section available in the top view has true shape.

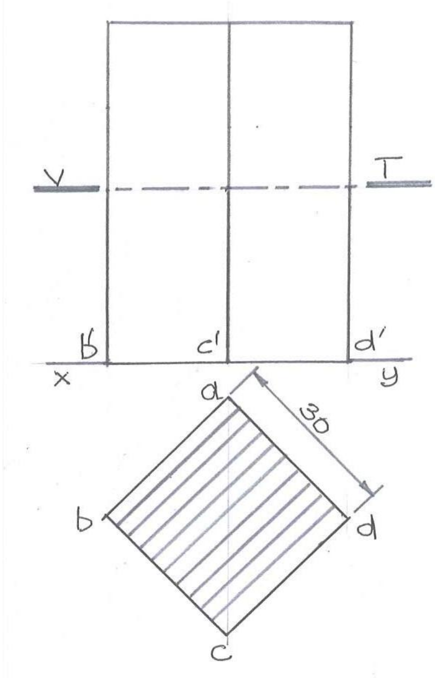


Figure 8.2 Square prism, rectangular surfaces equally inclined to VP, is sectioned by a horizontal plane

Problem 8.3

A square prism, side of base 30 mm and height 60 mm, is resting on HP with its base, axis perpendicular to HP, rectangular surfaces equally inclined to VP. The solid is cut a by a plane perpendicular to VP and inclined 45^o to HP, passing through the mid-point of the axis. Draw the projections of the sectioned square prism. Also show the true section in an auxiliary top view.

Solution

A square (***abcd***) of sides 30 mm may be drawn below x-y, taking every side as inclined 45^o to x-y. This is the top view of the prism.

After projecting the front view, the V-T of the cutting plane may be drawn 45^o inclined to x-y, along the mid-point of the axis. Since the upper part is removed, the bottom half may be finished.

The plane cuts the vertical edges at different heights from x-y, and cutting points, in the top view, ***1, 2, 3*** and ***4*** coincide with the corners of the base, respectively ***a, b, c*** and ***d***. The whole square is the section, which may be indicated by section lines (Figure 8.3).

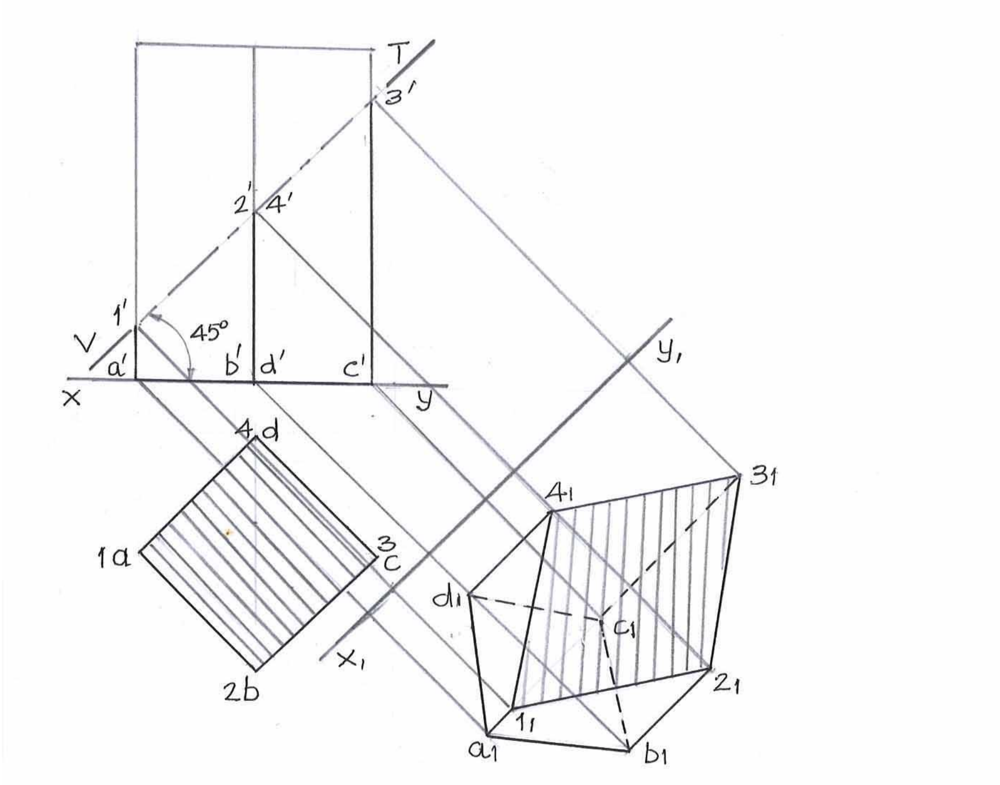


Figure 8.3 Square prism, rectangular surfaces equally inclined to VP, is sectioned by an inclined plane

As the cutting plane is inclined to HP, the top view, a projection available looking inclined to the cutting plane, is not the true section. The true shape of the section can be obtained only by viewing perpendicular to the section plane. Here the true section has to be drawn along with an auxiliary top view.

x_1-y_1 may be drawn parallel to the cutting plane line. The distances of ***1, 2, 3, 4, a, b, c,*** and ***d*** may be taken from x-y and marked with respect to x_1-y_1. After joining these points the auxiliary top view is ready. The true section $\boldsymbol{1_1 2_1 3_1 4_1}$ may be indicated with section lines.

Problem 8.4

A hexagonal prism, side of base 30 mm and height 50 mm, is lying on HP with a rectangular face, axis is perpendicular to VP. The solid is sectioned by a vertical plane in such a way that the true section is the largest hexagon possible. Also find out the inclination of the cutting plane with VP.

Solution

As the hexagonal prism is lying on HP with a rectangular face, the solution may be started with front view.

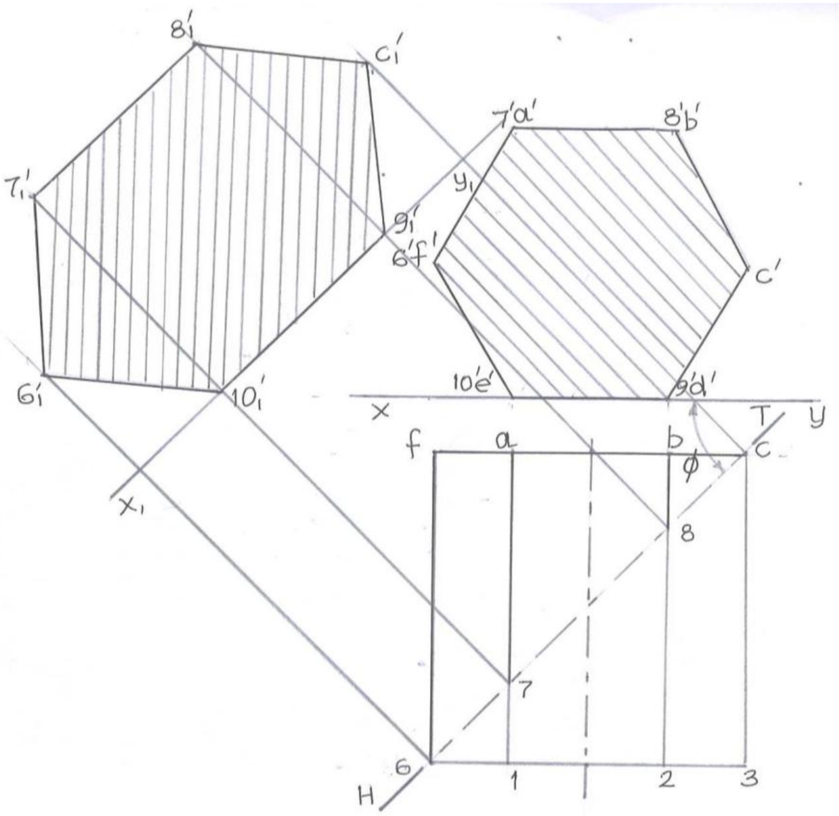


Figure 8.4 Hexagonal prism lying on HP, is sectioned making the largest hexagon

A regular heaxagon (***a'b'c'd'e'f'***) of sides 30 mm may be drawn above x-y, with one side on x-y. This is the front view of the prism. The corresponding top view may be projected. The top view is a rectangle ***cf63***. Indicate other longer edges ***a1, b2*** also in the top view. The largest heaxagon will be formed if the cutting plane passes through the corners C and 6. The vertical cutting plane (H-T in Figure 8.4) may be drawn passing through ***c*** and ***6***. The cutting plane passes through the longer edges at ***6, 7, 8, c, 9, 10***. The portion of the prism nearer to x-y in the top view may be finished.

The points (***6, 7, 8, c, 9, 10***) are marked in the front view on respective longer edges. As the plane passes through all the longer edges, the front view of the section will be a regular hexagon. The front view may be finished and section may be represented by section lines (Figure 8.4).

The inclination of H-T with x-y gives the inclination of the plane with VP (Φ).

While projecting perpendicular to H-T, the true shape of the section is available. x_1-y_1 may be drawn parallel to x-y. The distances of ***6', 7', 8', c', 9', 10'*** from x-y may be taken and marked from x_1-y_1. After joining $6_1'$, $7_1'$, $8_1'$, c_1', $9_1'$, $10_1'$ the true shape of section is available.

Problem 8.5

A suare pyramid, side of base 40 mm and height 80 mm, is cut by a horizontal plane through the mid-point of the axis. Draw the projections of the sectioned pyramid. Assume that the soild is resting on HP with its base, axis perpendicular to HP, sides of the base equally inclined to VP.

Solution

A square ***abcd*** of side 40 mm may be drawn below x-y, every side may be making 45^o with x-y. The geometric centre ***o*** may be marked and corners may be joined with ***o***.
a, b, c and ***d*** may be projected to x-y. The axis may be drawn with a length of 80 mm. ***b', c'*** and ***d'*** may be joined with ***o'***, which gives the front view of the full pyramid. The V-T of cutting plane may be drawn parallel to x-y through the mid-point of the axis. The portion of the pyramid which contains the base may be finished, which is the front view of the sectioned pyramid.

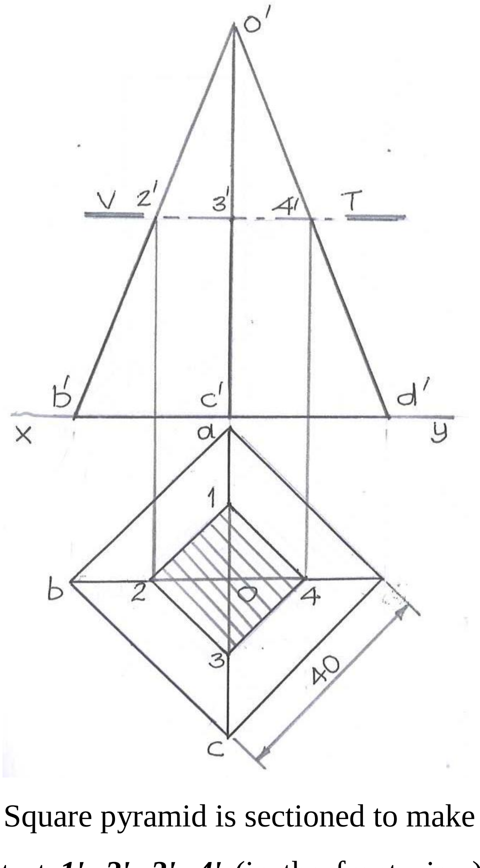


Figure 8.5 Square pyramid is sectioned to make a frustum

The slant edges are cut at ***1', 2', 3', 4'*** (in the front view) by the cutting plane. These points may be projected to the top view, ***1, 2, 3, 4***. The points ***2*** and ***4*** can be directly projected to ***ob*** and ***od***. As the cutting plane is horizontal it cuts all the slant edges at equal heights. Hence the length of the top view of all sectioned slant edges will be equal. The points ***1*** and ***3*** may be marked on respective slant edges making arcs with radius ***o4*** (or ***o2***). The square ***abcd***, the section ***1234***, and portions of slant edges ***a1, b2, c3*** and ***d4*** may be finsihed to get the top view of the frustum of the pyramid (Figure 8.5).

Problem 8.6

A square pyramid, side of base 50 mm and height 60 mm, is resting on HP with its base, axis perpendicular to HP and one side of base parallel to VP. The solid is sectioned by a plane inclined 45^o to HP and perpendicular to VP. Draw the projections of the sectioned object. Also show the true section. The plane bisects the axis.

Solution

A square of side 50 mm, ***abcd***, may be drawn below x-y with one side parallel to x-y. The corners of the square, ***a, b, c*** and ***d*** may be joined with the geometric centre. This is the top view of the square pyramid.

The front view, ***b'c'-o'*** may be drawn by direct projection taking an axis length of 60 mm.

V-T of the cutting plane may be drawn at an angle of 45^o with x-y passing through the mid-point of the axis. The portion of the pyramid containing the base may be finished to get the front view of the sectioned pyramid.

The plane cuts all the four slant edges respectively at ***1, 2, 3*** and ***4***. These points may be projected to the slant edges in the top view and may be joined to obtain the section. ***abcd, 1234*** and portions of slant edges, ***a1, b2, c3*** and ***d4*** may be finished. The section ***abcd*** may be represented with the section lines. Now the top view of the sectioned pyramid is available (Figure 8.6).

x_1-y_1 may be drawn parallel to V-T. The distances of ***1, 2, 3*** and ***4*** may be measured from x-y and may be marked from x_1-y_1. While joining ***1234*** the true section is available.

