Engineering Mechanics

Continued…(5)

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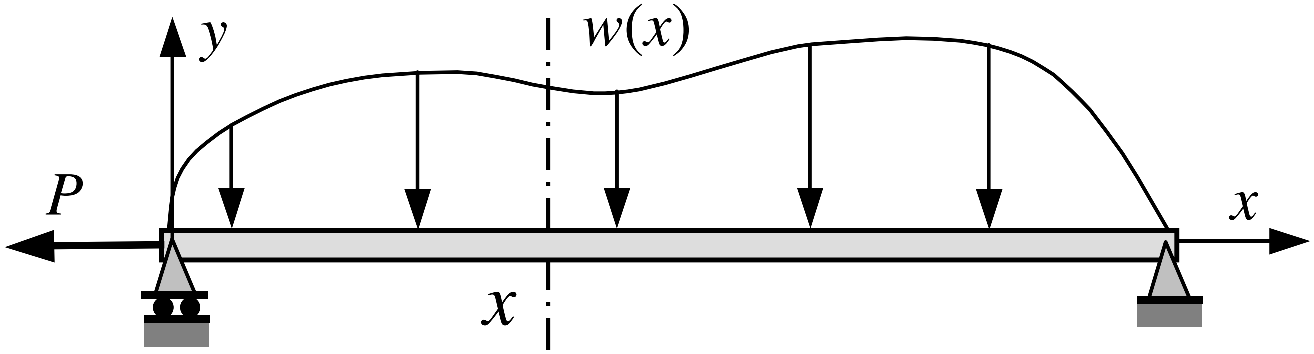
B Section Forces in Beams

Beams are thin prismatic members that are loaded transversely.

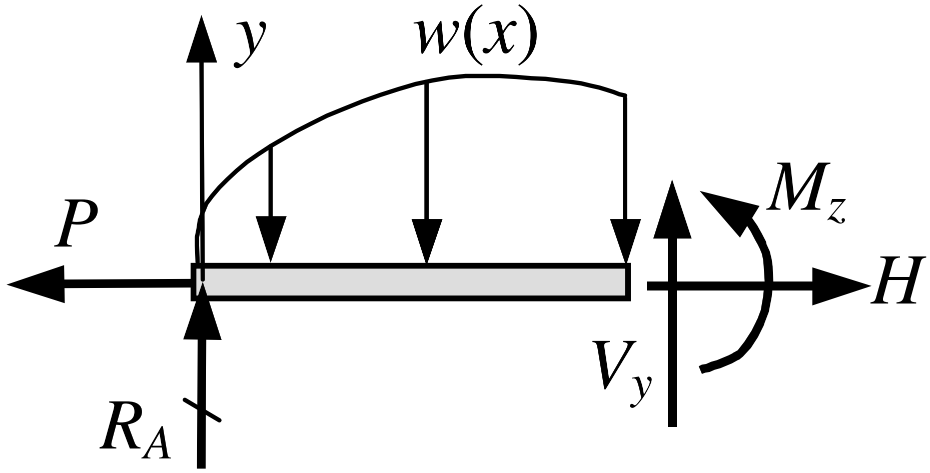
Shear Force, Axial Force and Bending Moment

Consider the beam shown below:



Consider a cut at a distance of *x* from the left end of the beam.

At the cut surface *x*, we have a single force and a single couple as the resultant of the distributed forces.



Resolve the force into V_y and H

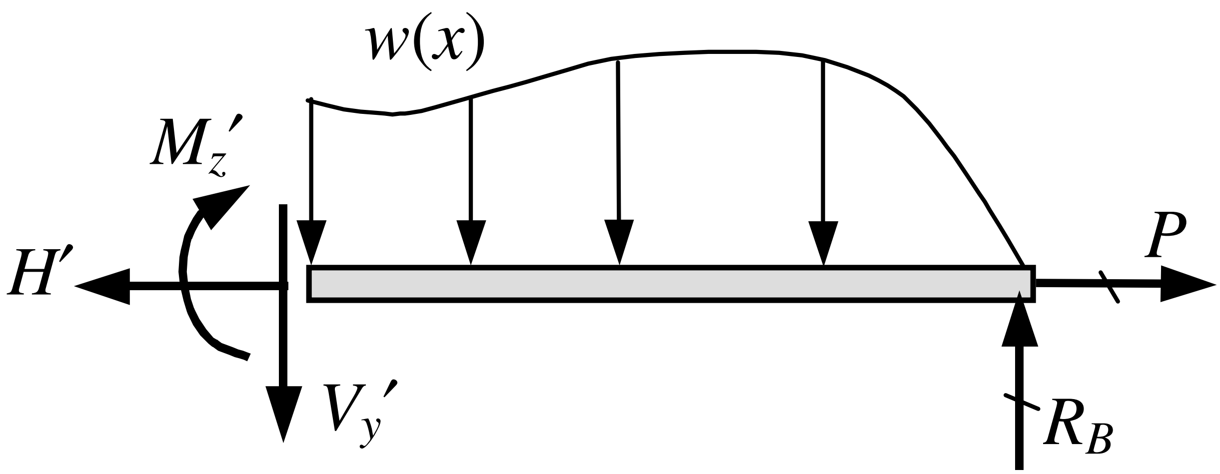
The couple is M_z

H –is the axial force (AF),

V_y –is the shear force (SF), and

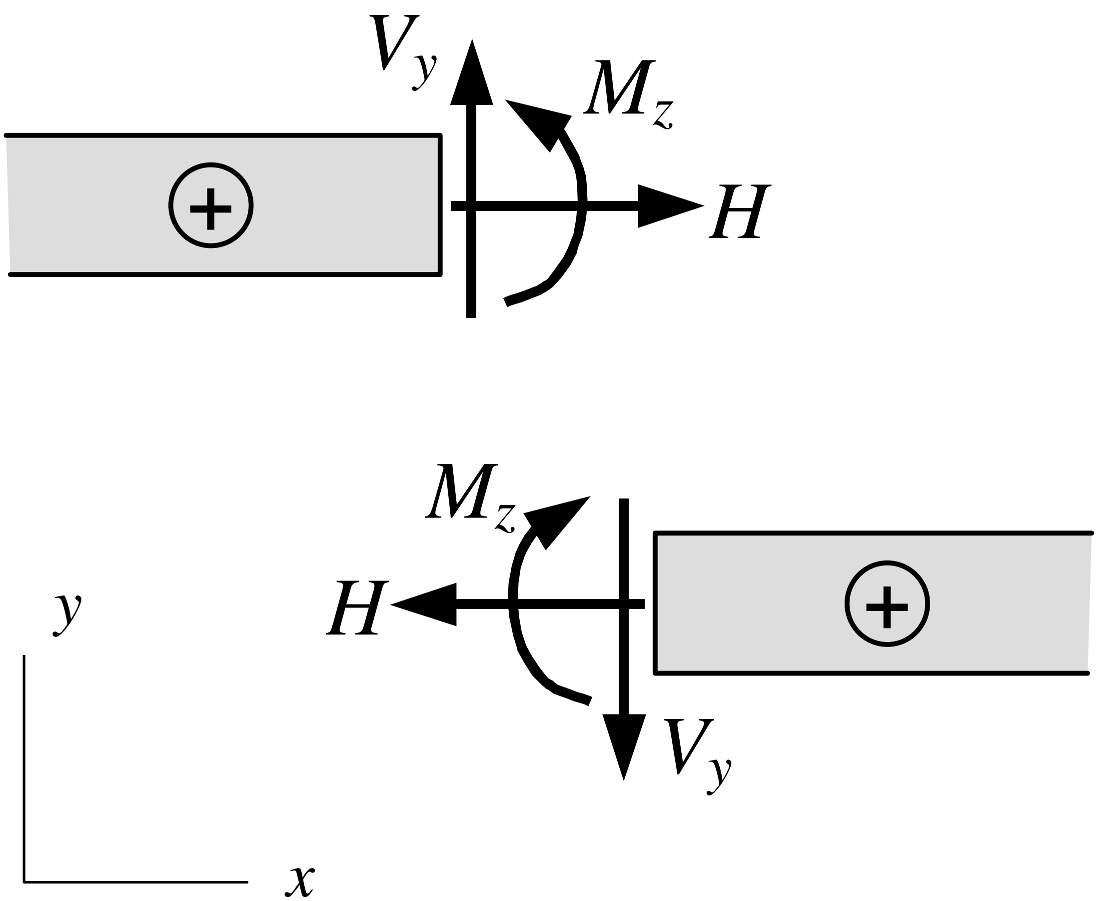
M_z –is the bending moment (BM).

The other part of the beam looks like this:



Sign Convention:

A force component at a section is positive if the area vector of the cross-section and the force component both have senses either in the positive or in the negative directions of the reference axes.