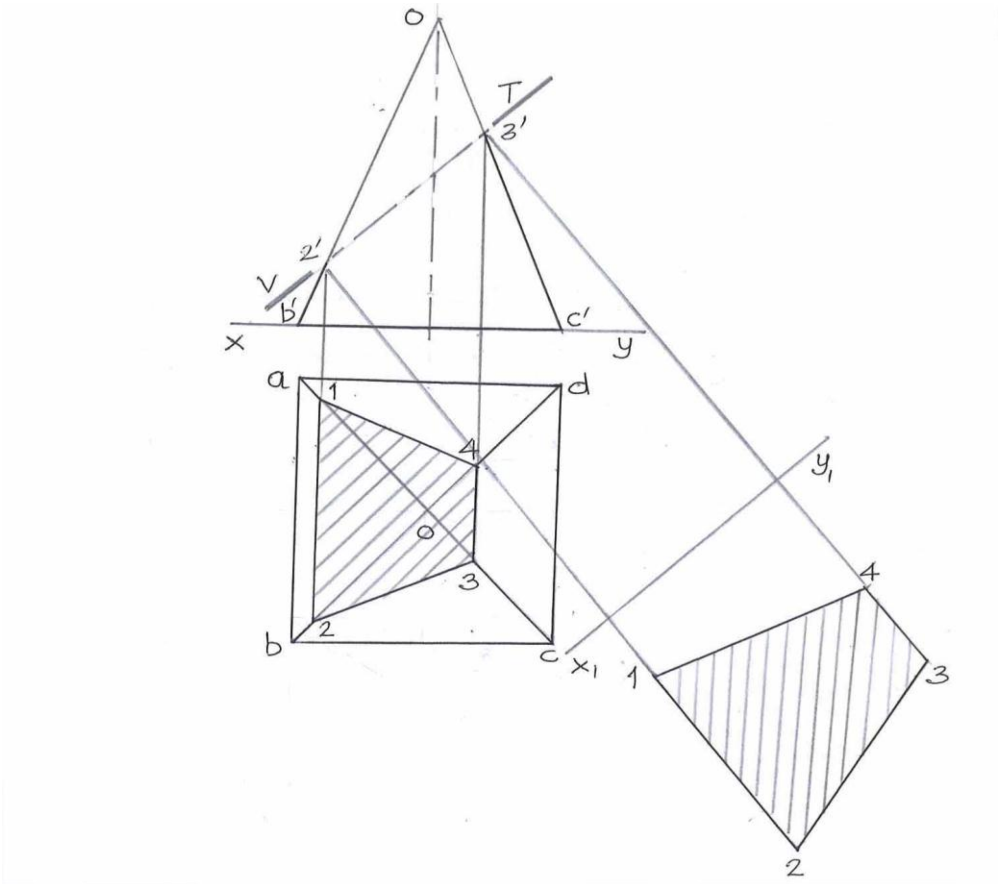


Figure 8.6 Square pyramid is sectioned by a plane 45° inclined to HP

Problem 8.7

A square pyramid, side of base 30 mm and height 60 mm, is resting on HP with a triangular face, axis is parallel to VP. The solid is sectioned by a horizontal plane, passing through the mid-point of the axis. Darw the projections of the portion of the pyramid containing the vertex of the solid.

Solution

A square (***abcd***) of sides 30 mm may be drawn below x-y, with one side perpendicular to x-y. The corresponding front view may be drawn. The front view may be rotated in such a way that one triangular face - face made by the side of base perpendicular to VP (perpendicular to x-y in the top view) and the vertex, may be coincided with x-y. The corresponding top view may be drawn.

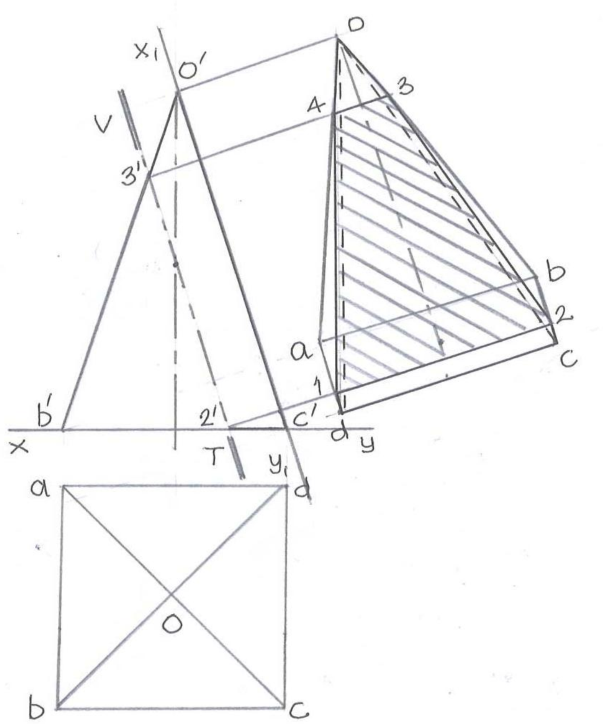


Figure 8.7 Square prism lying on HP with a slant face is sectioned by a horizontal plane

V-T of the cutting plane may be drawn passing through the mid-point of the axis. The cutting plane cuts three slant faces and the base. The points of cutting are ***1, 2*** (sides of the base) and ***3, 4*** (the slant edges). The part of the pyramid containing the vertex in the front view may be finished.

The cutting points in the front view may be projected to the top view directly. ***1, 2, 3, 4*** may be joined and may be represented as section using section lines. The top view of the sectioned pyramid may be finished (Figure 8.7).

Problem 8.8

A pentagonal pyramid, side of base 40 mm and height 60 mm, is resting on HP with its base, axis perpendicular to HP and one side of base parallel to VP. The solid is sectioned by a plane 45^{o} inclined to HP and perpendicular to VP, passing through a corner of the base. Draw the projections.

Solution

A pentagon side 40 mm, ***abcde***, may be drawn below x-y with one side parallel to x-y. ***a, b, c, d, e*** may be joined with the geometric centre to get the top view of the solid.

All the points may be projected to get front view. Taking an axis length of 60 mm the front view of the pyramid may be completed.

V-T may be drawn at an angle of 45^{o} to x-y which passes though one corner (corner B - in the front view, ***b′***)

The points at which the plane cuts the slant edges may be indicated ***1′, 2′, 3′, 4′,*** and ***5′***, respectively. The truncated pyramid (portion below the V-T of section plane) may be finished.

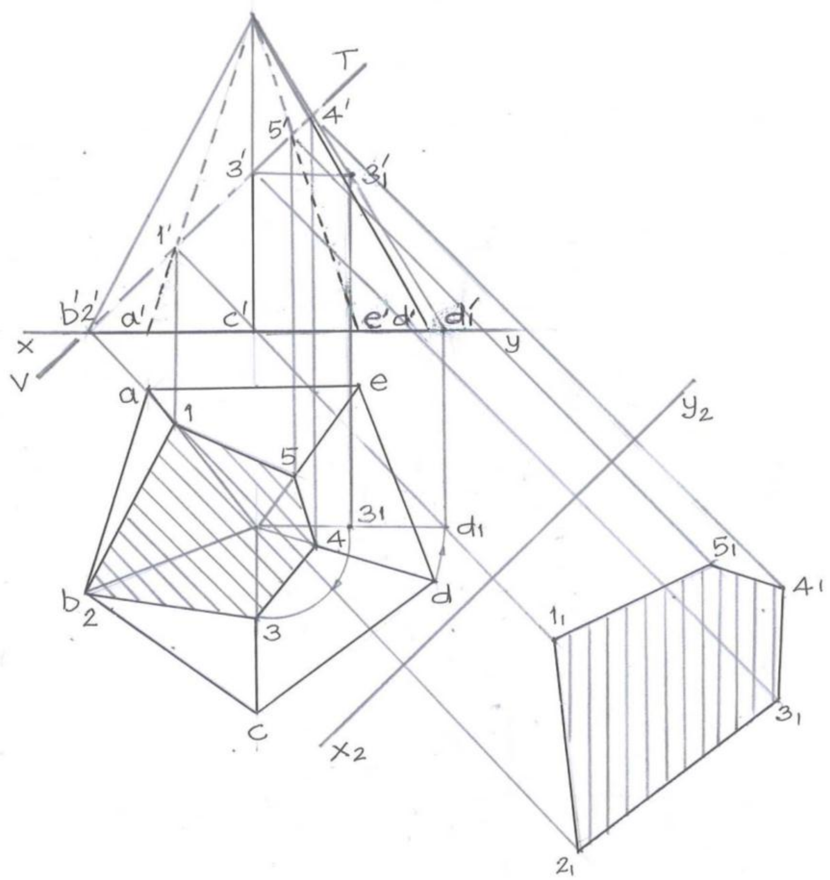


Figure 8.8 Projections of a truncated pentagonal pyramid

The points where the cutting plane passes over the slant edges may be projected to the top view. Except 3 which is on OC, can be directly projected.

While making OC (or any slant edge) parallel to x-y in the top view; the corresponding front view will give true length of OC (***od$_1$***, any slant edge) and the position of ***3*** also (***3$_1$***) on OC, while top view will be unchanged. 3_1 may be projected to ***od$_1$***. ***o3$_1$*** as radius, ***o*** as centre an arc is made to ***oc***, which will give the position of ***3***. Joining ***1, 2, 3, 4*** and ***5*** the contour of the section is available which may be finished and indicated with section lines. The top view of the truncated pyramid may be finished (Figure 8.8).

x_1-y_1 may be drawn parallel to V-T. The distances of points ***1, 2, 3, 4*** and ***5*** may be measured from x-y and be marked from x_1-y_1. While joining $\boldsymbol{1_1 2_1 3_1 4_1 5_1}$ the true shape of section is available.

Problem 8.9

A cone, base diameter 50 mm and height 60 mm, is sectioned by a plane 45^o inclined to the base removing the apex. Draw the projections of the truncated cone if the cutting plane passes through a point on the circumference of the base. Also show the true shape of the section.

Solution

As the position of the cone is not given in the question the simplest position of the cone may be assumed. The cone is assumed to be resting on HP with its base, axis perpendicular to HP.

A circle of diamter 50 mm may be drawn below x-y, which will indicate the top view of the cone. The circle may be divided in to 12 equal parts and indicate the points ***1, 2, 3, …12*** on the circle. The line joining every point (***1, 2, 3***, ..etc.) with the centre will represent a generator of the cone in the top view.

The front view of the cone may be projected. The points ***1, 2, 3, … 12*** may be projected to the front view and may be joined with ***o'*** to show 12 (minimum) generators. The V-T of the cutting plane may be drawn passing through a point on the circumference (Figure 8.9).

The cutting plane passes through all the generators as shown in the figure (***1***, $\boldsymbol{2_{11}}$, $\boldsymbol{3_{11}}$ …etc. respectively. The bottom part (removing apex) of the cone may be finished in the front view.

All the points where the cutting plane meets with the corresponding generators may be projected to the top view also (***1***, $\boldsymbol{2_{11}}$, $\boldsymbol{3_{11}}$, ..etc.). $\boldsymbol{4_{11}}$ and $\boldsymbol{10_{11}}$ cannot be projected directly. $\boldsymbol{4_{11}}$ or $\boldsymbol{10_{11}}$ (identical points) may be projected parallel to x-y to ***o'7'*** ($\boldsymbol{4'_{12}}$), which may be projected to get its top view ($\boldsymbol{4_{12}}$). ***o*** as centre $\boldsymbol{o\text{-}4_{12}}$ as radius, arcs are made to ***o-10*** and ***o-4*** (respectively $\boldsymbol{10_{11}}$ and $\boldsymbol{4_{11}}$). The points $\boldsymbol{12_{11}3_{11}4_{11}\ldots 12_{11}}$ may be joined to get the section in the top view. The top view of the truncated cone may also be finished.

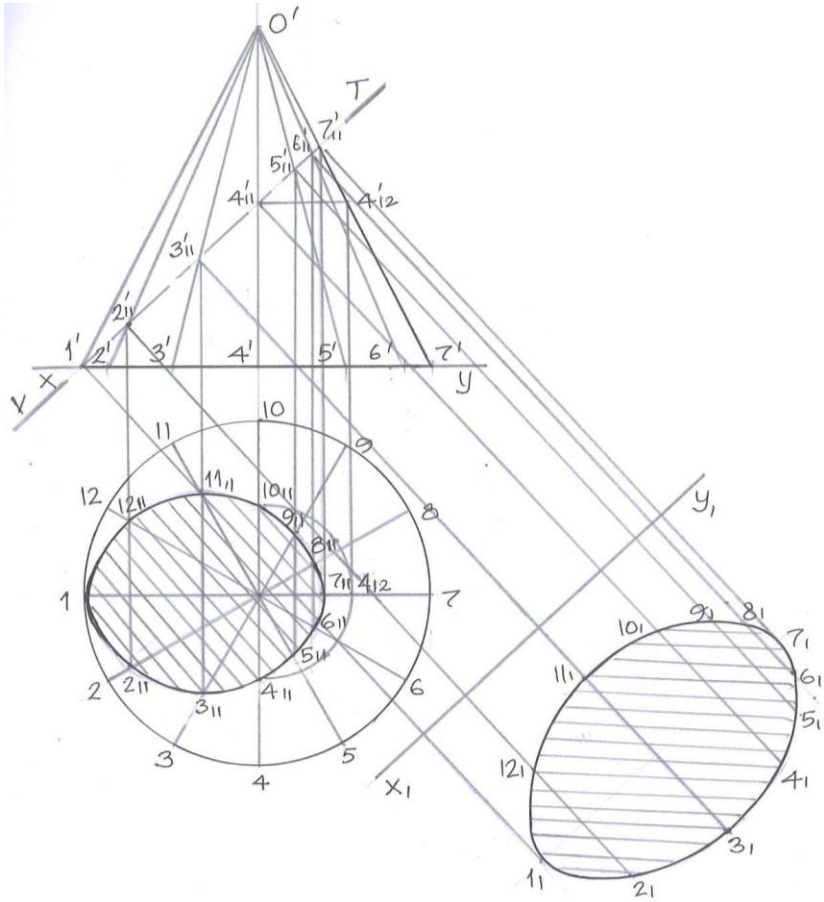


Figure 8.9 Projections of a truncated cone

x_1-y_1 may be drawn parallel to V-T. The distances of points indicating the section (***$12_{11}3_{11}4_{11}\ldots12_{11}$***) may be measured from x-y and be marked with respect to x_1-y_1. Joining ***$1_12_13_14_1\ldots.12_1$*** will give the true section.

Problem 8.10

A cone, base diameter 50 mm and height 60 mm, is resting on HP with its base, axis perpendicular to HP. It is sectioned by a plane perpendicular to both HP and VP. Draw the projections of the portion of the cone containing the apex. Assume that the cutting plane is 5 mm away from the axis. Also show the true section on a side view.

Solution

A circle of diameter 50 mm may be drawn below x-y, which is the top view of the cone. The circle may be divided into 12 equal parts and indicate the divisions on the circle (***1, 2, 3, … 12***).

The corresponding front view may be drawn and the divisions on the base circle may be indicated. The H-T and V-T of the cutting plane may be shown, which is a common line perpendicular to x-y passing through both top view and front view, 5 mm away from the axis.

The generators on the half surface of cone (O1, O2, …O7) where the cutting plane passes need to be shown (Figure 8.10).

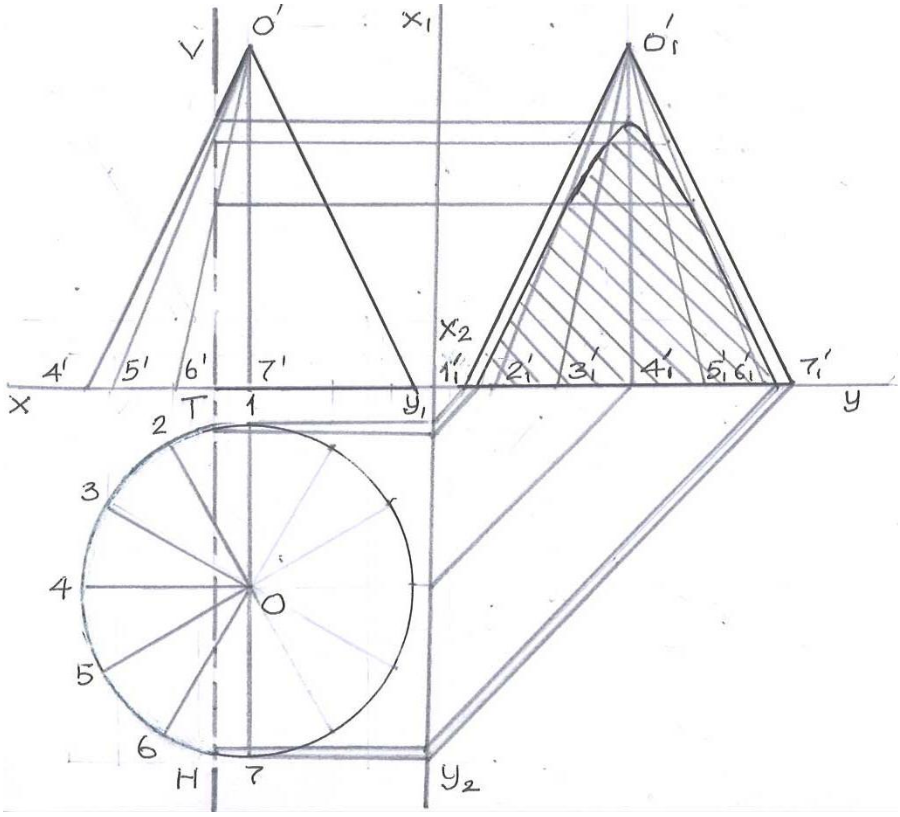


Figure 8.10 Cone sectioned parallel to the axis

The generators O2, O3, O4, O5, O6 and the base circle (at two points) are cut by the section plane. The portion of the cone which contains the apex may be finished both in the top and front views.

After drawing side view from left, the cutting points may be projected either from front view or from top view to the side view. The points at which plane passes through the base circle can be projected from the top view as given in Figure 8.10.

The side view may be finished and the section may be represented with section lines.

Chapter 9

Development of Surfaces

Chapter 9

Development of Surfaces

9.1 Development of Surfaces

If an object is made of thin sheet or foil, its surface can easily be developed by cutting it through any line on the surface and by unfolding systematically, stretching to a single plane, as a single area. This area of the external surface of any object is known as the development of the object.

For example, a square prism made of thin paper may be considered. It has (i) two end faces of identical square in shape and (ii) four equal rectangular surfaces, which are attached together to form the square prism. By cutting through any of the longer edges and through shorter edges also, the area of material used for making the square prism can be found out. The reverse has more relevance; the development of surface will give the minimum area of material required for making an object. By spreading out or developing the whole external surface area the so called 'development of square prism' is available (refer Figure 9.1).

Pure solid objects cannot be really developed as per earlier statement. In such cases, solid objects are assumed to be covered by thin foil and the covered foil alone will be developed; hence, known as 'development of surface'.

While developing the surface, the developed area gives the true surface area of the object. Hence, every line appearing on the developed surface will have true length.

As development of surface gives the minimum area required for making objects, it finds application in sheet metal industry, carton manufacturing, and similar industries where the surface area of the object is to be pre-determined to avoid wastage of material.

9.2 Illustrated Examples

Problem 9.1

Draw the development of the surface of a square prism, side of base 30 mm and height 50 mm. Assume that the prism is resting on HP with its base, all rectangular faces are equally inclined to VP.

Solution

All the problems from development are solved beginning with the projections of the object. The development of surfaces contains true area and all lines have true lengths. The true dimensions are obtained from the projections of the objects and are used to complete the development of external surfaces of the objects.

A square of sides 30 mm may be drawn below x-y, keeping all sides inclined 45^o to x-y. All the corners are indicated in the top view.

The corresponding front view is also drawn.

The end faces are parallel to HP and hence the top view will give true sides of the square. All longer edges are parallel to VP, true length of longer edges are available in the front view. Using the true sides from top view and longer edges taken from front view, the development of square prism can be drawn.

For developing the surface, it may be assumed that the surface is cut through a longer edge and conveniently the shorter edges also. In this problem, the surface is cut through A1. Then cutting is continued through A-B, A-D, D-C in the base. While in the top face, cutting is continued along 1-2, 1-4, 2-3.

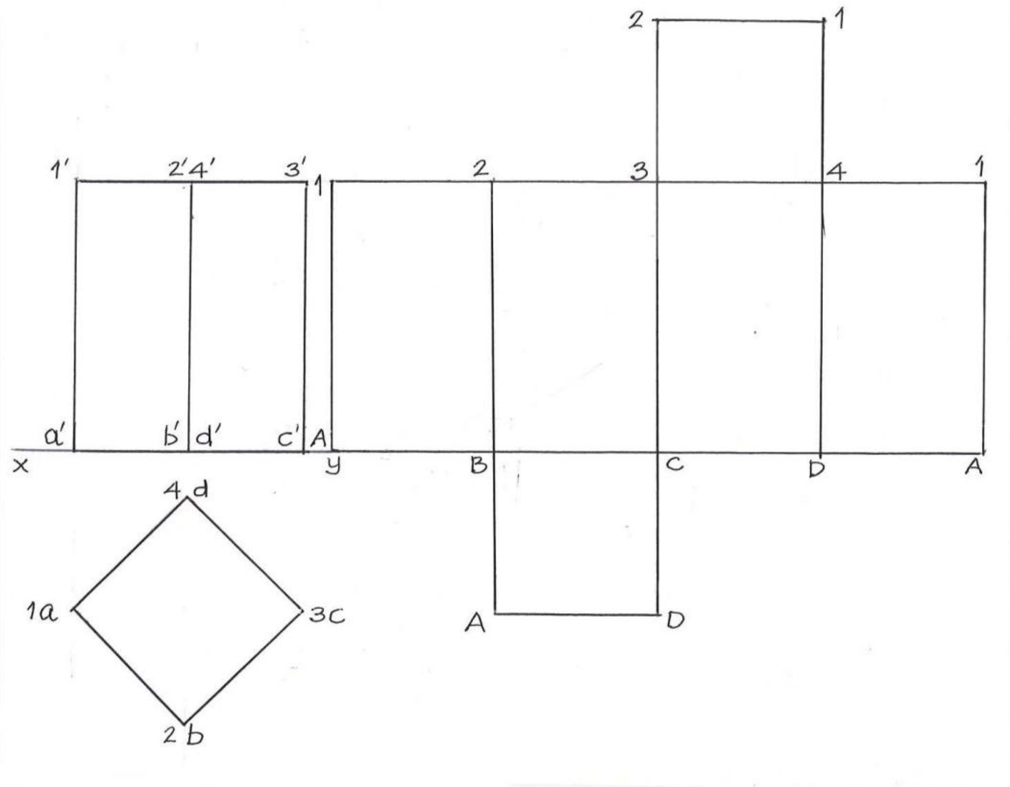


Figure 9.1 Development of a square prism

As the longer edges give true length, it can directly be projected. A line parallel to x-y may be drawn through ***1'*** (or ***3'***). Draw a line A-1 perpendicular to x-y to the line drawn parallel to x-y through ***1'***. Taking the side of the square as radius, A as centre an arc is made on x-y, to get B. In the similar process C, D, and A may be obtained. Perpendiculars may be drawn through B, C, D and A to x-y to get 2, 3, 4, and 1.

While joining A-1, B-2, C-3, D-4 and A-1 four rectangular faces are obtained in their true shape. As no cut is made through B-C, the square base may be attached at BC and similarly the top end face may be attached at 3-4. The four rectangular faces along with end faces may be finished to get the development of surface of the square prism.

Problem 9.2

A square prism, side of base 30 mm and height 50 mm, is resting on HP with its base, all rectangular faces are equally inclined to VP. The solid is sectioned by a plane 45^o inclined to HP and perpendicular to VP, passing through the mid-point of the axis. Draw the development of the sectioned prism.

Solution

This problem basically contains three parts; (i) projections of the prism, (ii) indicating the sectioned part and (iii) drawing the development of the sectioned prism.

A square of 30 mm is drawn below x-y, with all sides 45^o inclined to x-y. After completing the top view, the front view of the prism may be drawn.

The V-T of the section plane may be drawn inclined 45^o to x-y, passing through the mid-point of the axis. The bottom half may be finished indicating that the upper half is removed.

In the top view, the section appears to be square. The whole square may be indicated as section using section lines.

It may be noted that in the problems from development, the section need not be indicated using section lines; only the contour of section may be shown. The section lines indicate the material cut or this is applicable in the case of pure solid. Pure solid cannot be developed. In the case of pure solid objects, the solid is assumed as covered by thin sheets and that too alone be developed or only the external surface is developed. When an object made of thin sheet is sectioned only the contour of section is available.

In addition, in sectioned objects, the development of sectioned portion has more importance. If the section is on the lateral rectangular surfaces, not on the end faces, in many cases, the end faces are dropped while developing.

The length of longer edges, ***c'3'***, is directly projected towards side. Starting from C, taking the length of side from top view, D, A, B, C may be marked. At a height of

C3 from x-y, a line is drawn parallel to x-y. 4, 1, 2, 3 may be marked vertically above D, A, B, C, respectively. The longer edges C-3, D-4, A-1, B-2, C-3 may be drawn. The points (5, 6, 7, 8) through which the cutting plane respectively passes through these longer edges are marked in the front view. These points may be transferred to the respective lines.

C-7, D-8, A-5, B-6, C-7, 7-8, 8-5, 5-6, 6-7, C-D, D-A, A-B, B-C may be finished to get the development of the sectioned prism (Figure 9.2).

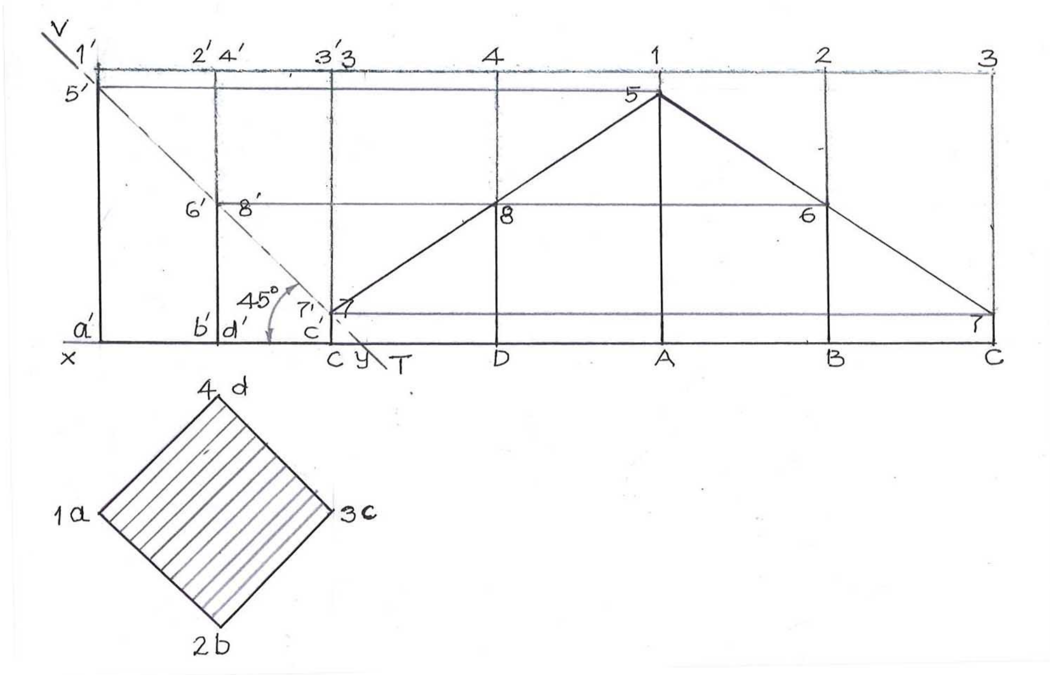


Figure 9.2 Development of sectioned square prism

Problem 9.3

A hexagonal prism, side of base 25 mm and height 60 mm, is resting on HP with a rectangular face, axis is perpendicular to VP. The prism is sectioned by a plane perpendicular to HP, inclined 45^{o} to VP, bisecting the axis. Draw the development of the sectioned prism.