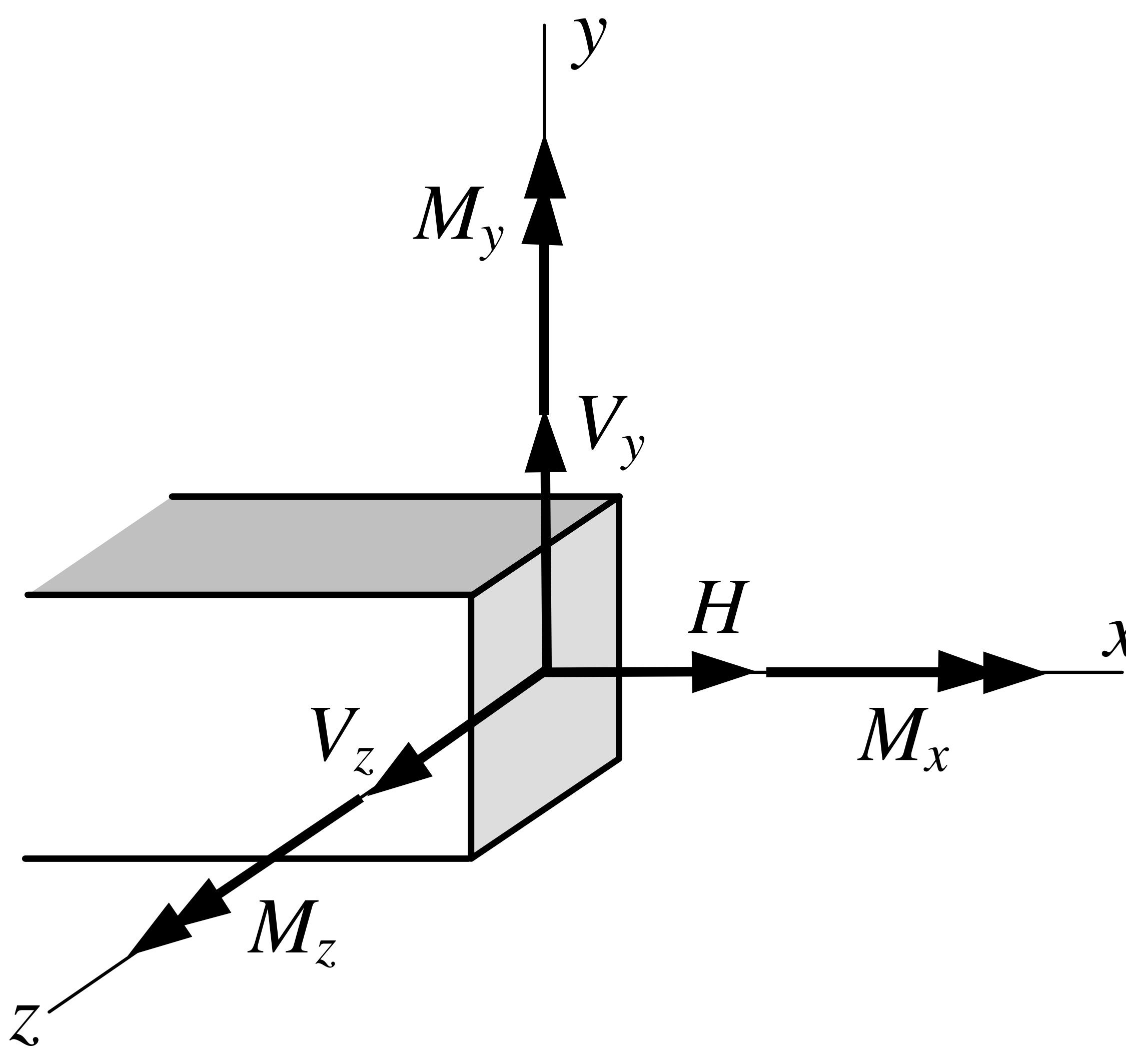
For 3D beams—there are

Two shear forces (V_y and V_z),

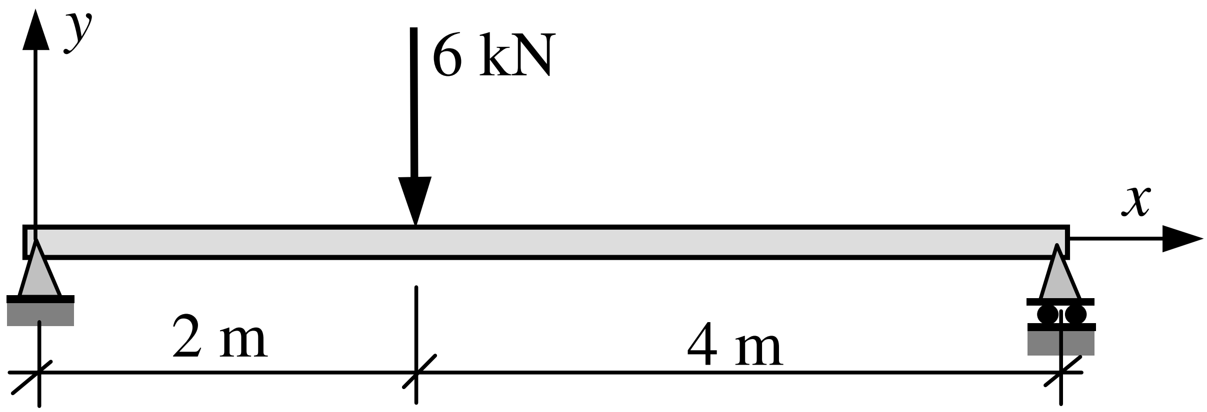
Two bending moments (M_y and M_z),

One axial force (H), and

One torsional moment (M_y).



Example: Draw the shear force and bending moment diagrams of the simply supported beam shown below.

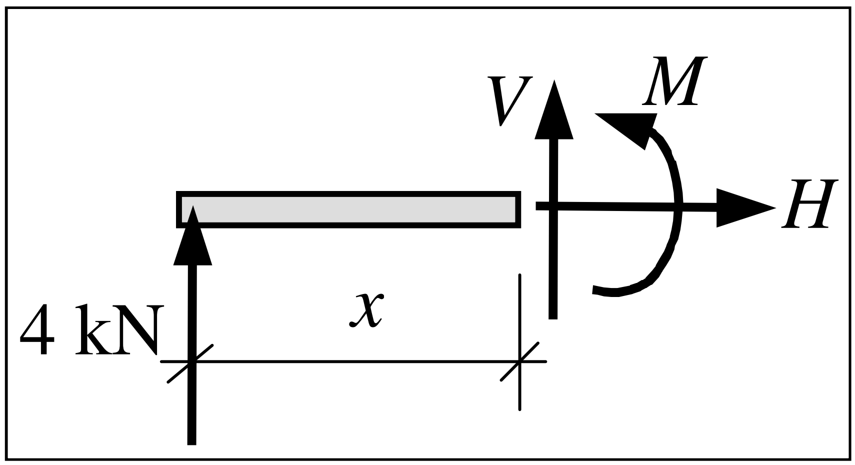


$\underline{0 < x < 2\text{ m}}$:

$V_y = -4$ kN;

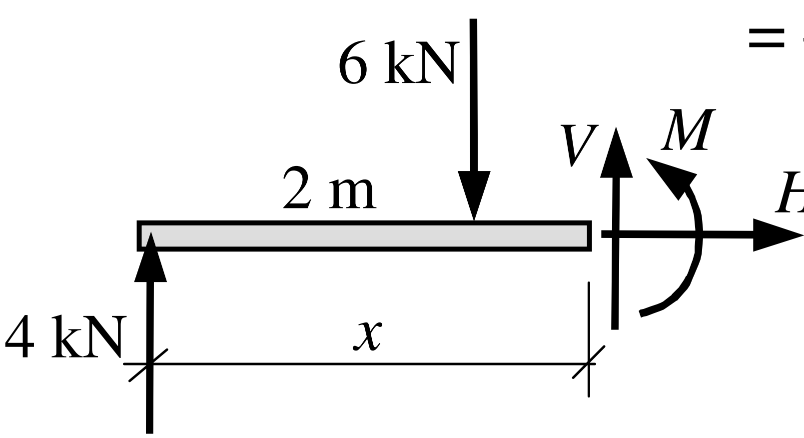
$M_z = 4\,x$ kN m

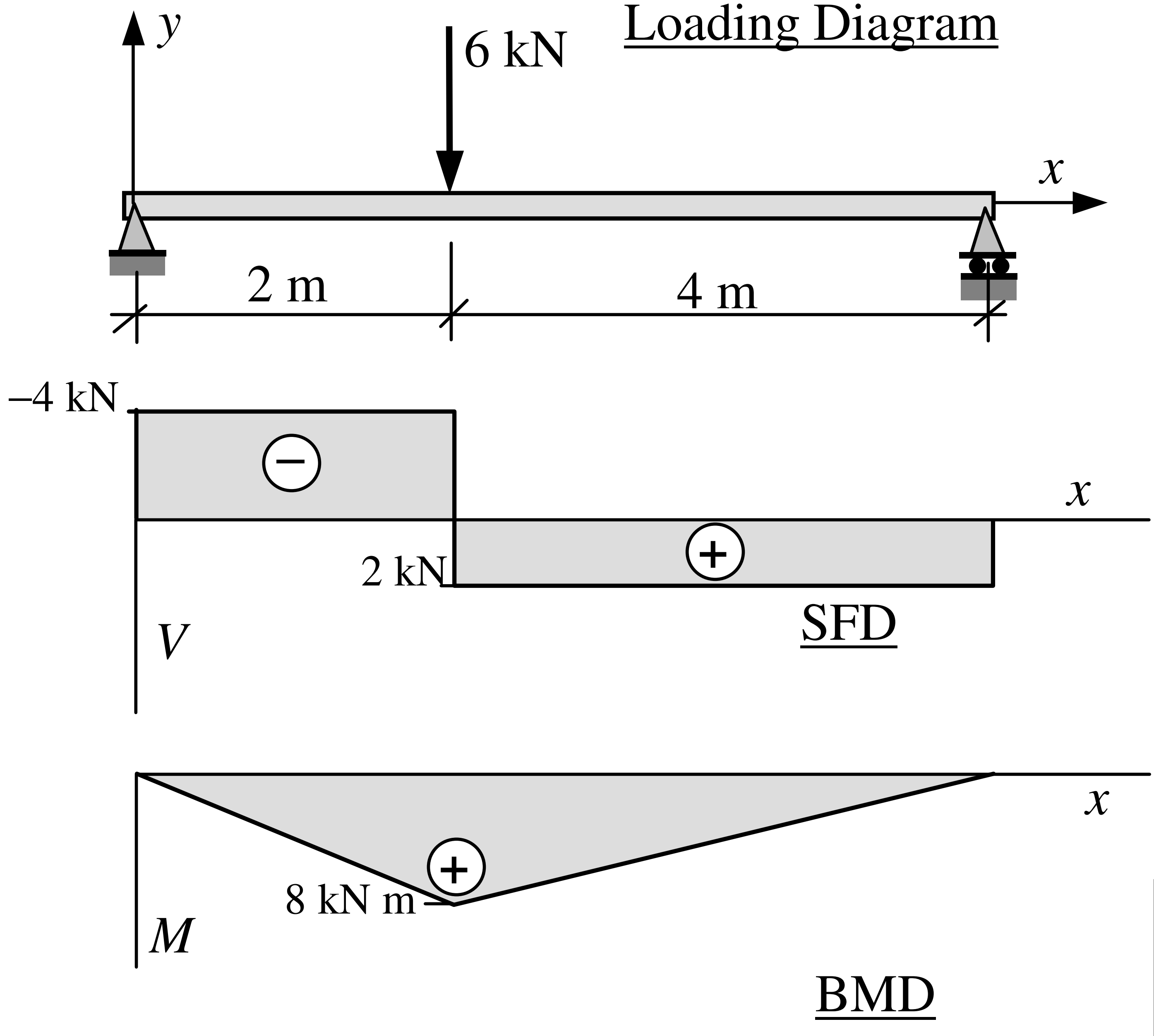
$H = 0$



$\underline{2 > x > 6\text{ m}}$: $V = 2$ kN; $M = 4\,x - 6\,(x - 2)$

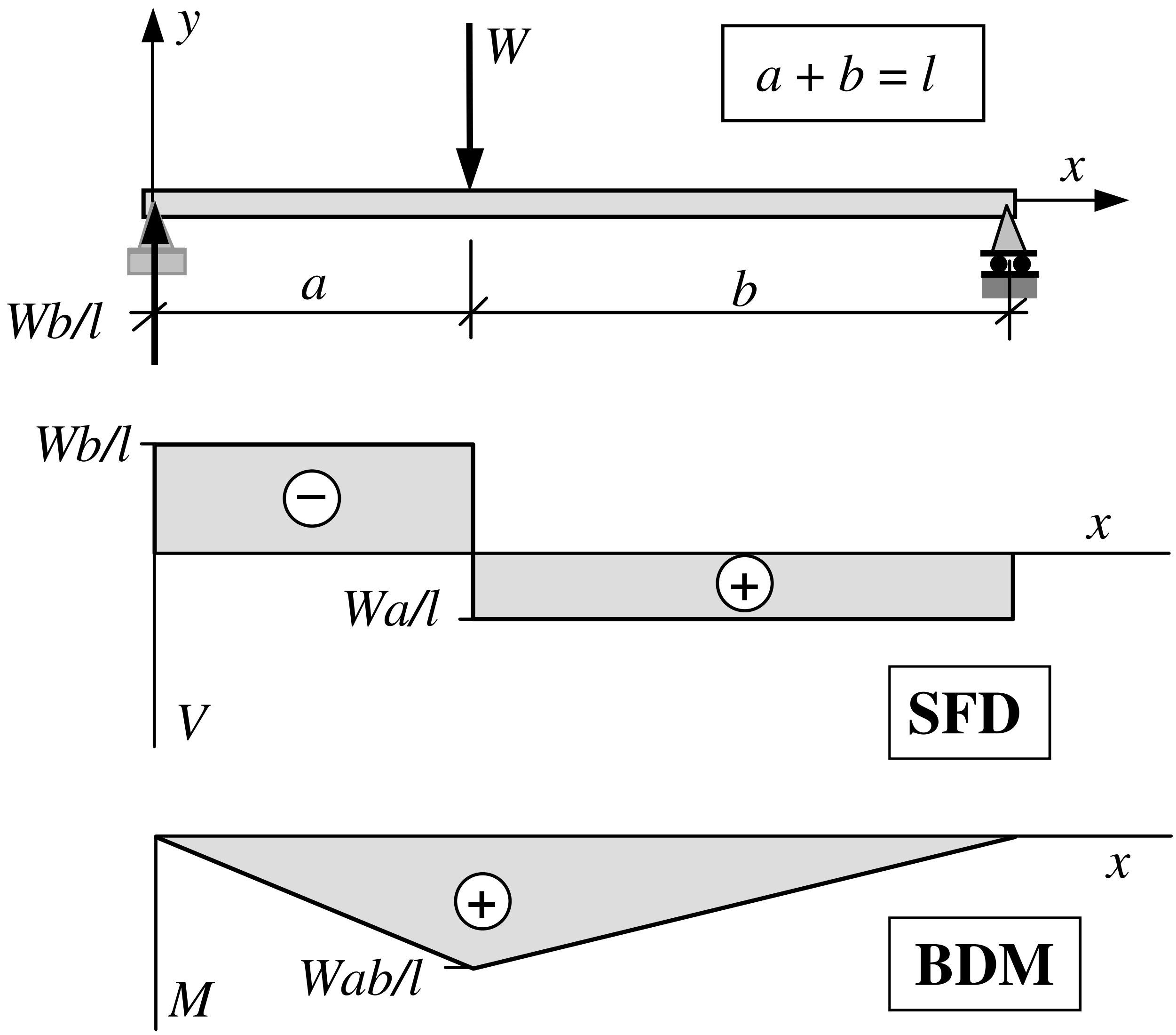
$= -2\,x + 12$ kN m



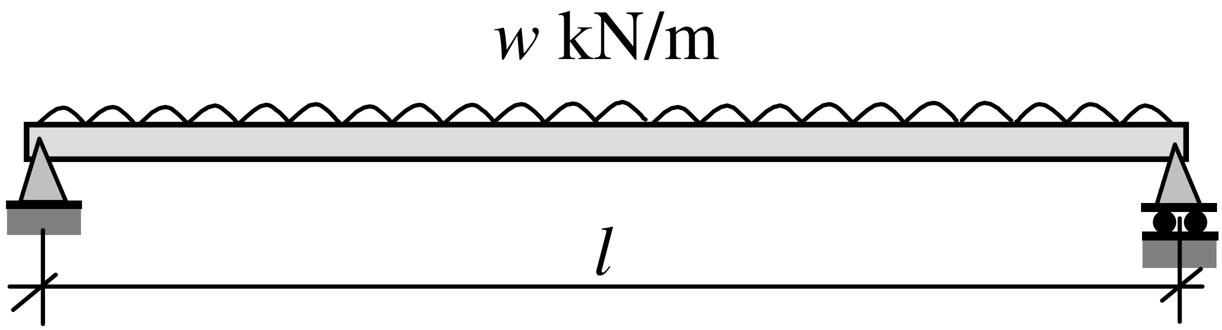


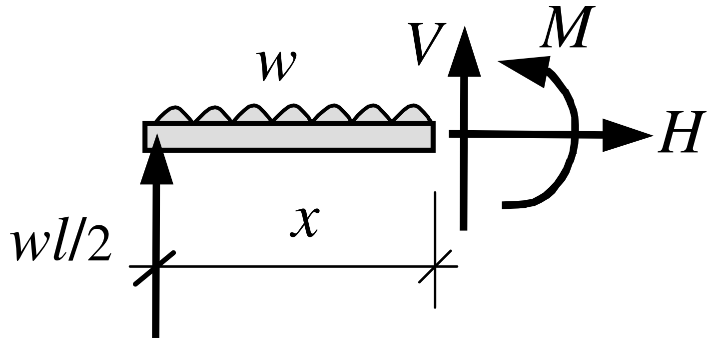
SFD—Shear Force Diagram

BMD—Bending Moment Diagram



Draw the SFD and the BMD for a s.s. beam subjected to a u.d.l.