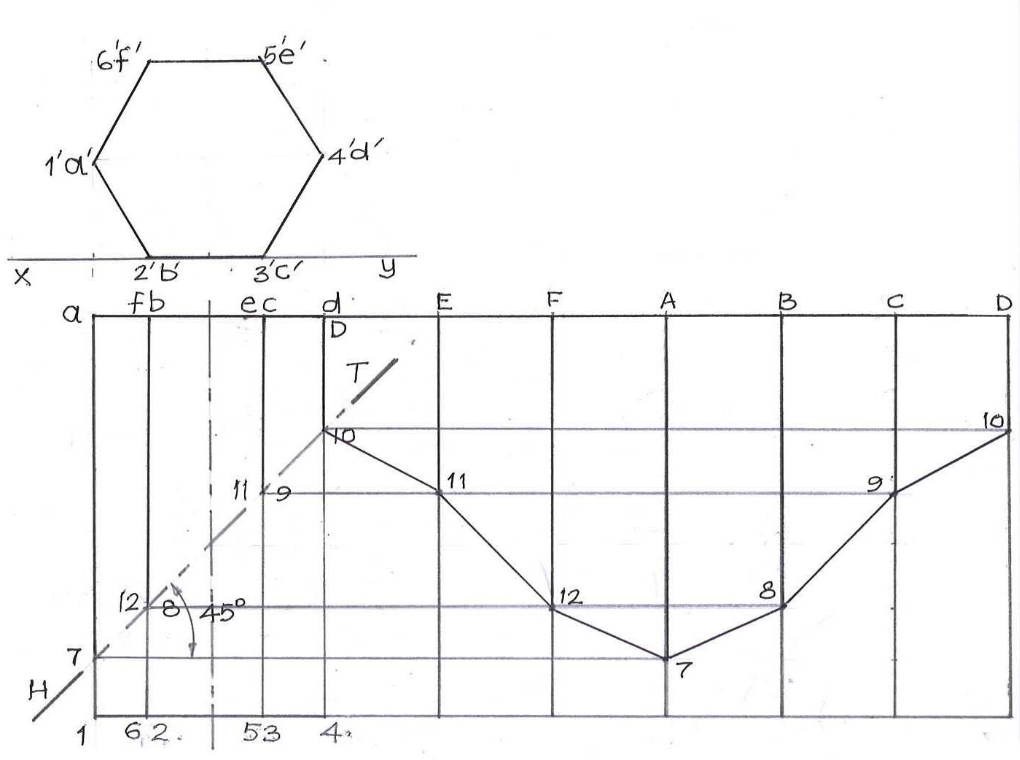


Figure 9.3 Development of sectioned hexagonal prism

Solution

A hexagon of sides 25 mm may be drawn above x-y with one side on x-y, which is the front view of the hexagonal prism.

The corresponding top view may be projected. H-T of the cutting plane may be drawn 45^o inclined to x-y and passing through the mid-point of the axis. The plane cuts all the longer edges, A-1, B-2, C-3, D-4, E-5, F-6, respectively at 7, 8, 9, 10, 11, 12. The true shape of the lateral surface (no harm in dropping the end faces) may be drawn directly projecting longer edges from the top view and shorter edges from front view. The points 7, 8, 9, 10, 11, 12 at which the plane passing through the longer edges may be marked directly drawing lines parallel to x-y.

D-E-F-A-B-C-D-10-9-8-7-12-11-10-D, E-11, F-12, A-7, B-8, C-9 may be finished to get the development of the sectioned hexagonal prism.

Problem 9.4

A pentagonal pyramid, side of base 30 mm and height 60 mm, is resting on HP with its base, axis perpendicular to HP and one side parallel to VP. The pyramid is sectioned by a plane inclined 45^o to HP and perpendicular to VP. Draw the development of the truncated pyramid, if the plane bisects the axis of the pyramid.

Solution

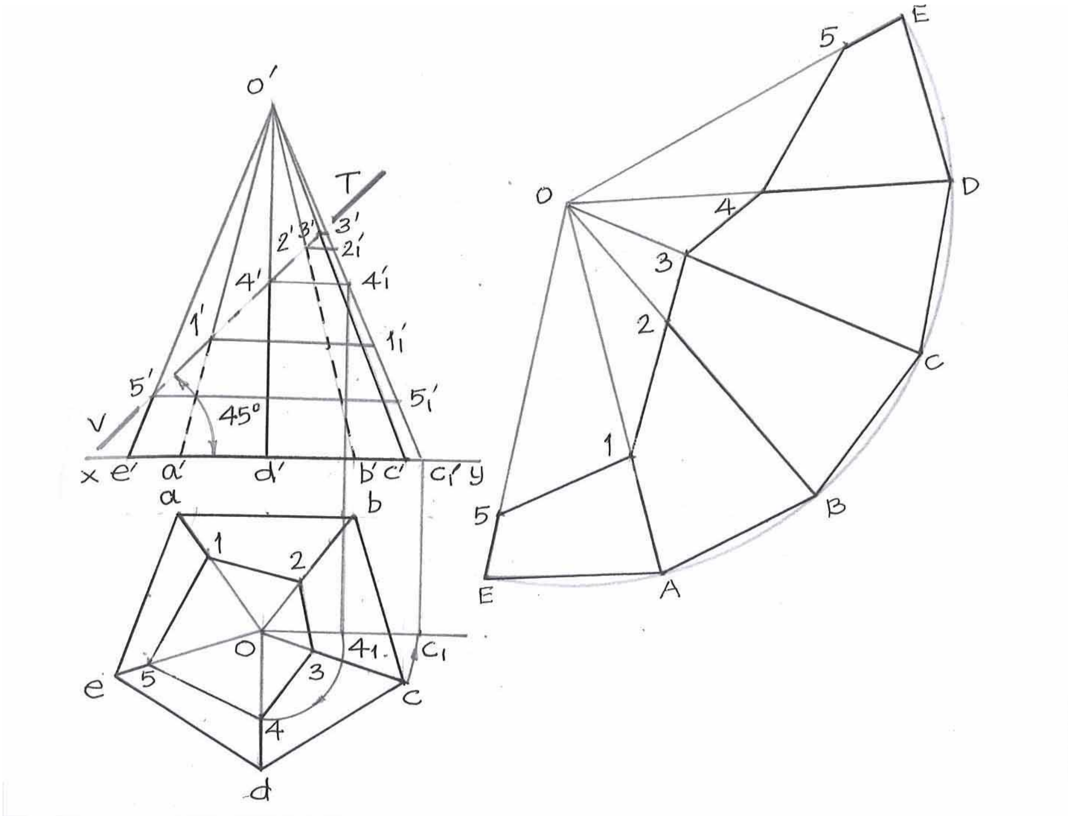


Figure 9.4 Development of sectioned pentagonal pyramid

A pentagon of side 30 mm may be drawn below x-y with one side parallel to x-y. The corners may be joined with the geometric centre. This is the top view of the pyramid. All the corners may be projected to get the front view. The front view of the pyramid may be completed with a height of 60 mm.

V-T of the cutting plane may be drawn at 45^{o} inclined to x-y, passing through the mid-point of the axis of the object. The slant edges are cut at 1, 2, 3, 4, 5 respectively on O-A, O-B, O-C, O-D, O-E. All the cutting points on slant edges may be projected to the top view. The cutting point ***4'*** may be projected after transferring it ($\boldsymbol{4_1}$') to $\boldsymbol{o'c_1'}$ (=$\boldsymbol{o'd_1'}$), then to $\boldsymbol{oc_1}$ and by making an arc with radius $\boldsymbol{o4_1}$ and centre ***o***.

The contour of the section ***123451*** may be finished along with the base and ***a1***, ***b2***, ***c3***, ***d4***, ***e5***.

While developing the lateral surface, only five slant faces – five isosceles triangles having base, side of the base of the solid and other two sides, slant edges of the object, are available. As all the lines have true lengths, true length of the slant edges may be obtained. OC is already made parallel to VP (***oc*** is parallel to x-y, $\boldsymbol{oc_1}$) or

$o'c_1'$ gives true length of the slant edge.

A curve of radius $o'c_1'$ may be drawn with O as centre. Assuming that development is made cutting through the slant edge O-E, E may be marked on the arc. Points E, A, B, C, D, E may be marked taking equal radius of side of the base of the solid. E, A, B, C, D, E may be joined with O and may be finished. This is the development of the pentagonal pyramid (Figure 9.4).

The points 5, 1, 2, 3, 4, 5 may be located on the corresponding slant edges in the development using their actual distances from the vertex, which can be obtained by transferring these points to the true length of slant edge ($o'c_1'$). Thus $o'1_1'$ gives the true distance of point 1 from vertex O, and the point 1 is marked on the slant edge OA in the development such that O1 = $o'1_1'$ and so on.

5-1, 1-2, 2-3, 3-4, 4-5, 5-E, E-D, D-C, C-B, B-A, A-E, E-5, 1-A, 2-B, 3-C, 4-D may be finished.

Problem 9.5

A square pyramid, side of base 30 mm and height 50 mm, is resting on HP with its base, axis perpendicular to HP, one side of base parallel to VP. The pyramid is sectioned by a plane perpendicular to VP and parallel to a triangular face bisecting the axis. Draw the development of the sectioned object which contains the vertex.

Solution

A square of side 30 mm may be drawn below x-y, with one side parallel to x-y. All the corners are joined with the geometric centre ***o*** so that the top view is completed.

The front view of the pyramid may be drawn by taking a height of 50 mm.

When one side of the base is parallel to VP (in the top view parallel to x-y) two sides of base will be perpendicular to VP (in the top view two sides are perpendicular to x-y). Two slant faces are perpendicular to VP. V-T of the cutting plane may be drawn parallel to a slant face perpendicular to VP, passing through the mid-point of the axis. The portion of the pyramid which contains the vertex may be finished.

It is observed that except the face OAB, other three slant faces are sectioned by the cutting plane. The points at which the plane cuts the edges may be indicated (1 on AD, 2 on BC, 3 on OC, 4 on OD). The position of ***1*** on ***ad*** and ***2*** on ***bc*** give true lengths, either from front view or top view (If the edges of the base are inclined to VP, the true lengths of sectioned edges may be obtained from the top view). ***3*** and ***4***

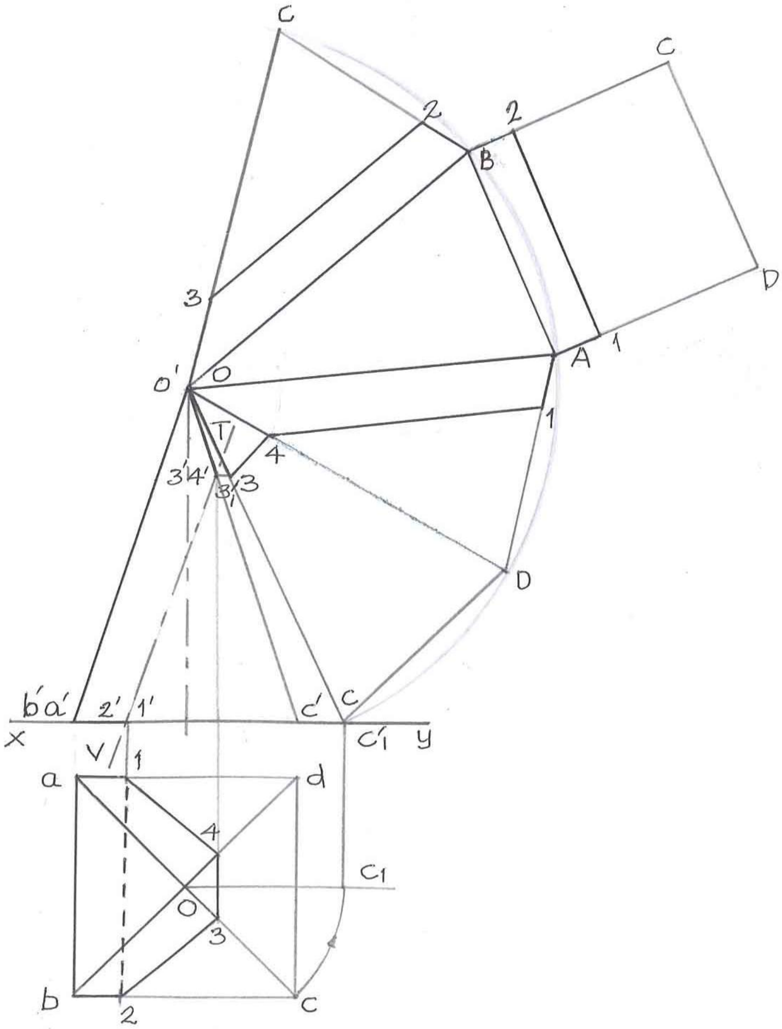


Figure 9.5 Development of sectioned square pyramid

are points on longer edges which are inclined to both HP and VP. Hence the true length of the sectioned slant edges can be found out by making those parallel to one principal plane, here VP, $\boldsymbol{oc_1}$. The points ***3*** and ***4*** may be transferred parallel to x-y to $\boldsymbol{o'c_1'}$. ***o***' (O) as centre, $\boldsymbol{o'c_1'}$ as radius an arc is drawn. Starting from C, mark D, A, B and C taking uniform spacing of side of the base. While joining O-C, O-D, O-A, O-B, O-C the complete lateral surface is available. O as centre $\boldsymbol{o'3'_1}$ as radius, the points 3 on OC and 4 on OD may be marked. 1 and 2 are symmetrical points on AD and BC. The distance ***a1*** may be taken and 1 be marked on AD from A and 2 on BC from B.

After sectioning the base, A12B is the existing part of the base. A12B may be attached to AB, taking dimensions from the top view. O-3-4-1-A-1-2-B-2-3-O, O4, OA, OB, AB may be finished to get the development of the sectioned square pyramid (refer Figure 9.5).

Problem 9.6

A cylinder, base diameter 42 mm and height 60 mm, is resting on HP with its base, axis is perpendicular to HP. A plane perpendicular to VP, and inclined 60^o to HP cuts the solid passing through a point on the axis 40 mm above the base. Draw the development of the part containing the base.