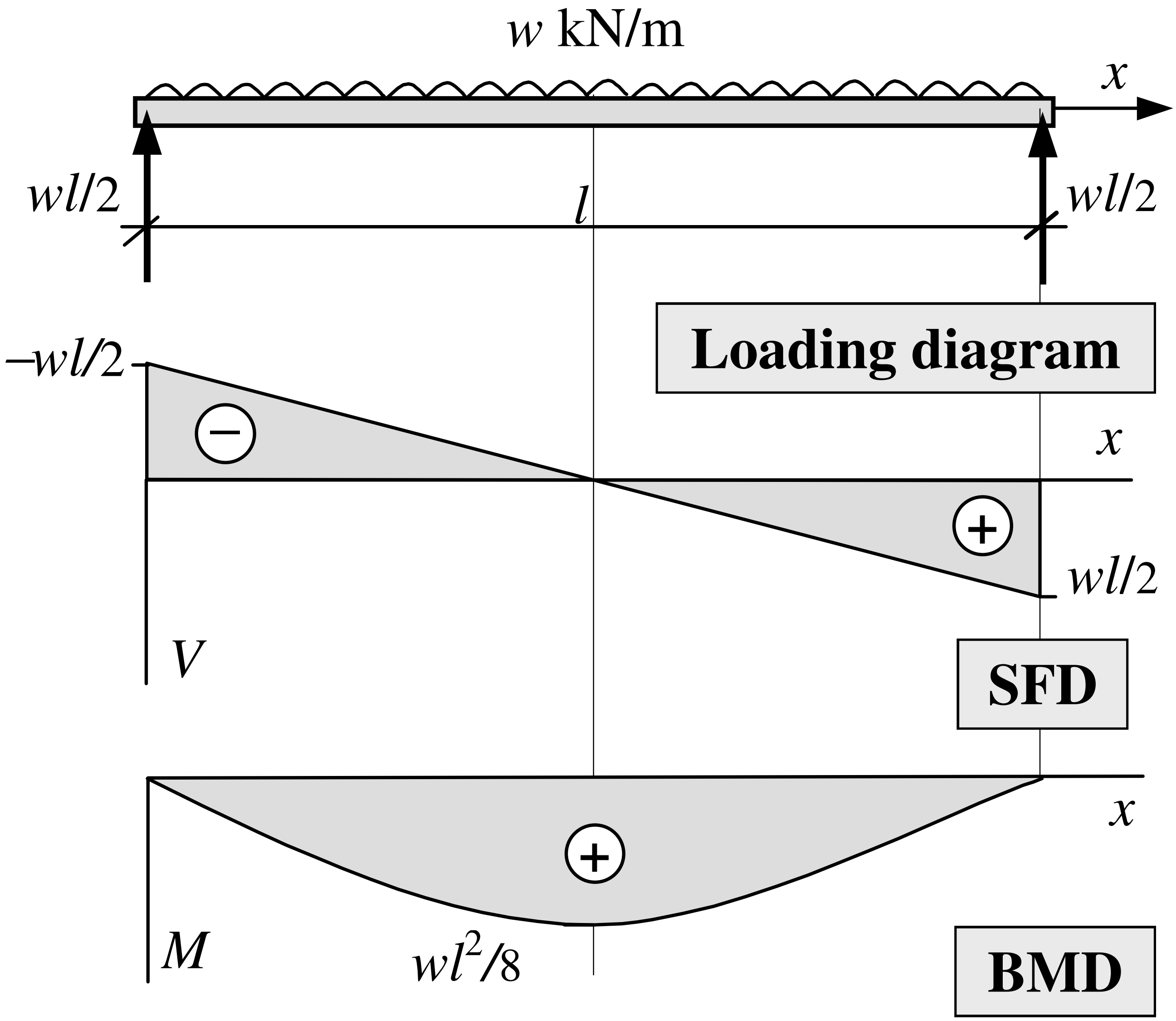


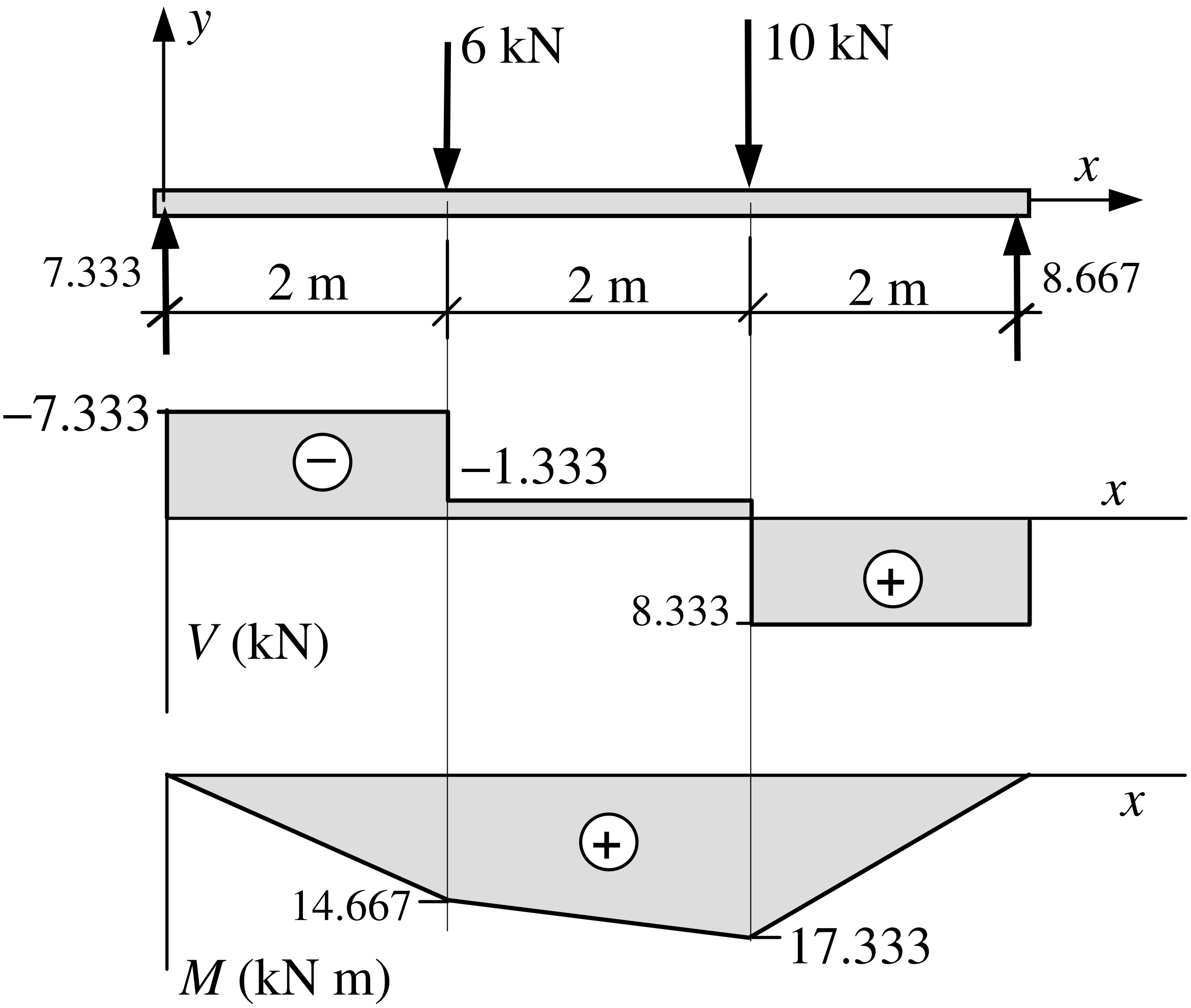


$0 < x < l$:

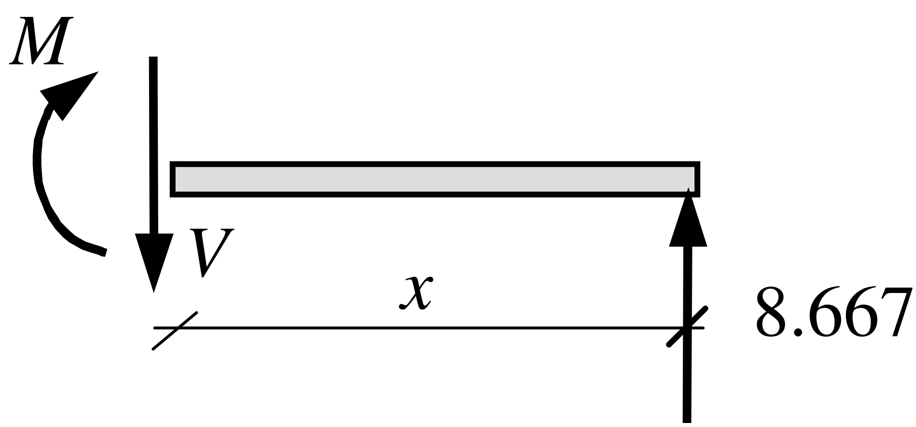
$V = -wl/2 + w\,x$

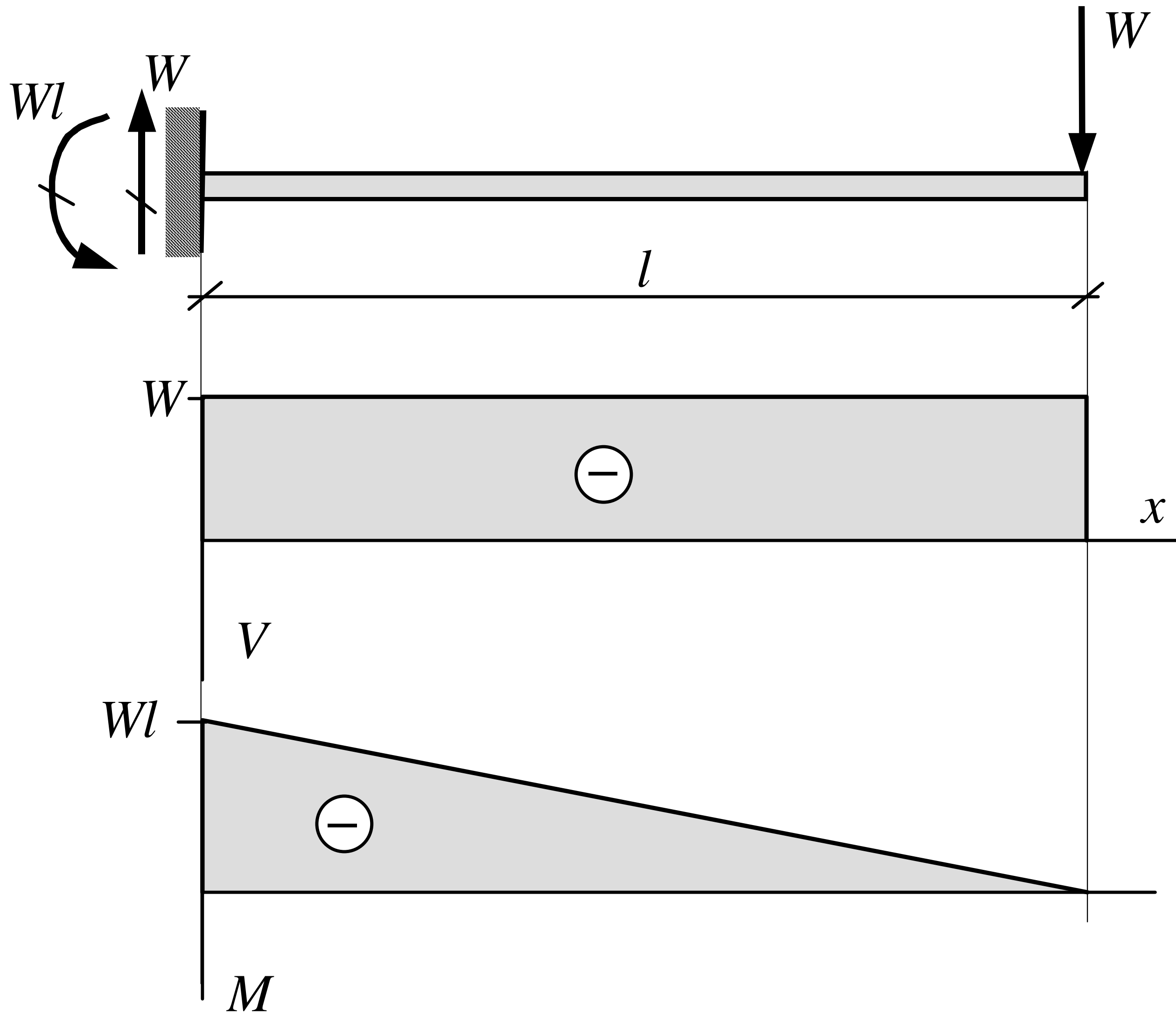
$M = wl/2.\,x - w\,x.x/2 = w\,x/2\,(l - x)$



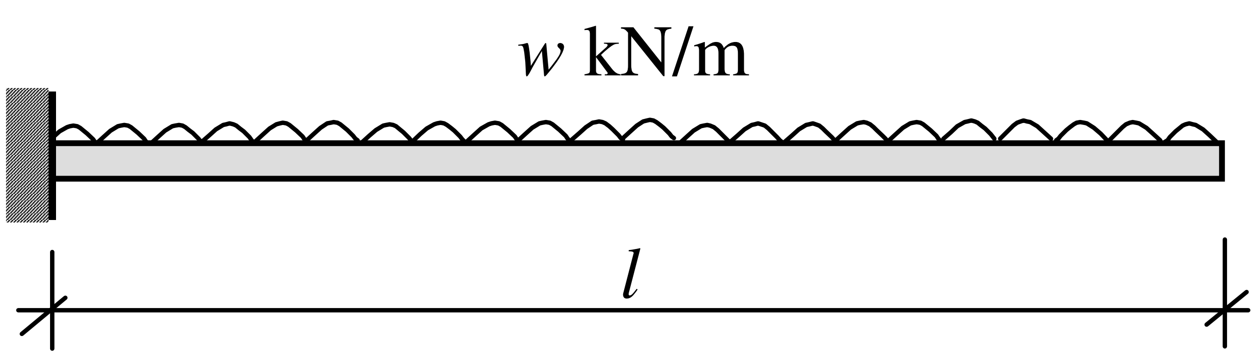

y
6 kN
10 kN
x
7.333
2 m
2 m
2 m
8.667
−7.333
−1.333
x
8.333
V (kN)
x
14.667
17.333
M (kN m)