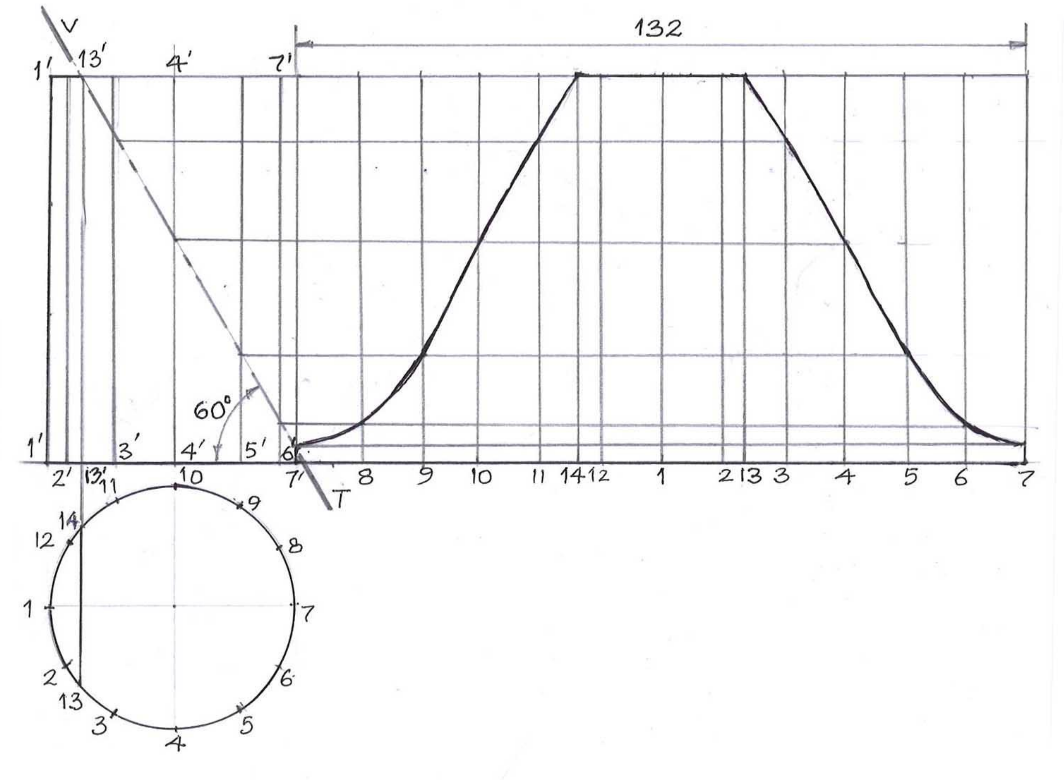


Figure 9.6 Development of sectioned cylinder

Solution

A circle of diameter 42 mm may be drawn below x-y, which is the top view of the cylinder. The circle may be divided into 12 equal parts. The corresponding front view may be drawn, and the generators may be shown according to the divisions in the top view. The point on the axis 40 mm above base may be located. The V-T of the cutting plane may be drawn inclined 60^o to x-y and passing through the point located on the axis. The major part of the cylinder, which contains the base, may be finished to get the front view of the sectioned cylinder.

The section plane cuts the top end face; it cuts the circumference at 13 and 14.

The cylinder has no definite lines on the surface. The imaginary generators 1-1, 2-2, etc. are shown on the surface. As the generators are parallel to VP, the cutting points on generators can directly be plotted on the developed surface. While the cutting points on the end face may be taken from the top view.

It is known that when the surface of the cylinder is developed, a rectangle having πD (D: diameter of the cylinder) as horizontal dimension and height as vertical dimension is available.

Taking the height of the cylinder as height, 132 mm (approximately) as horizontal dimension a rectangle may be drawn. The horizontal dimension (equals to the length of circumference) may be divided into 12 parts, the whole area appearing to be 12 rectangular strips. All points made by cutting the generators may be located on the respective generators directly by drawing parallel lines to x-y.

13 and 14 may be shown on the top view of end face.

The angle between 2-13 and 12-14 may be measured and arc length may be calculated. These arc lengths may be indicated on the upper edge of the developed surface, from 2, 2-13 and from 12, 12-14.

This can be easily completed approximately. The arc length 2-13 and 12-14 may be measured, and 13 and 14 may be indicated directly on the upper edge of the developed surface. The upper edge 13-14 and the bottom edge 7-7 may be finished. The cutting points shown on the lateral surface may be joined by smooth curve and be finished to get the development of the sectioned cylinder.

Problem 9.7

A cone, base diameter 50 mm and height 70 mm, is resting on the HP with its base, axis perpendicular to HP. It is sectioned by a plane perpendicular to VP and inclined 60^o to HP. Draw the development of the part containing the base. Assume that the cutting plane passes through a point on the circumference.

Solution

A circle of diameter 50 mm may be drawn below x-y. The circle may be divided into 12 equal parts. Every division may be joined with the centre to indicate the generators in the top view.

The corresponding front view may be drawn taking a height of 70 mm. The generators may be shown in the front view also.

The V-T of the cutting plane may be shown at an angle of 60^o to x-y, passing through a point on the base circle. The points at which the cutting plane passes over the generators may be marked. All the cutting points may be projected to the top view also. The points on O-4 and O-10 may be marked by taking it parallel to x-y in the front view, $\boldsymbol{4_1'}$. The corresponding top view $\boldsymbol{4_1}$. $\boldsymbol{o\text{-}4_1}$ as radius, ***o*** as centre arcs may be cut to ***o-4*** and ***o-10***. The contour of the section in the top view is an ellipse.

To draw the development of lateral surface of the sectioned cone, the surface may be cut through O-1 (otherwise the developed area will be divided into two parts or these areas will be connected through a point only). The full area of the lateral surface may be a segment, which may be drawn first and then the required area of the sectioned cone may be represented.

For making the area exact, the included angle of the segment may be calculated, which is equal to $\theta=2\pi S/R$ (S: slant height, R: radius of the base circle).

$\theta = 122^o$ in this problem. ***o'7'*** as radius ***o'*** (O) as centre an arc is constructed and the angle 122^o may be marked. The arc length may be divided into 12 equal parts. The imaginary generators may be marked on the area starting from O-1, O-2, O-3, …O-12, O-1.

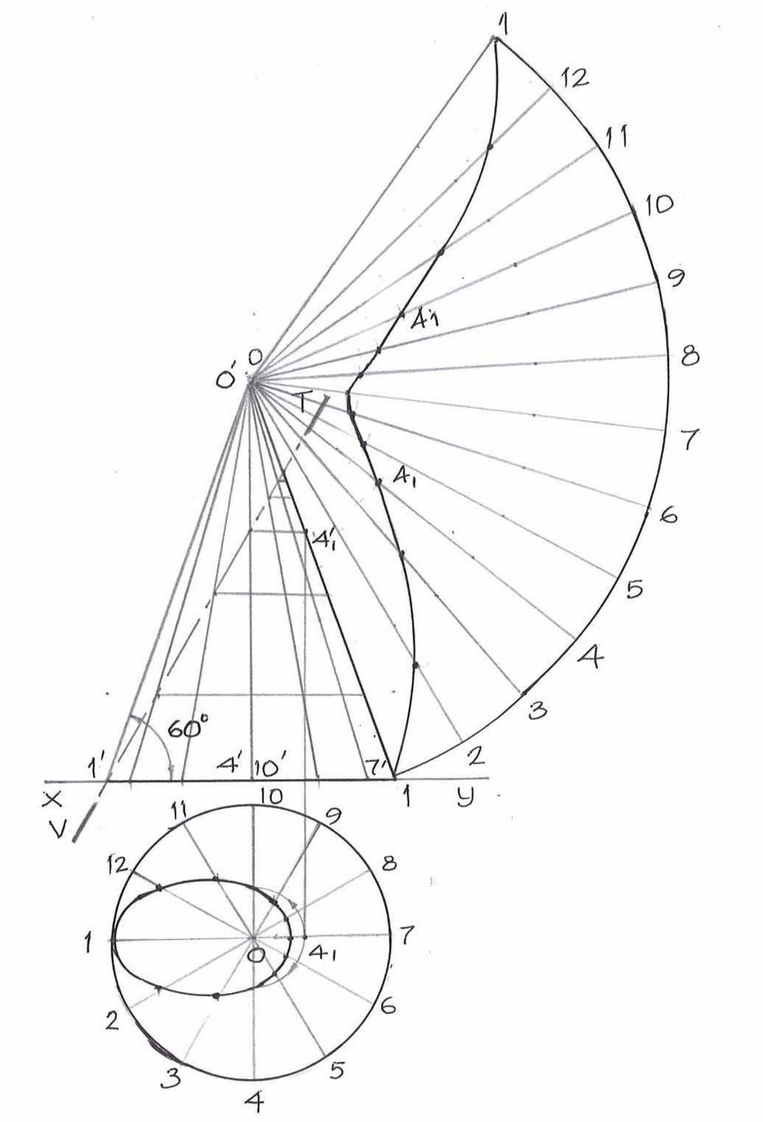


Figure 9.7 Development of sectioned cone, the plane passes through all the generators

In the front view the points on generators through which the cutting plane passes may be transferred to the extreme end generator (***o'7'***) taking parallel to x-y, so as to get the true lengths. All these points may be marked on the respective generators in the developed area. While joining all the points marked and finishing the outer contour (generators need not be finished as these are imaginary) the development of the sectioned cone can be obtained.

Problem 9.8

A cone, base diameter 50 mm and height 70 mm, is resting on HP with its base, axis perpendicular to HP. The cone is sectioned by a plane perpendicular to both HP and VP, parallel to the axis. The plane cuts 5 mm away from the axis. Draw the development of the part containing the apex of the cone.

Solution

A circle of diameter 50 mm may be drawn below x-y. The circle may be divided into 12 equal parts. The corresponding front view may be drawn and the divisions on base circle from top view may be transferred to the front view.

The H-T and V-T of the cutting plane may be shown parallel to the axis and 5 mm away, as the plane is perpendicular to HP and VP.

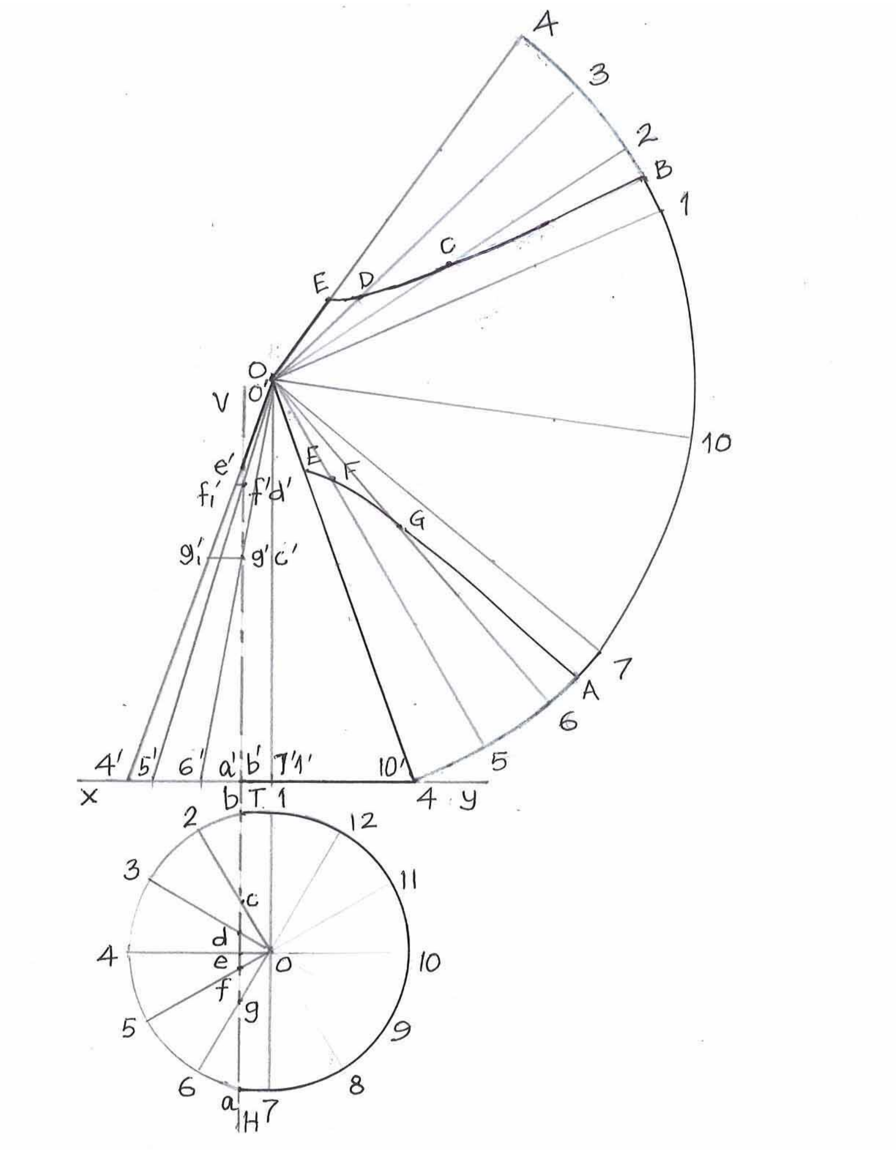


Figure 9.8 Development of a cone, sectioned parallel to the axis

The generators may be drawn on the side where the plane cuts the object. The plane cuts a few generators and the base circle (Figure 9.8).

The point through which the cutting plane passes through the generators may be indicated on the true length of the generator (on ***o'-4'***). An arc of radius ***o'-4'*** may be drawn with ***o'*** as centre. Draw the segment through an angle of 122^o. As no generator is removed fully the cutting can be through any of the generators for developing the surface. The segment is included in O-4 and O-4. The arc may be divided into 12 equal parts and generators may be indicated through the points 1, 2, 3, 4, 5, 6, 7. The points through which the plane is passing over generators (C, D, F, G) may be transferred to the generator O4 (c'_1, d'_1, f'_1, g'_1 on ***o'4'***). All these points may be indicated on the respective generators taking true dimensions, $o'f'_1$, $o'g'_1$ etc. While joining these points and finishing the outer contour, the development of the present sectioned cone is available.

Chapter 10

Isometric Projection

Chapter 10
Isometric Projection

10.1 Pictorial Projections

Pictorial projection (refer Chapter 1) contains only one projection (view) which will be sufficient to understand the object. It has a three dimensional effect and hence in most of the cases the object will be clearly understandable in pictorial projection. The commonly used pictorial projections are: (i) perspective projection, (ii) isometric projection, and (iii) oblique projection. Only isometric projection is discussed in this chapter.

10.2 Isometric projection

Basically isometric projection is the front view borrowed from orthographic projections from a typical position of the object.

A cube may be assumed to be resting on HP with a corner, having one solid diagonal perpendicular to VP. The front view (refer Figure 10.1) of the cube in the specific position is treated as the 'isometric projection' of the cube. In this projection, the characteristics of the cube are used to draw the isometric projection of any object.

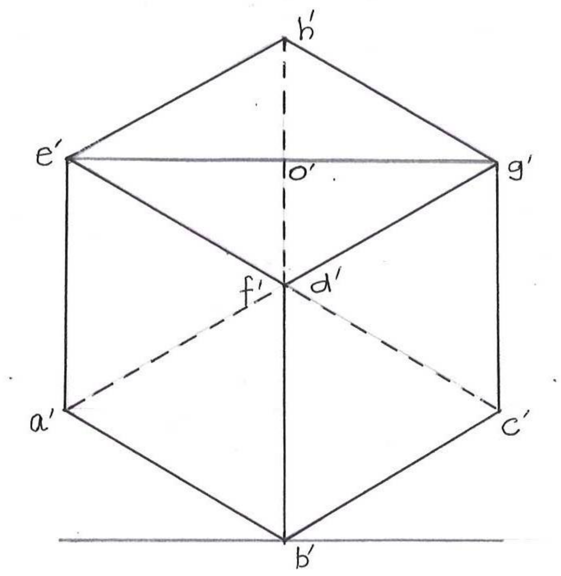


Figure 10.1 Isometric projection of Cube

The same projection is available when the cube is placed on HP and viewing along a solid diagonal.