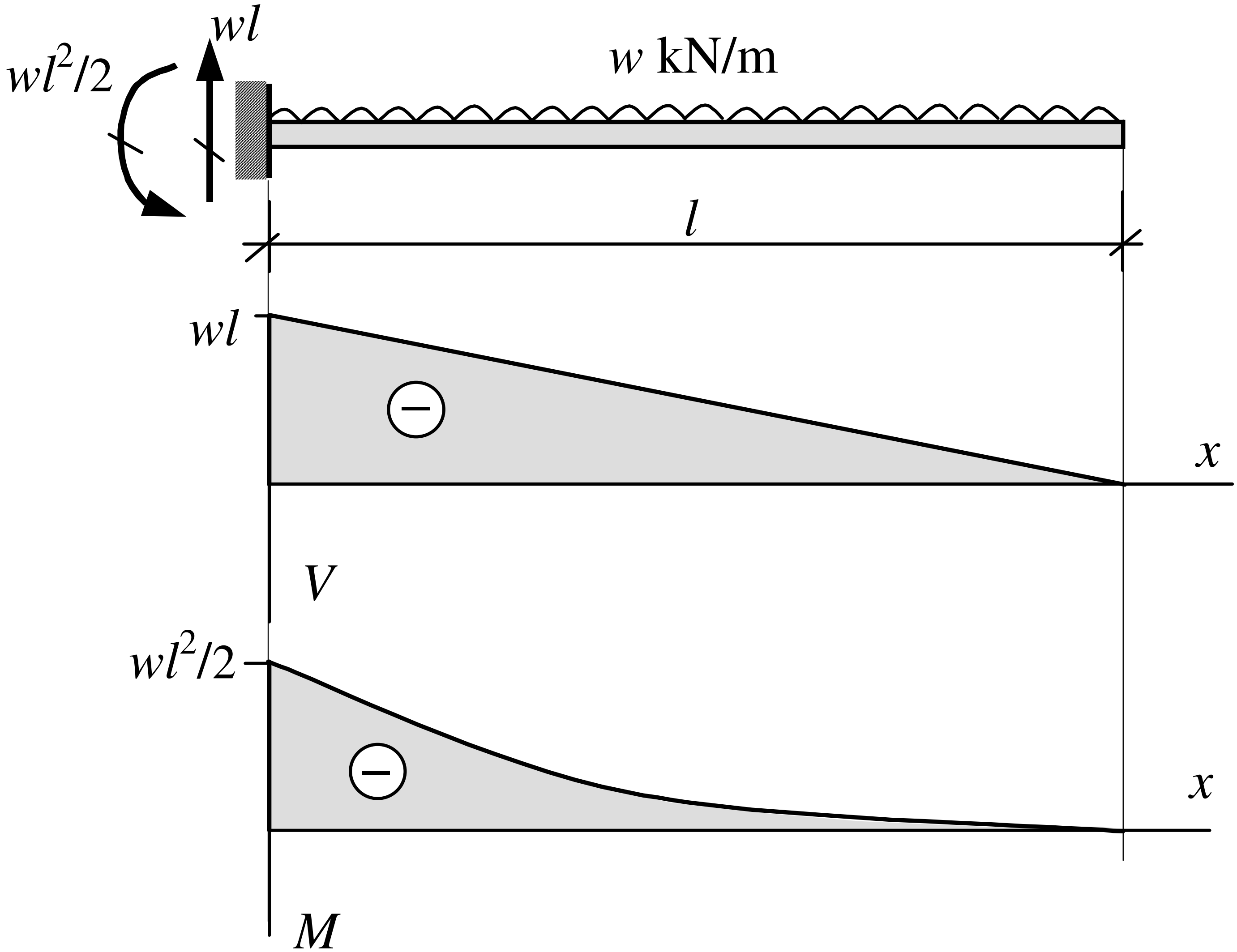
$4 < x < 6$


M
V
x
8.667



Exercise: Draw SFD and BMD of:

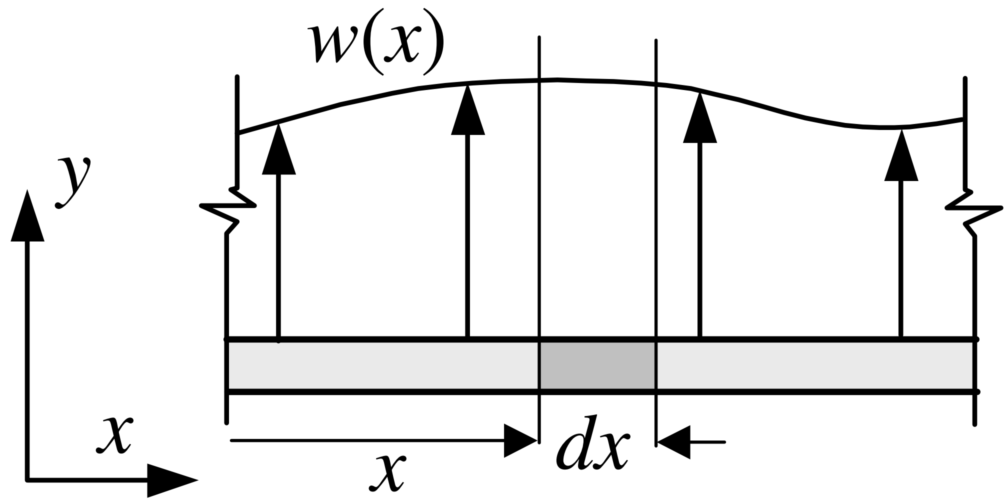



wl
$wl^2/2$
w kN/m
l
wl
x
V
$wl^2/2$
x
M

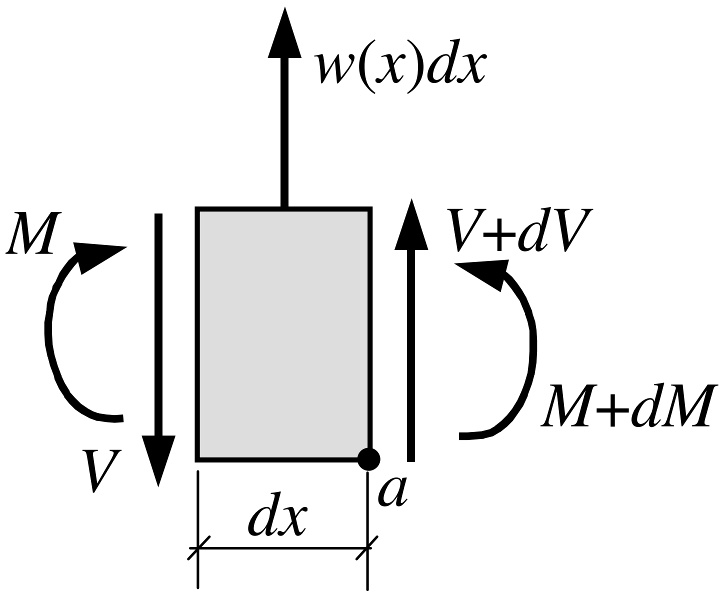
Differential Relations for Equilibrium

There exists relations between the loading, the SF and the BM at any section.

Consider the equilibrium of an infinitesimal strip of a beam as shown below:



Note that the loading is assumed to be positive if it acts along the positive *x* direction.



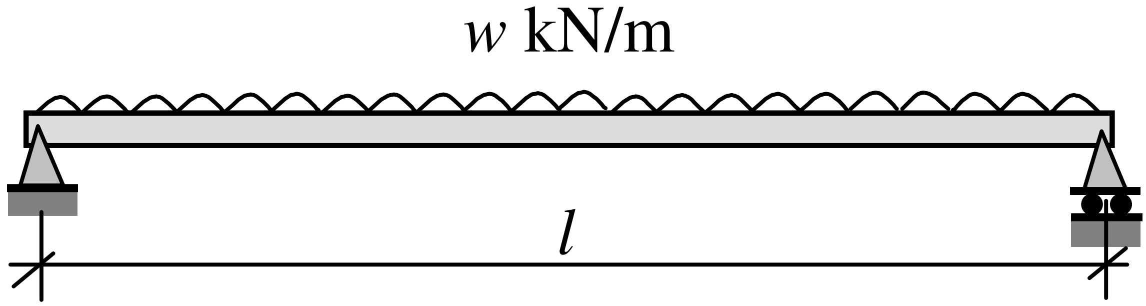
$$\Sigma F_y = 0: \quad -V + (V + dV) + w(x)\, dx = 0$$

$$\therefore \quad \boxed{\frac{dV}{dx} = -w}$$

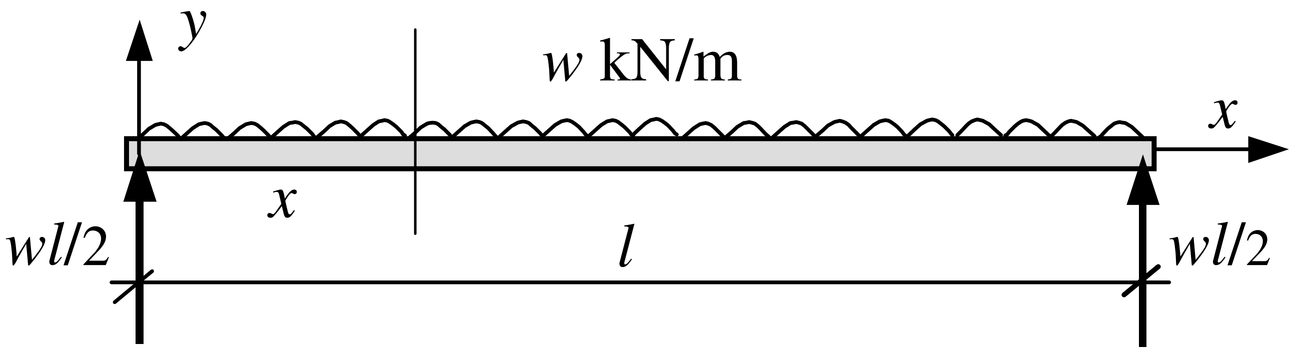
$$\Sigma M_a = 0:$$

$$V\, dx - M - w(x)\, dx\, dx/2 + (M + dM) = 0$$

$$\therefore \quad \boxed{\frac{dM}{dx} = -V}$$



Obtain the SF and BM distributions in the s.s beam shown above subjected to a udl.



At x, $w(x) = w$;

$$\frac{dV}{dx} = -w \; ; \therefore \; V = wx + A$$

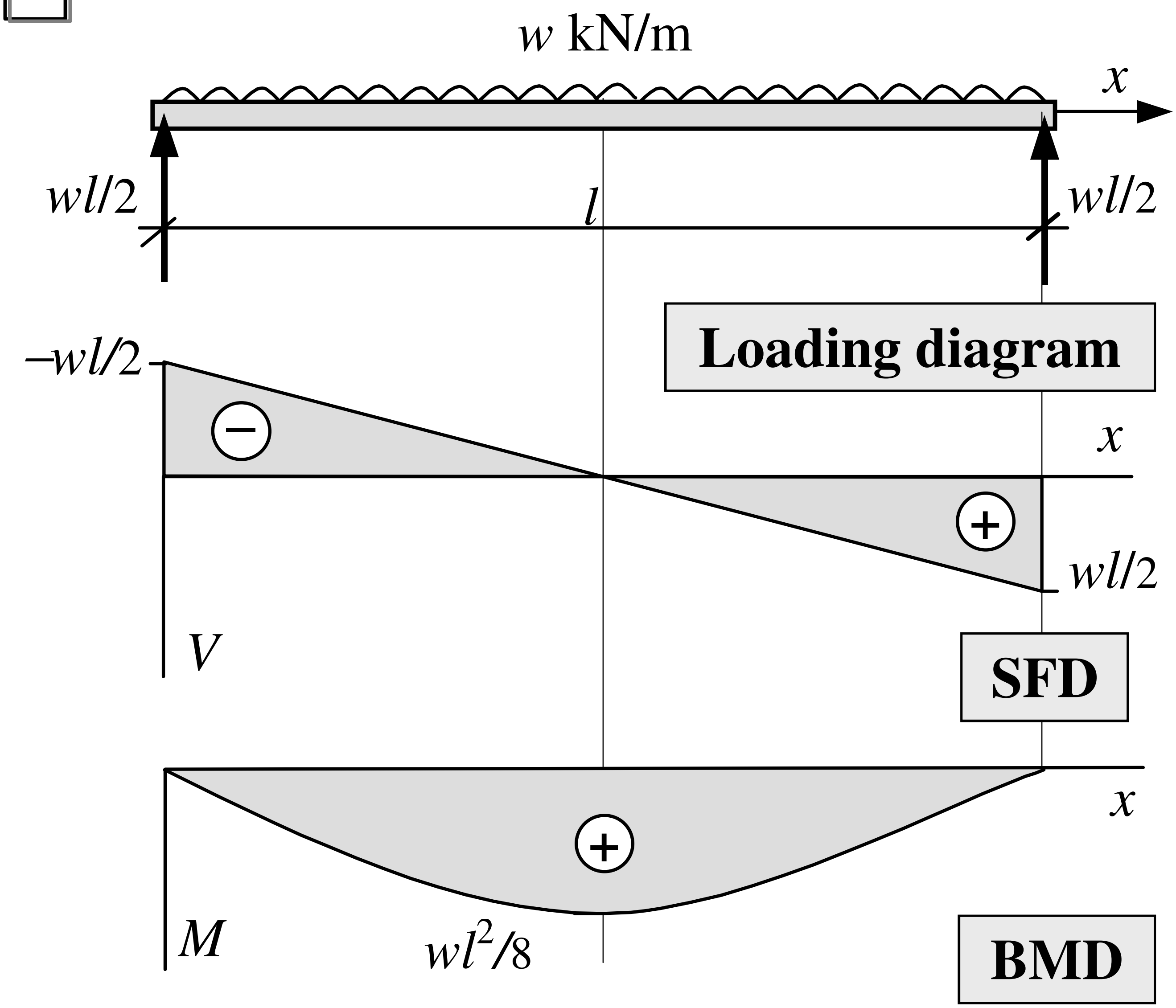
At $x = 0$, $V = -wl/2$; $\therefore$ $A = -wl/2$;

$$\therefore \; \underline{\underline{V(x) = w(x - l/2)}}$$

$$\frac{dM}{dx} = -V \;\; = -\,w(x - l/2)$$

$$\therefore \; M(x) = \underline{\underline{wx^2/2 - wl\,x/2 + B}};$$

At $x = 0$, $M = 0$; $\therefore$ $B = 0$;



Note:

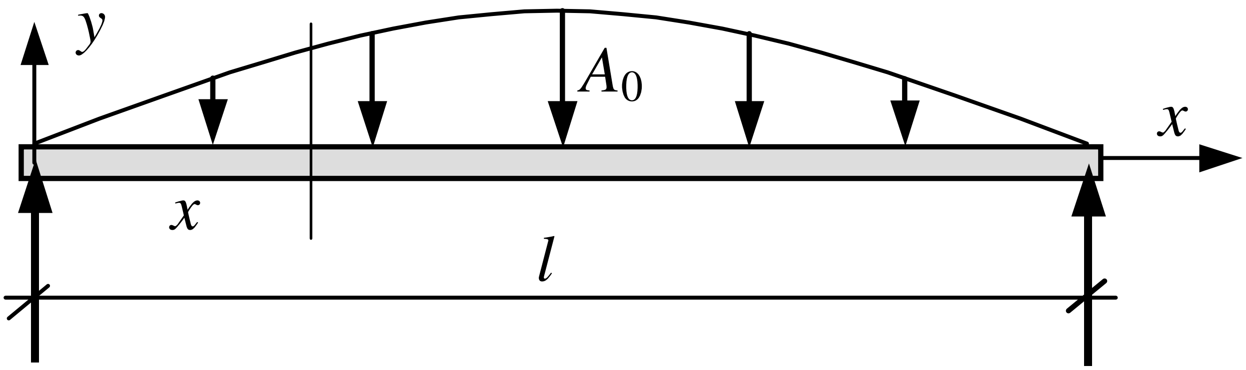
Maximum BM at point of zero SF

Maximum SF at point of zero load

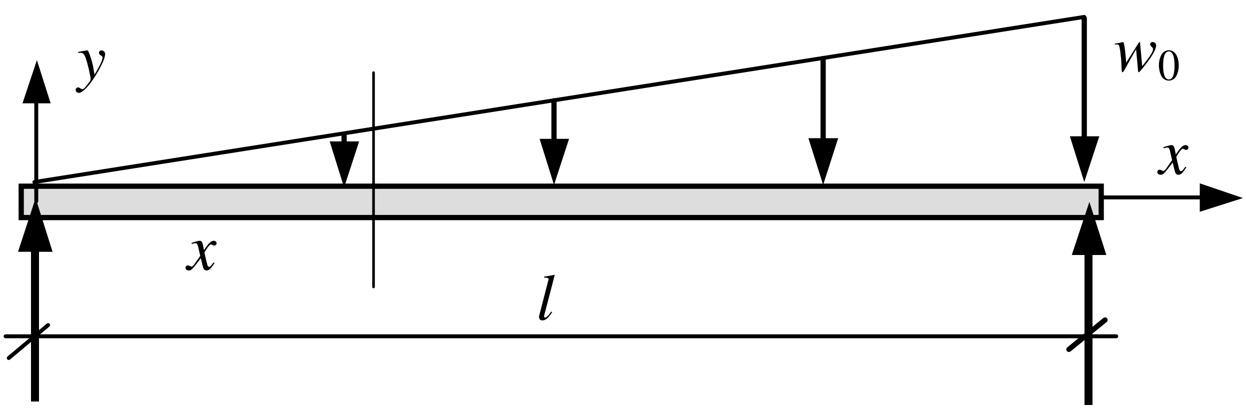
Exercise 1: A s.s beam is loaded with a distributed load given by

$$w(x) = A \sin \frac{\pi x}{l}$$

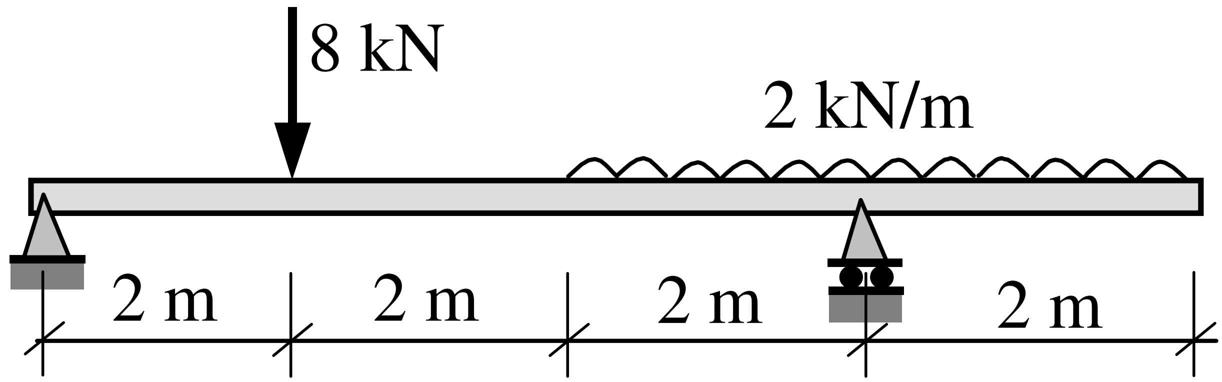
Obtain the SF and BM distributions.

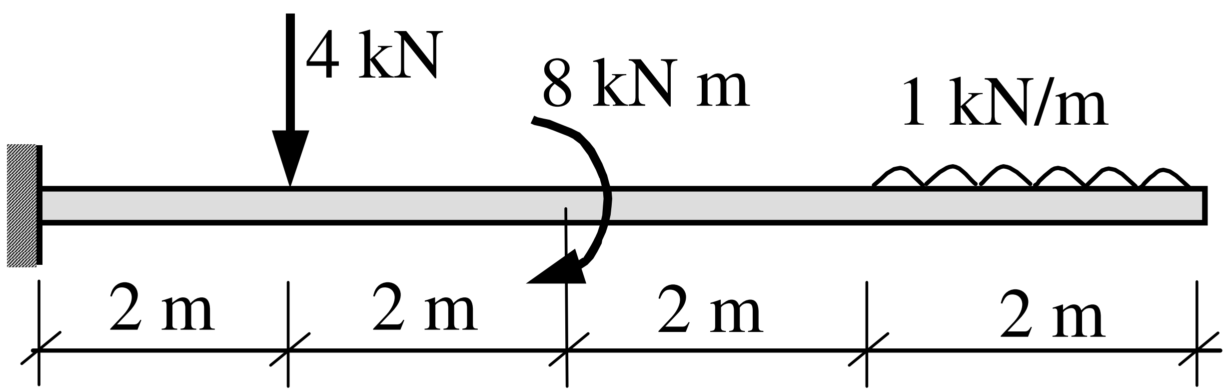


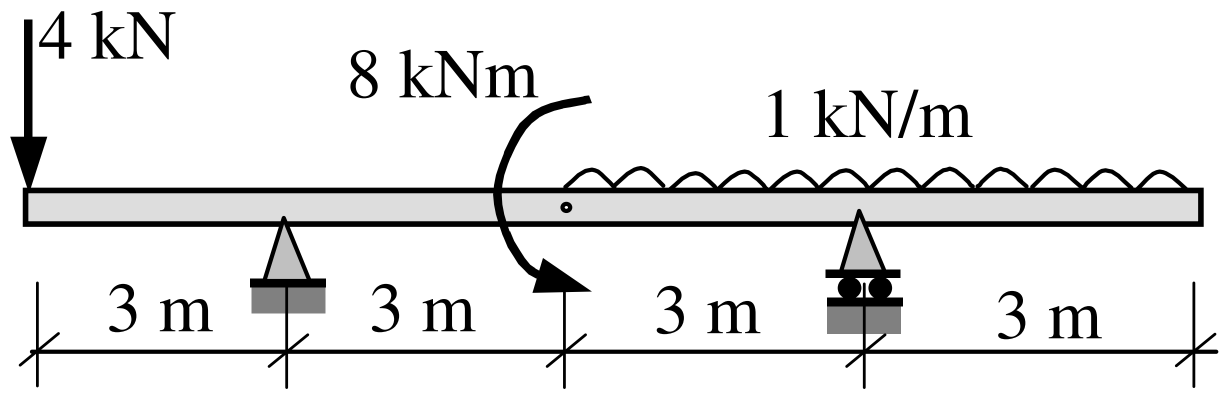
Exercise 2: A s.s beam is loaded with a distributed load given by $w(x) = w_0\, x/l$. Obtain the SF and BM distributions.