While studying the projection closely, the following conclusions can be derived.

(i) All the horizontal edges are visible as inclined 30^o to the reference line

(ii) All the vertical edges are visible as perpendicular to the reference line

(iii) All the lines (horizontal and vertical) on the object are reduced to some extent, called as 'isometric length'.

While constructing the isometric projection of any object, the above properties or constraints are followed.

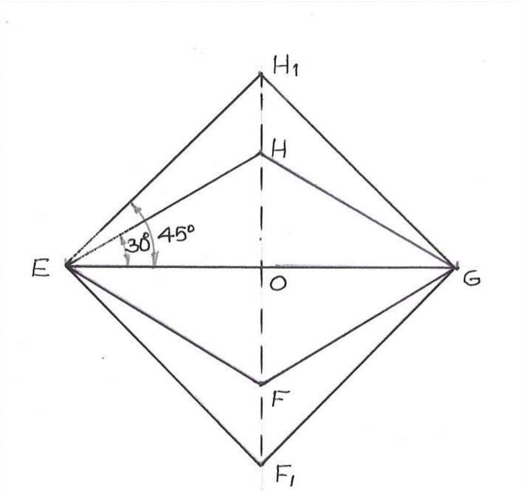


Figure 10.2 Relationship between true length and isometric length

The relationship between the true length and isometric length is established from Figure 10.2. The top face (not only top face, every face) of the cube (Figure 10.1) appears to be a rhombus (***e'f'g'h'***) in the isometric projection. The real shape of the face is square. When the solid diagonal FD is perpendicular to VP, diagonals of top face (EG) and base (AC) are parallel to both HP and VP. In both the views these will appear as true. EG as diagonal, the true shape of face, square (EF_1GH_1), is constructed. EO, being a common side, the relationship between EH and EH_1 can be determined from trigonometry.

EH=0.816 EH_1 or the isometric length = true dimension multiplied by 0.816.

The first process in drawing isometric projection is the construction of the 'Isometric scale' (Refer Figure 10.3). Two lines, EO and OH_1, may be drawn with equal length and mutually perpendicular. E and H_1 are joined. Angle OEH_1 will be 45^o. EH is drawn at an angle of 30^o to OE (angle OEH). When EH_1 is the true length, EH is the corresponding isometric length. EH_1 may be divided into any number of equal or un-equal divisions as per requirement. Through every division lines may be drawn parallel to OH_1 and may be intersected to EH. Every division on EH, corresponding to divisions in true length (in EH_1) will give isometric length.

In a particular drawing, the isometric scale may be constructed according to the maximum true dimension in the drawing.

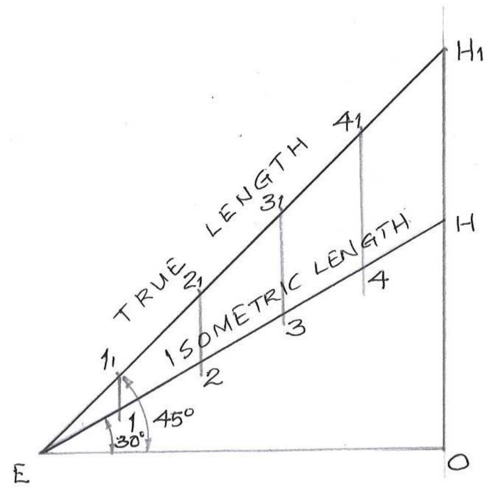


Figure 10.3 Isometric Scale

10.3 Isometric View or Isometric Drawing

The basic properties of isometric projections are discussed in Section 10.2. Among these, one important property is 'reduction of true dimensions to isometric dimensions'. In many cases, for convenience, while drawing isometric projection, true dimensions are used in place of isometric dimensions. The figure that is drawn using true dimensions in place of isometric dimensions is known as 'Isometric view or Isometric drawing'.

It may be noted that if sphere is present in the object considered, then always isometric projection is suggested.

10.4 Conversion of Multi-view Representation into Isometric Projection

The problems from isometric projection are solved as 'conversion of multi-view representation into isometric projection'. Hence, in every problem of isometric projection, the first step is drawing minimum two views of the object, and then the corresponding isometric projection or isometric view is constructed following the various characteristics of isometric projection or view.

10.5 Isometric Projections of Planes

Problem 10.1

A square lamia, sides 40 mm, is placed in such a way that its plane is parallel to

HP and one side is perpendicular to VP. Draw the isometric projection.

Solution

The first step is drawing two views of the object in its position. A square of side 40 mm (***abcd***) may be drawn below x-y, keeping one side perpendicular to x-y. The corresponding front view (***b'c'***) is also drawn.

To get isometric projection of the square, the square may be observed at an angle along BD (***bd*** in top view) or CA (***ca*** in top view). By referring the projections, it is understood that in the position of the square lamina, its all four sides are horizontal. All the sides will be visible as 30^o inclined to the reference (horizontal line) line and all the sides will be reduced to isometric length in the isometric projection.

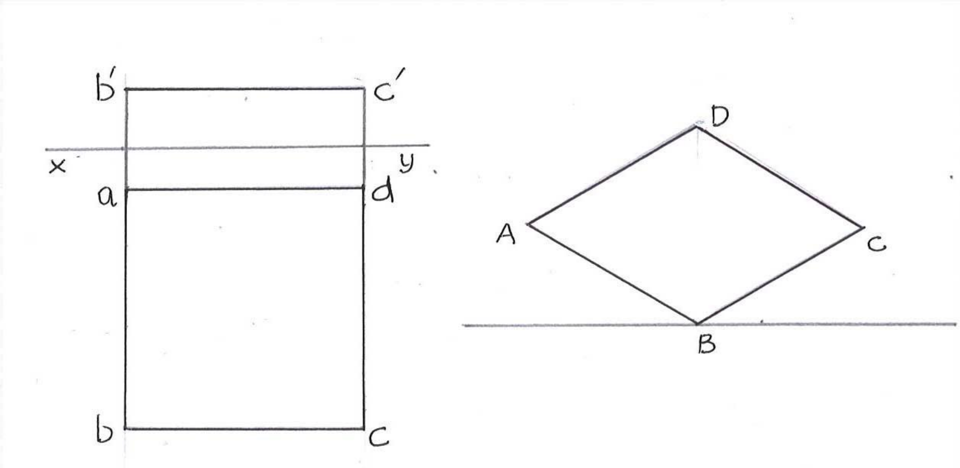


Figure 10.4 Two views of square lamina (left);
Isometric projection of square lamina (right)

The second step in the problem is construction of isometric scale taking the largest dimension of the object as the maximum true length in the isometric scale.

If many problems from isometric projections are to be solved, then a common isometric scale may be used taking the highest true dimension among the maximum dimensions of different objects. It is assumed that an 'isometric scale' is constructed with the highest dimension as true length for illustrating the problems in this chapter.

Being a square lamina, having true side 40 mm, this is the only dimension. The isometric length of 40 mm may be read from the isometric scale. Point B may be marked on the horizontal line. BA = BC = isometric length of 40 mm, may be constructed at 30^o inclination to the horizontal reference line. AD and DC may also be drawn 30^o inclined to the reference line using the dimension taken earlier. The square is visible as a rhombus, ABCD, in the isometric projection, which may be finished (Figure 10.4).

Problem 10.2

A rectangle, length 60 mm and breadth 40 mm, is placed with the shorter sides of plane perpendicular to HP. Draw the isometric projection if the plane is parallel to VP.

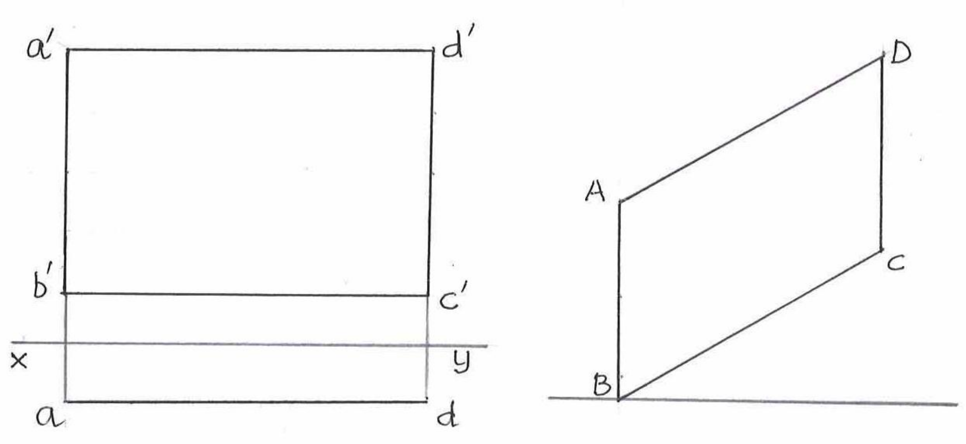


Figure 10.5 Isometric projection of a rectangle (right) with its plane vertical

Solution

The front view (***a'b'c'd'***) of the rectangle is drawn keeping the shorter sides perpendicular to x-y. The corresponding top view (***ad***) is also drawn. Here AB and CD are perpendicular to HP while BC and DA are horizontal lines. Hence, BC and DA must be inclined 30^o to the reference line, and AB and CD must be perpendicular to the reference line in the isometric projection.

The isometric lengths corresponding to 60 mm and 40 mm are read from the scale. Point B is marked on the reference line. BC = isometric length of 60 mm, is drawn 30^o inclined to the reference line. BA = isometric length of AB, may be drawn perpendicular to the reference line (refer Figure 10.5). The parallelogram may be completed and finished drawing AD in its isometric length 30^o inclined to the reference line and CD perpendicular to the reference line.

Problem 10.3

A triangle, sides 60 mm, 50 mm and 35 mm, is placed with its plane horizontal, with the longest side parallel to VP. Draw the isometric projection of the triangle

Solution

Th top view (**abc**) of the triangle is drawn with the longest side ***ab*** parallel to x-y. The corresponding front view (***a'c'b'***) is also drawn.

In the triangle all three sides are horizontal and hence it is difficult to construct the isometric projection following the basic property, 'all horizontal lines will appear 30^o to the reference line' or it will not appear like that.

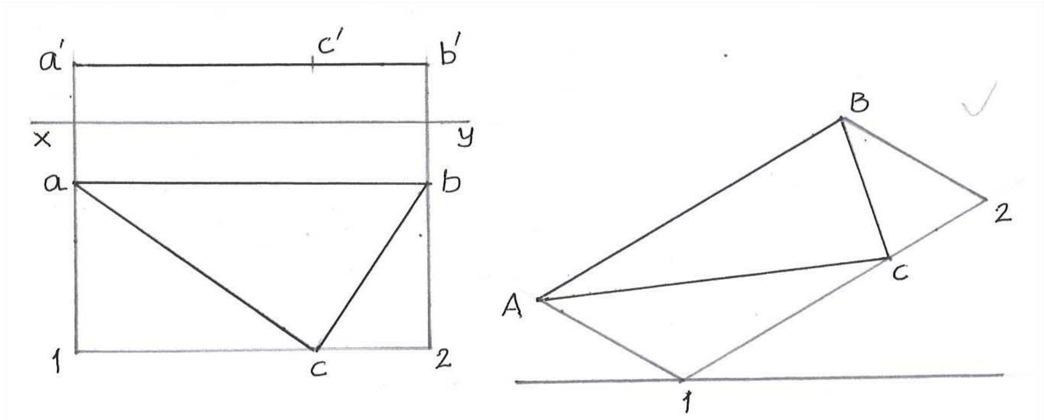


Figure 10.6 Isometric projection of the triangle (right) with its plane horizontal

The triangle is inscribed in a rectangle, ***ab21***. Taking the iometric dimensions of ***ab*** and ***a1***, the isometric projction of the rectangle is constructed, following the basic characteristics of the isometric projection. C is markd on 12 taking the isometric distance of ***1c***. Joining A and B with C, and finishing ABC, the isometric projection of the triangle is completed.

Problem 10.4

A circle of diameter 50 mm is placed with the plane horizontal. Draw its isometric projection.

Solution

A circle may be drawn having diameter 50 mm below x-y, which is the top view and its front view will be a straight line parallel to x-y.

As, the circle has no definite edges and corners, drawing isometric projection following the method described is difficult.

The circle may be inscribed in a square, ***1234*** (top view). The surface of the circle is horizontal and the plane of the square is also horizontal. As all the four sides of the square are horizontal, isometric projection of the square (rhombus) may be drawn taking isometric length of the side (refer Figure 10.7).

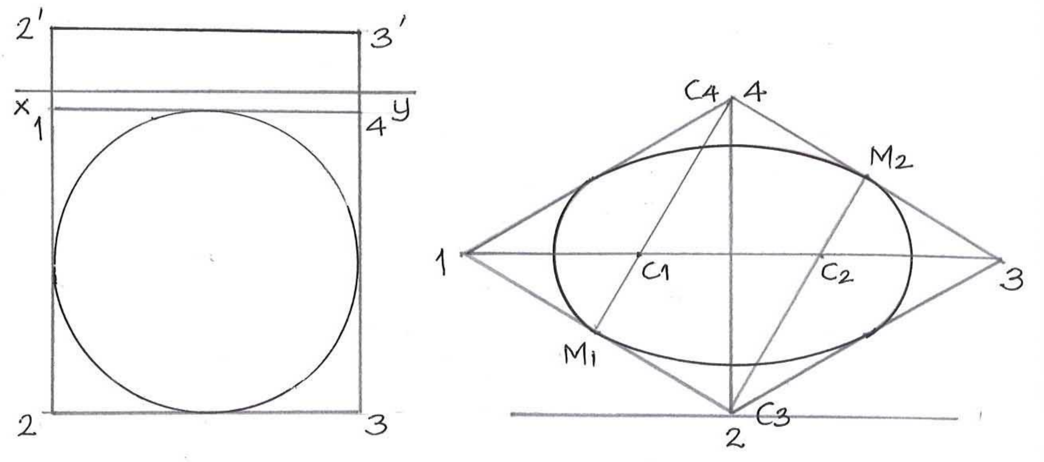


Figure 10.7 Isometric projection of the circular lamina (right) with its plane horizontal

When the circular lamina is viewed at an angle, it will appear as an ellipse. The ellipse may be constructed in the rhombus which is already drawn. 'Four centre method' can be employed for constructing the ellipse. M_1 and M_2 are mid-points of 1-2 and 3-4, respectively. The lines joining 4-M_1 and 2-M_2 will intersect the main diagonal (1-3) at C_1 and C_2, respectively. Now, C_1, C_2, 2 (C_3), 4 (C_4) are the four centres which are used for completing the ellipse.

While drawing arcs with C_1 as centre C_1M_1 as radius; C_2 as centre, C_2M_2 as radius; C_3 as centre, C_3M_2 as radius and C_4 as centre, C_4 M_1 as radius, the ellipse is completed.

10.6 Isometric projection of solids

Problem 10.5

A pentagonal pyramid, side of base 30 mm an height 60 mm, is resting on HP with its base, axis perpendicular to HP, one side of the base parallel to VP. Draw the isometric view of the pyramid.

Solution

Since isometric view is drawn no need of isometric length; hence no use of isometric scale.

A regular pentagon (***abcde***), 30 mm side, may be drawn below x-y, with one side (***ae***) parallel to x-y. The corresponding front view is also drawn. The pentagon (refer Figure 10.8) is inscribed in a rectangle, ***1234***.

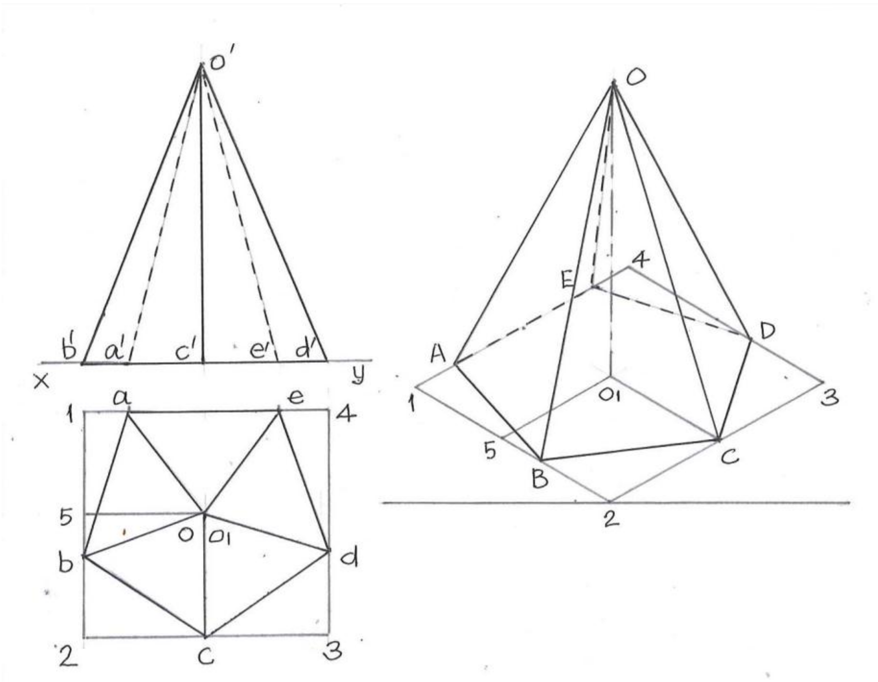


Figure 10.8 Isometric view of pentagonal pyramid (right) axis vertical

The isometric view of the pentagon is constructed first. As the plane of the pentagon is horizontal, the sides of the rectangle 12, 23, 34, 41 are also horizontal. The parallelogram 1234 may be drawn taking all the edges (Figure 10.8) 30^o inclined to the reference line using true dimensions. B on 12 and D on 34 may be marked using the distance ***1b*** from the top view. A and E may be marked on 14 using the distance ***1a.*** C is marked as the mid-point of 23. The geometric centre of the base, O_1 may be marked by drawing lines through 5 parallel to 23 and through C parallel to 12. A vertical line may be erected from O_1 and taking a height of 60 mm (O_1O) O may be marked. O may be joined with A, B, C, D, and E to get the isometric view of the pyramid. In this position, AE, ED and EO are hidden lines, which need not be shown in the isometric view (This is applicable to isometric projection also).

Problem 10.6

A hexagonal prism, side of base 25 mm and height 50 mm, is resting on HP with a rectangular face, axis is perpendicular to VP. The solid is cut by a vertical plane inclined 45^o to VP passing through the mid-point of the axis. Draw the isometric view of the sectioned prism.

Solution

A hexagon of sides 25 mm may be drawn above x-y with one side on the x-y. The corresponding top view may also be drawn. The H-T of the cutting plane may be indicated at 45^o inclined to x-y passing through the mid-point of the axis. The points at which the cutting plane meets with longer edges are marked (1, 2, 3, 4, 5, 6). The portion of the prism nearer to x-y may be finished. The section may be shown in the front view also using section lines. Isometric view is drawn as the section is visible directly.

The hexagonal prism may be inscribed in a cuboidal box (Figure 10.9 a). The isometric view of the cuboid will be drawn first and the sectioned prism will be drawn as inscribed in the box.

The isometric view of the cuboidal box, 7-8-9-10-7-8-9-10, is drawn. All the longer edges, with ends A, B, C, D, E, F may be drawn after taking dimensions from front view. On longer edges A1, B2, C3, D4, E5, F6 may be marked taking true dimensions from the top view. Along with A1, B2, C3, D4 (hidden lines E5, F6 need not be indicated), 123456 and ABCDEF may be finished to complete the isometric view of the sectioned prism (Figure 10.9 b).

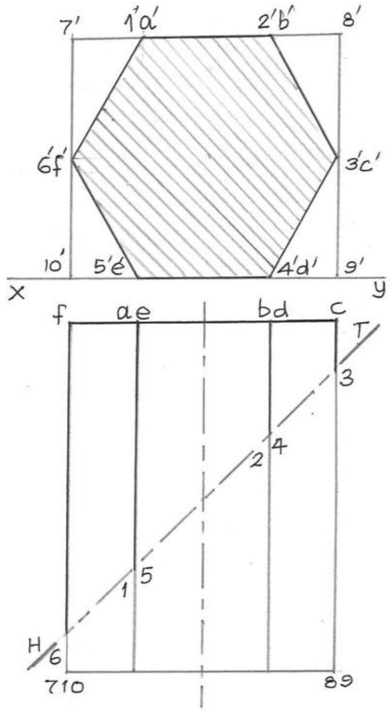


Figure 10.9 (a) Two views of sectioned hexagonal prism

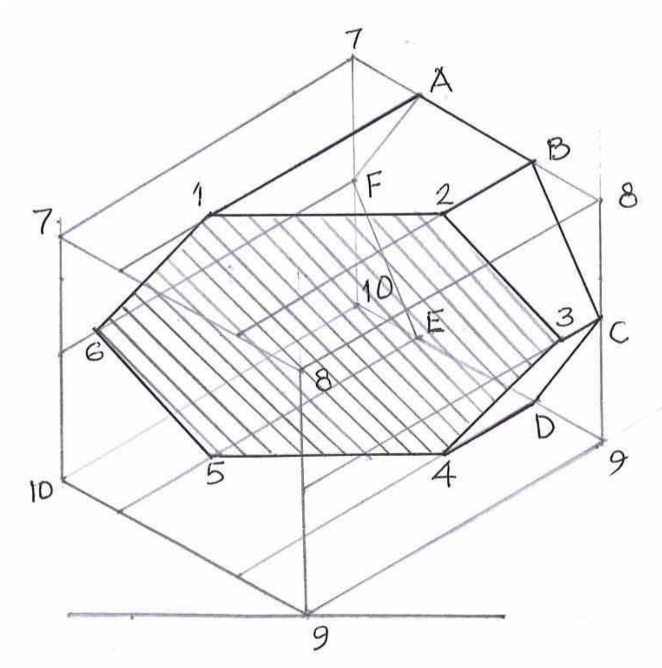


Figure 10. 9 (b) Isometric view of sectioned hexagonal prism

Problem 10.7

A square pyramid, side of base 50 mm and height 65 mm, is resting on HP with its base, axis is perpendicular to HP, a side of base parallel to VP. A plane perpendicular to VP and inclined 45^{o} to HP cuts the solid bisecting the axis. Draw the isometric view of the sectioned pyramid.

Solution

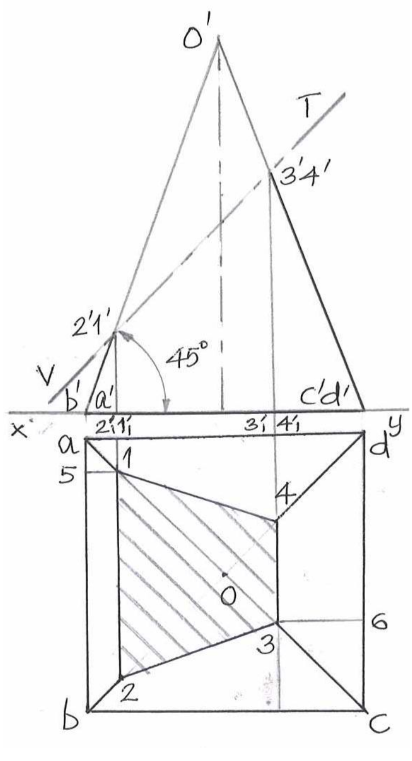


Figure 10.9 (a) Two views of sectioned pyramid

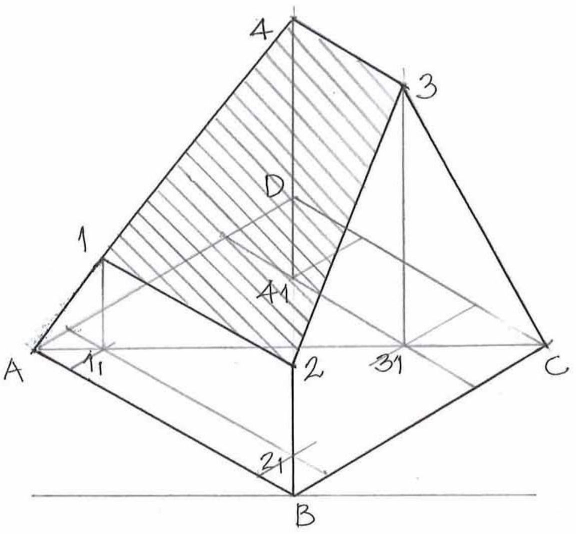


Figure 10.9 (b) Isometric view of sectioned pyramid

Solution

The top view may be drawn as a square (***abcd***) having sides 50 mm below x-y, taking one side parallel to x-y. ***a, b, c, d*** may be joined with the geometric centre ***o***. The corresponding front view may be completed.

V-T of the cutting plane may be drawn taking an angle of 45^o to x-y, passing through the mid-point of the axis. The front view of truncated pyramid may be finished. The cutting points on the slant edges may be marked on the top view and the top view may be finishd indicating the section.

1, 2, 3, 4 are points on the slant edges through which the cuting plane is passing. It may be noted that points ***1*** and ***2*** are symetrical, and ***3*** and ***4*** are symmetrical in the top view. The vertical lines dropped from 1, 2, 3, 4 to the base will help in drawing the isometric view. The vertical dropped from 1 will meet the base at 1_1, from 2 at 2_1, 3 at 3_1 and 4 at 4_1.

The square is visible as a rhombus in the isometric view. Taking true dimension, the rhombus is constructed.

The isometric view of the sectioned pyramid can be drawn by erecting verticals from 1_1, 2_1, 3_1 and 4_1 which are located on the base. The lengths of verticals may be taken from the front view, respectively $\boldsymbol{1_1'1', 2_1'2', 3_1'3', 4_1'4'}$.

While joining 1-A, 2-B, 3-C, 4-D and indicating the section in 1234, the isometric view is completed.

Problem 10.8

Draw the isometric view of a conical frustum with diameter of base 50 mm and top face 25 mm. Take the height as 50 mm.

Solution

The top view and front view of the conical frustum may be drawn (Being a conical frustum, even without drawing these views isometric view can be drawn).

Both circles in the top view may be inscribed in squares. Taking true dimension, the rhombus indicating the base may be drawn. The centre of the top face may be indicated taking true height from the centre of the base. The rhombus showing the top face may also be drawn.

By using four centre method, ellipses may be constructed in the rhombuses (top face and base). The ellipse in the top face may be finished. The visible part of the ellipse indicating base may also be finished along with the required generators (Figure 10.10).

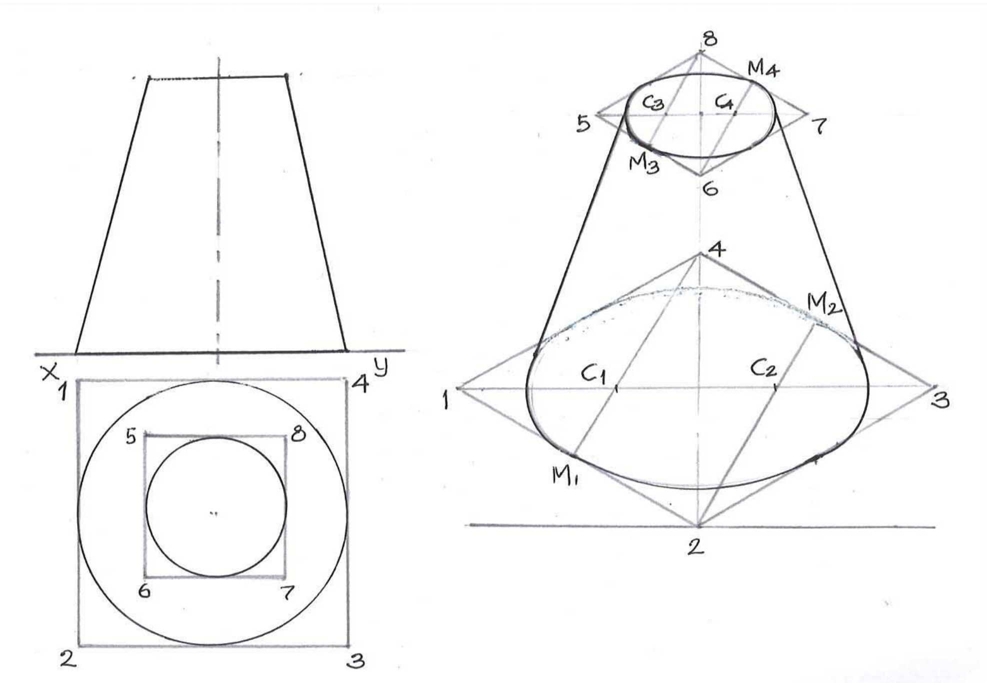


Figure 10.10 Isometric view of a conical frustum

Problem 10.9

Draw the isometric view of a pentagonal pyramid, side of base 40 mm and height 60 mm, which is lying on the HP with one of its triangular faces, axis is parallel to VP.

Solution

As in the above cases, the first step is drawing two views of the pyramid in the position. The pyramid is lying on HP with a triangular face and axis is parallel to VP. Two steps are required for drawing two views of the solid.

The solid is assumed to be resting on the HP with its base, one side of the base perpendicular to VP. Both the projections are completed. After making a triangular face on HP, the top view is also obtained.

The object may be inscribed in a box. The top view and front view of the box may be drawn as in Figure 10.11a. The isometric view of the box is drawn first and then, pentagonal pyramid may be drawn as inscribed in the box.

The isometric view of the box, top face 1234, and base 5678, is drawn. The central plane is marked through the mid-point of 14, 23, 58, 67. The centre of base O_1 and vertex O are falling on this plane. A and B are marked on 58 taking the distances from the top view. O may be marked at the mid-point of 67.

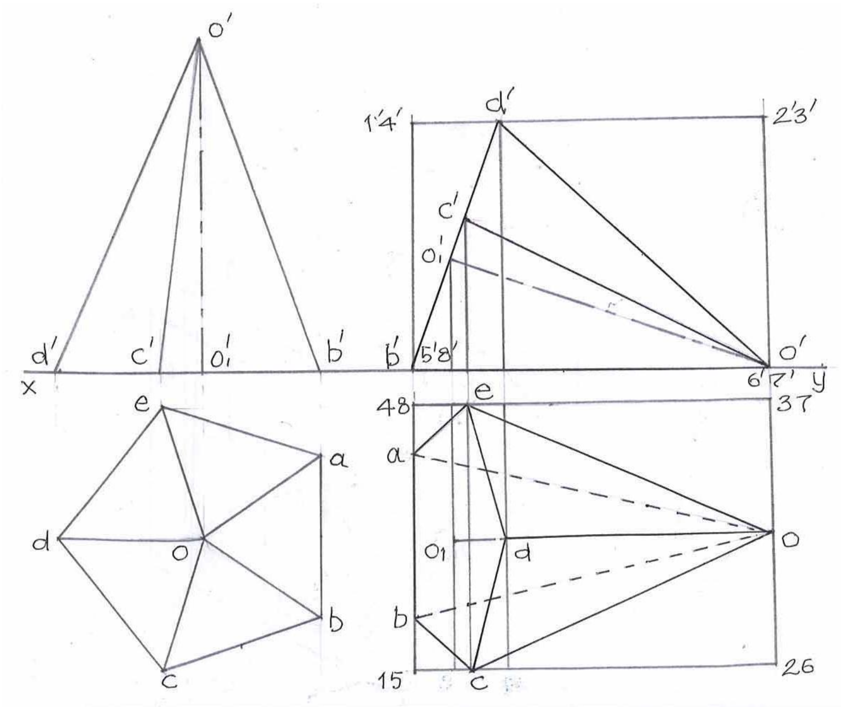


Figure 10.11 (a) Projections of pentagonal pyramid lying on HP with a triangular face

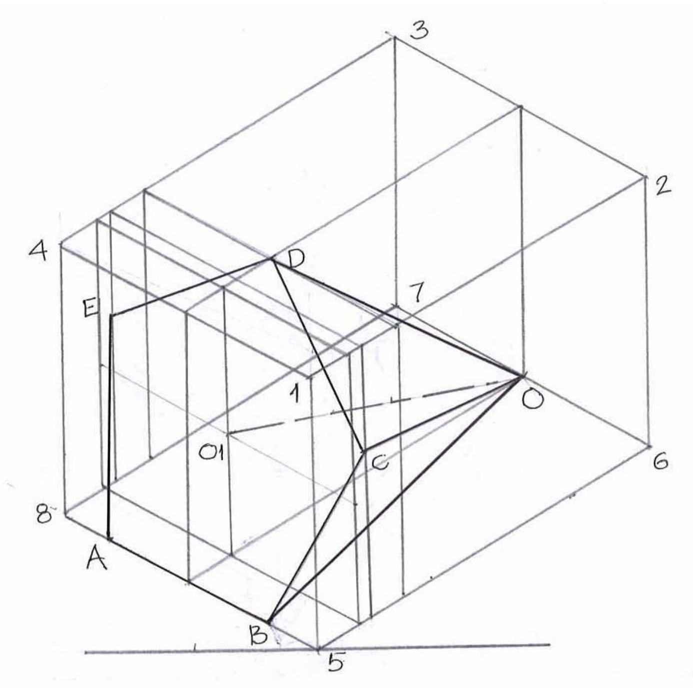


Figure 10.11 (b) Isometric view of pentagonal pyramid which is lying on HP

The planes parallel to 1485 through the centre of base O_1, C and E, and D may be drawn taking the distance from top view (or front view). The distance of ***o_1'*, *c'*** and ***e'***, and ***d'*** from x-y may be taken, and marked on the respective planes from the plane 5678. Joining ABCDE, DO, CO and BO the isometric view of the lying pyramid is available.

Problem 10.10

Draw the isometric projection of a sphere of diameter 48 mm which is placed centrally on the top face of a conical frustum. The base diameter of the frustum is 60 mm, top face diameter 48 mm and height 40 mm.

Solution

It may kindly be noted 'when sphere is observed from any angle, it will appear as circle, having true diameter. Hence, when sphere is present as an object for which isometric projection or view is to be drawn, only isometric projection shall be constructed and for this isometric scale is essential.

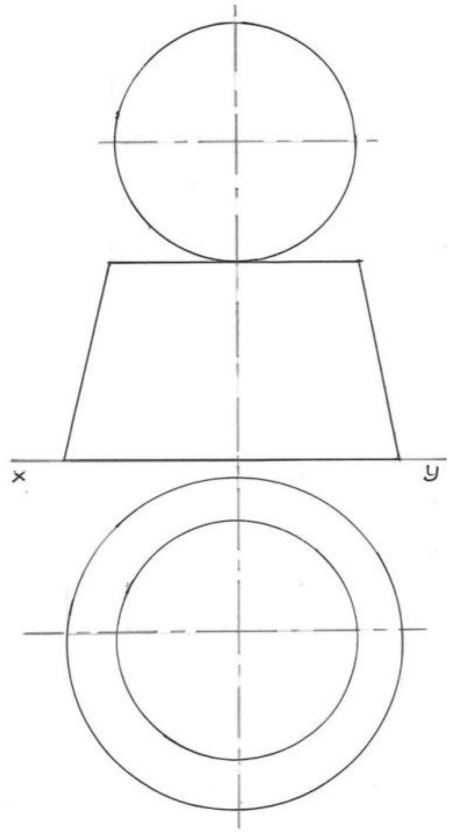


Figure 10. 12 (a) Two views of the sphere resting on conical frustum

Two views (top and front views) of the sphere resting on conical frustum may be drawn (Without these views also isometric projection of this combination can be drawn.). The top face of the frustum and sphere have equal in diameter. The sphere appears as a circle and the circles representing the base and top face of the frustum appear as ellipses. The diameter of the circle representing sphere and the major axis of the ellipse representing top face of the frustum must be equal.

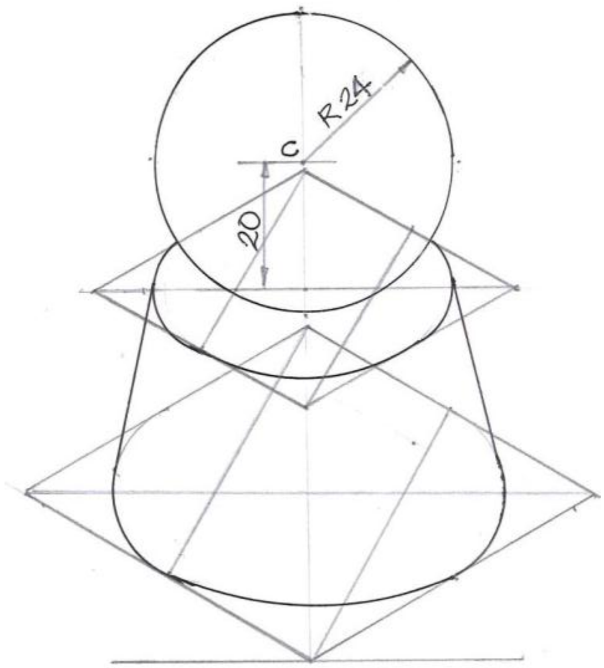


Figure 10. 12 (b) Isometric projection of sphere resting on conical frustum

The circles representing the base and top face may be assumed to be inscribed in squares. These squares appear as rhombuses in the isometric projection. The corresponding isometric dimensions may be obtained from the isometric scale and the rhombus representing the base may be constructed. Isometric dimension of the height may be marked from the geometric centre of the base and the rhombus representing the top face may also be drawn. Using four-centre method, the ellipses may be drawn in both rhombuses.

When the sphere resting at the centre of the top face is observed at an angle, the point at which the sphere is really resting on the top face is not visible. The 'vertical line joining the centre of the sphere and the point at which the sphere rests on the top face (radius=24 mm)' is reduced to its isometric length (20 mm). The geometric centre of the top face as centre, isometric length of radius of sphere (20 mm) as radius an arc is drawn on the vertical line drawn from the geometric centre of the top face. The point at which arc meets on the vertical is the centre of the sphere (C). C as centre, true radius = 24 mm, as radius a circle may be drawn. The circle and visible part of the frustum may be finished to complete the required isometric projection.

Problem 10.11

Draw the isometric view of the machined object whose three views are given.

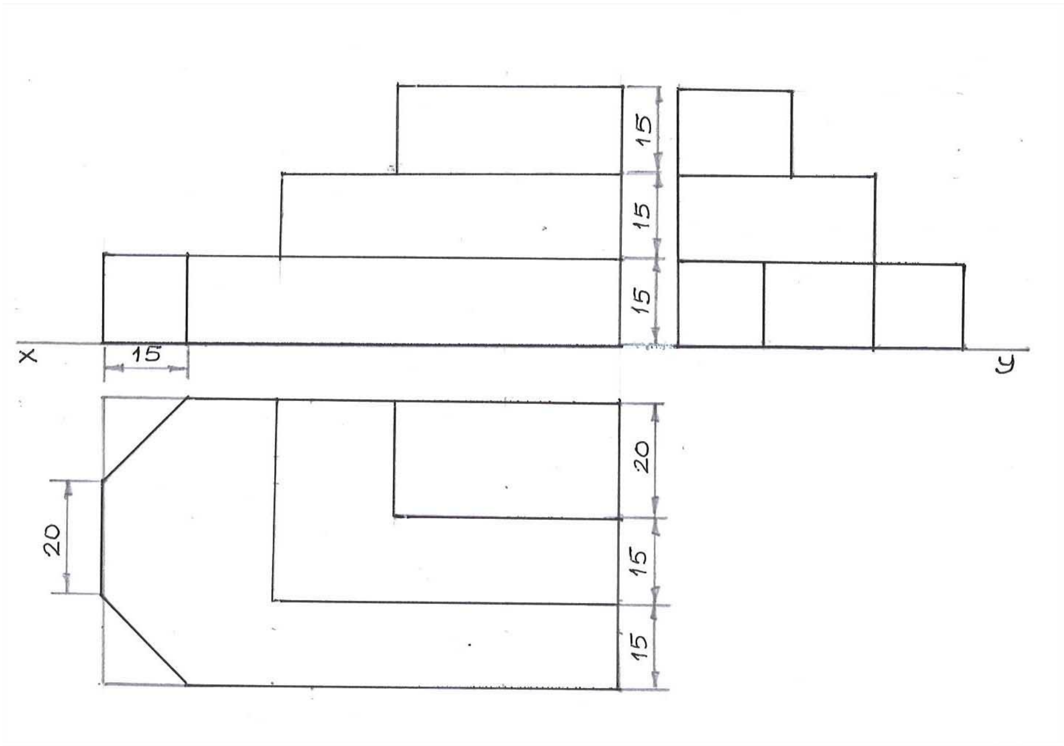


Figure 10.13 (a) Three views of a machined component

Solution

The isometric view of the machined component is shown in Figure 10.13 b showing all constructions.

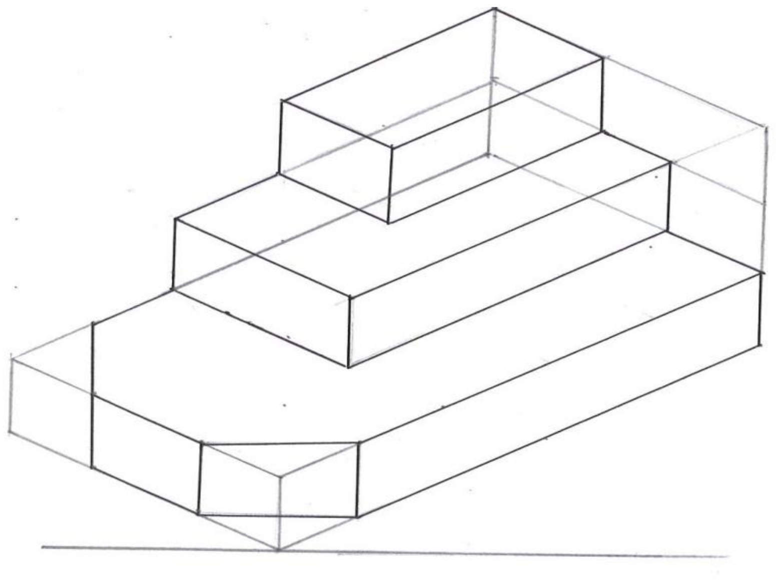


Figure 10.13 (b) Isometric view of the machined object

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