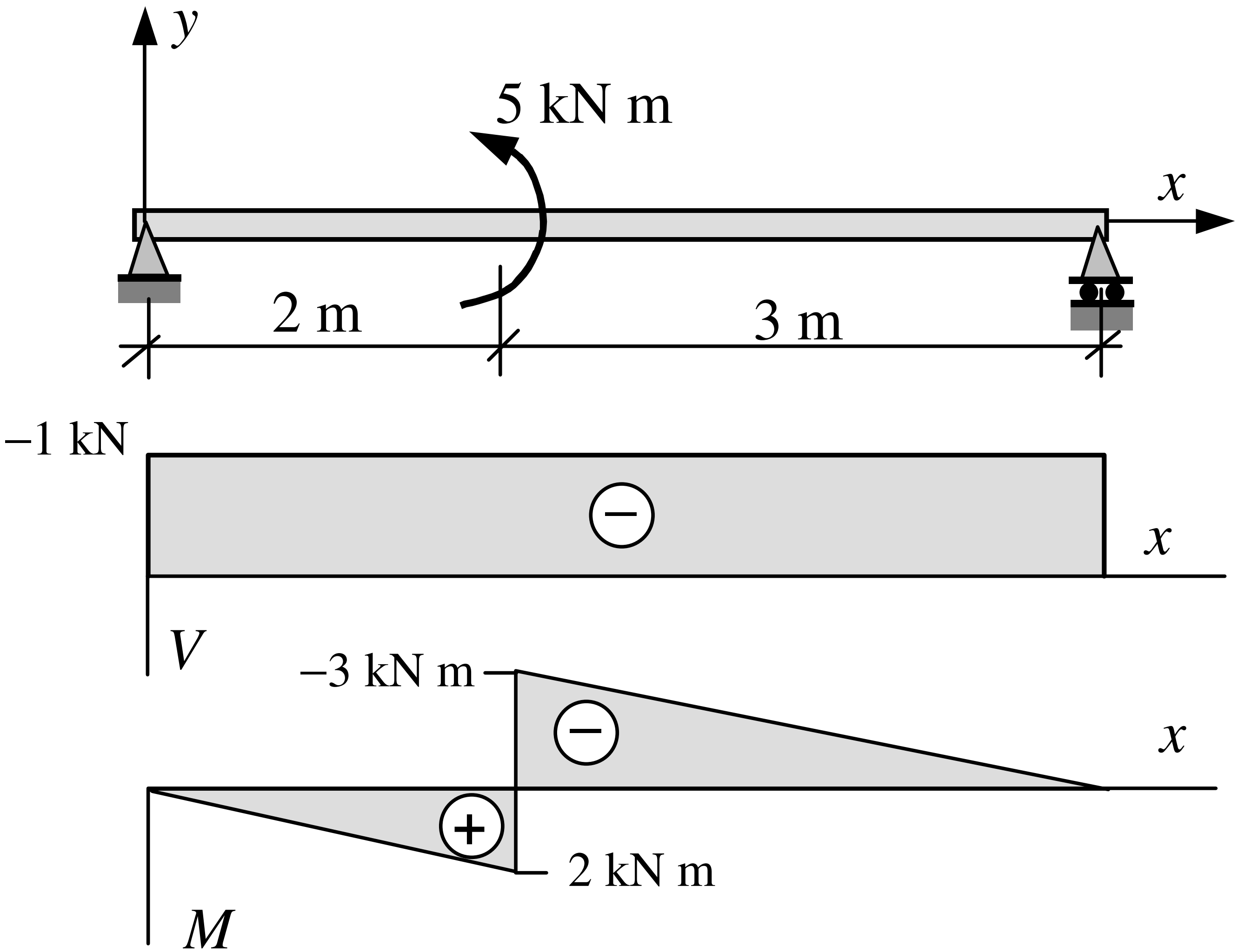


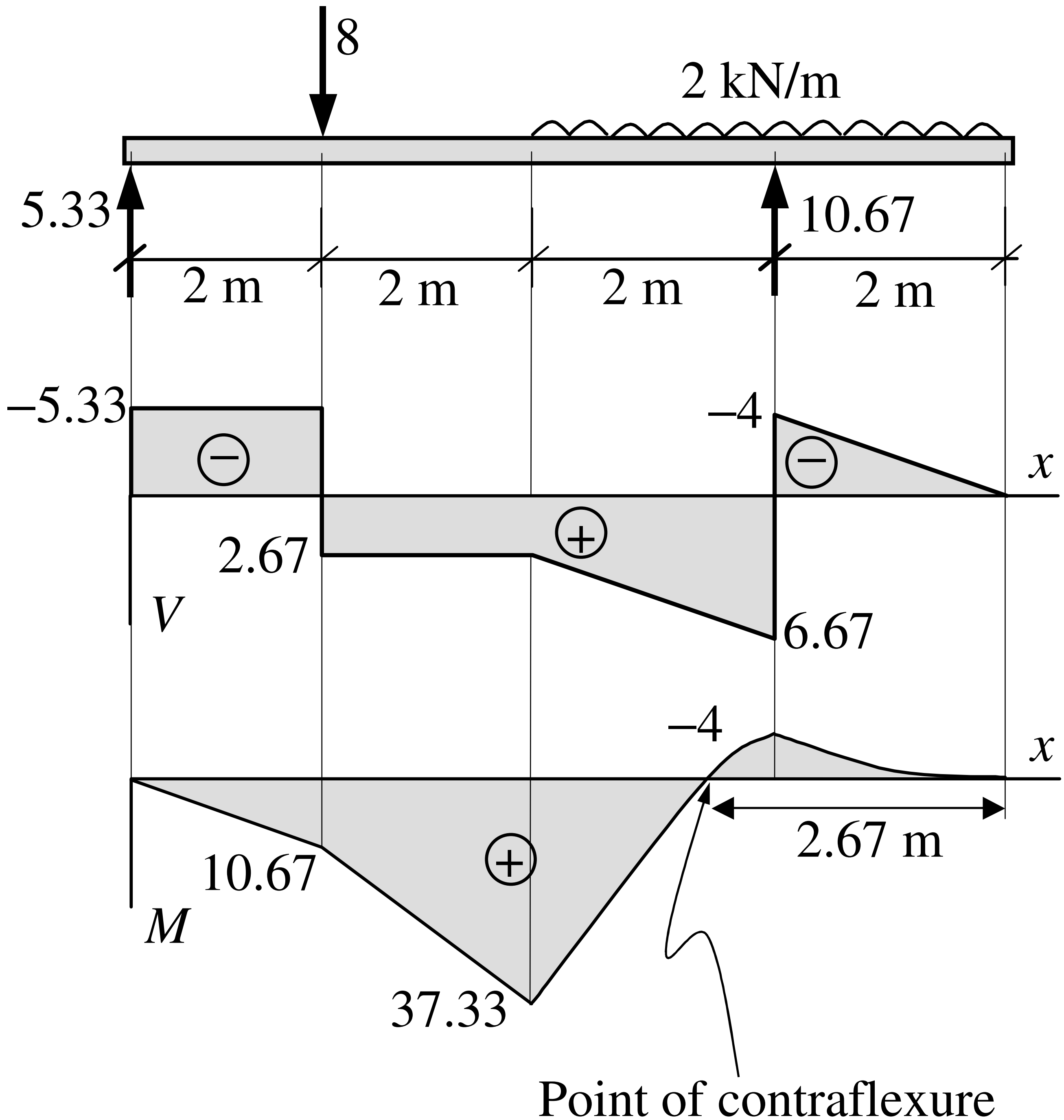
Draw the SFD and BMD of the following beams:

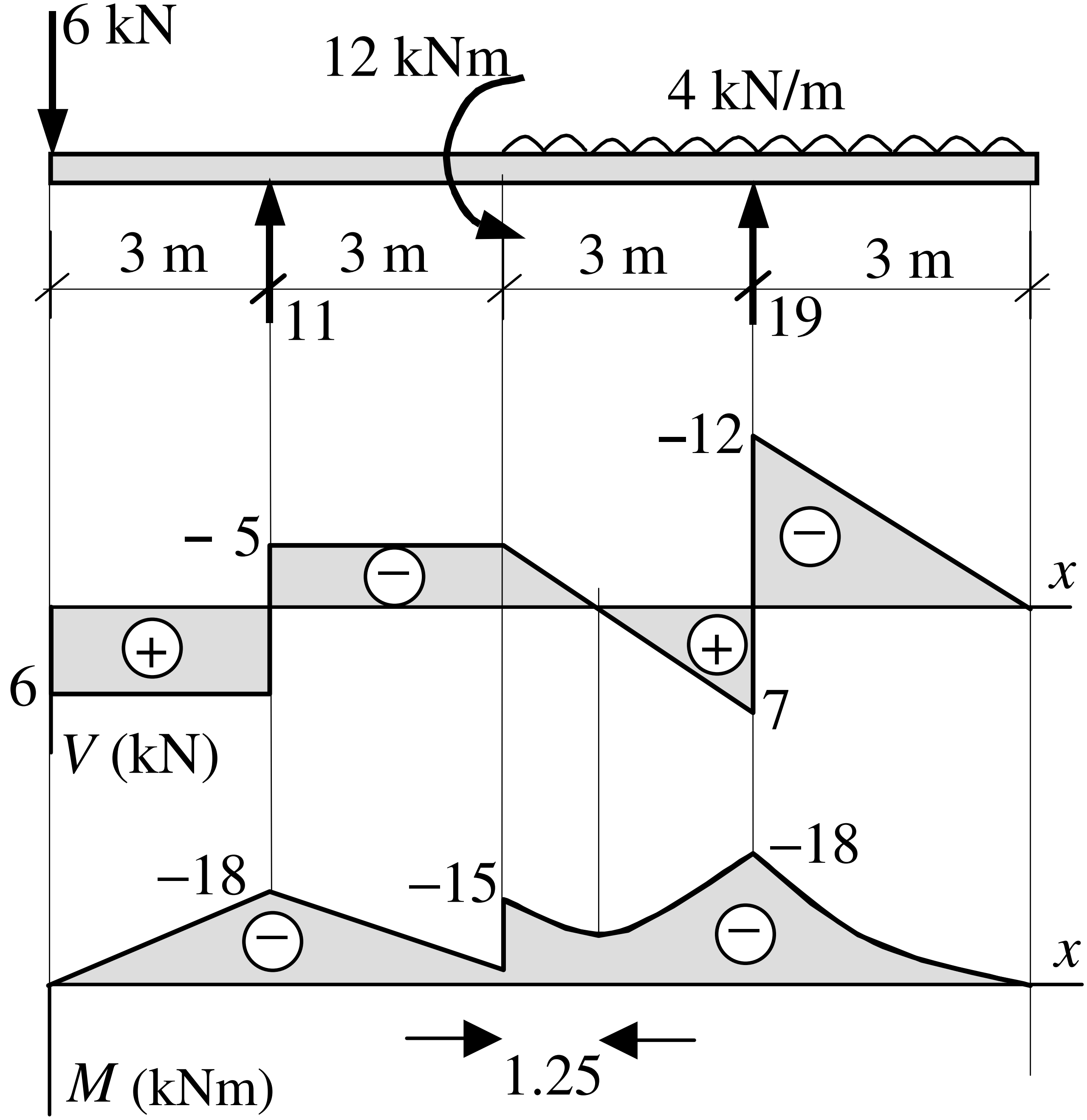







y
5 kN m
x
2 m
3 m
−1 kN
x
V
−3 kN m
x
2 kN m
M


8
2 kN/m
5.33
10.67
2 m
2 m
2 m
2 m
–5.33
–4
x
2.67
V
6.67
–4
x
2.67 m
10.67
M
37.33
Point of contraflexure


6 kN
12 kNm
4 kN/m
3 m
3 m
3 m
3 m
11
19
−12
− 5
6
7
x
V (kN)
−18
−15
−18
x
1.25
M (kNm)

Chains and Cables

Relatively flexible chains and cables are often used to support loads

Examples:

- Suspension bridges
- Transmission lines

Cable is assumed to be "perfectly flexible" and "inextensible"

"Flexibility" assumption means that

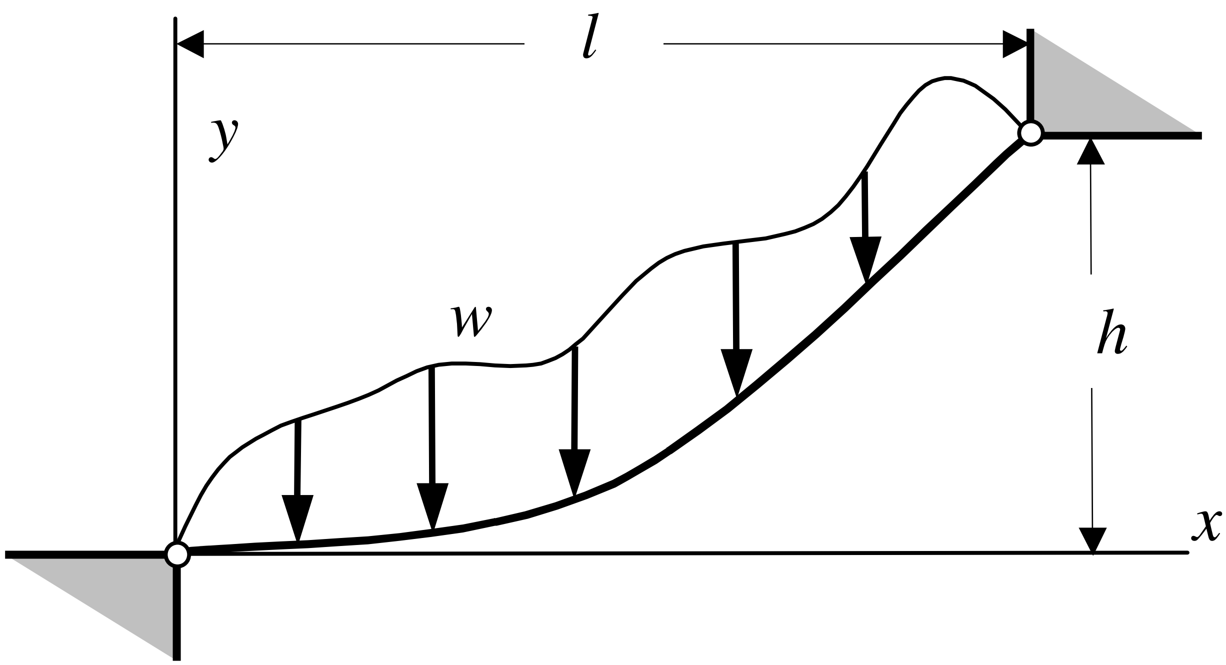
- at the centre of any cross-section of the cable only a tensile force is transmitted and no moments can be present
- this force must be tangential to the cable profile

"Inextensibility" assumptions denotes that the length of the cable remains a constant.

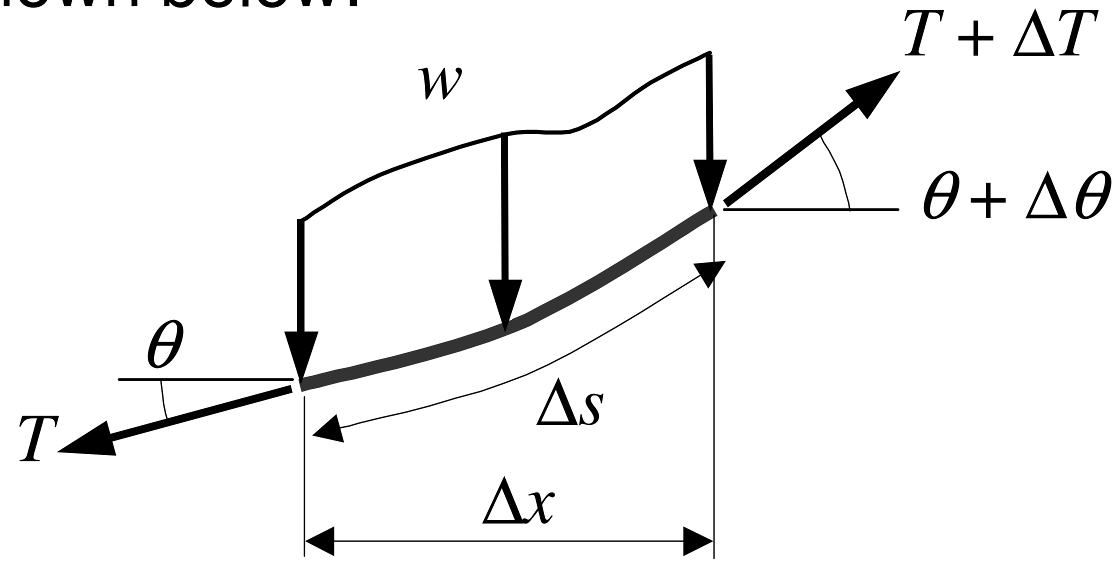
A Coplanar Cable with Loading as a Function of x (Parabolic cable):

Consider a cable suspended between two rigid supports *A* and *B* under the action of a loading *w*(*x*) given per unit length measured in the horizontal direction.

The cable and the load are assumed to lie in one and the same plane.



Consider an element of the cable as shown below:



$F_x = 0$:

$$-T\cos\theta + (T+\Delta T)\cos(\theta+\Delta\theta) = 0$$

$$\lim_{\Delta x\to 0} \frac{(T+\Delta T)\cos(\theta+\Delta\theta) - T\cos\theta}{\Delta x} = 0$$

or $$\frac{d}{dx}\left(T\cos\theta\right) = 0$$

$\therefore \underline{\underline{T\cos\theta = H}} =$ a constant, $\boxed{\mathbf{A}}$

where H is the horizontal component of the cable tension anywhere along the cable

$\Sigma F_y = 0$:

$$-T \sin\theta + (T + \Delta T)\sin(\theta + \Delta\theta) - w_{av}\,\Delta x = 0$$

or $\dfrac{d}{dx}\left(T\sin\theta\right) = w$

Integrating the above, we get

$$\underline{\underline{T\sin\theta = \int w(x)\,dx + C_1'}} \quad \boxed{\mathbf{B}}$$

B ÷ **A**, we get $\dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{H}\int w(x)\,dx + C_1$

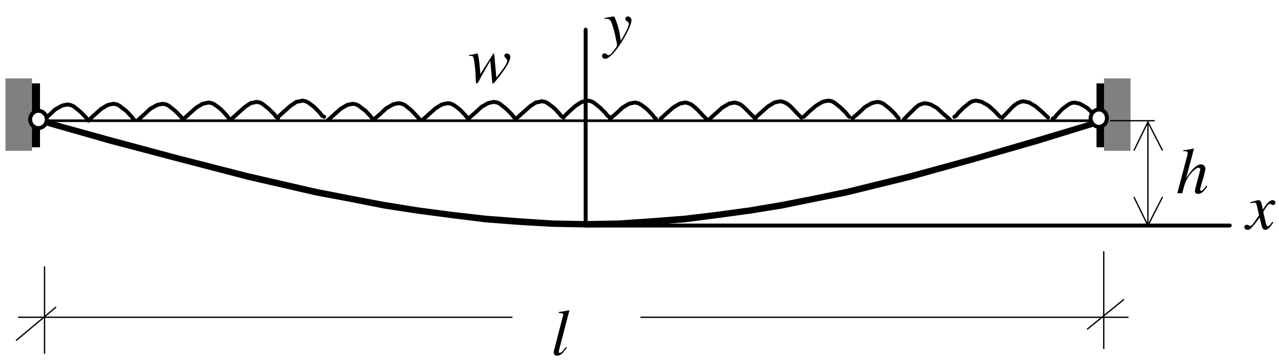
That is, $\tan\theta = \dfrac{dy}{dx} = \dfrac{1}{H}\int w(x)\,dx + C_1$

integrating the above, we obtain

$$y = \frac{1}{H}\int\left[\int w(x)dx\right]dx + C_1 x + C_2$$

y gives the equation of the deflection curve. C_1 and C_2 are found from the boundary conditions.

Example: Neglect the self weight of the cable and determine the shape, length and the maximum force in the cable.



We have $$y = \frac{1}{H}\int w\,x\,dx + C_1 x + C_2$$

$$= \frac{wx^2}{2H} + C_1 x + C_2$$

At $x = 0,\ y = 0 \Rightarrow C_2 = 0$

At $x = 0,\ dy/dx = 0 \Rightarrow C_1 = 0$

Therefore, the deflection curve is

$$y = \frac{wx^2}{2H}$$

At $x = l/2, y = h \Rightarrow \quad h = \dfrac{w}{2H}\dfrac{l^2}{4}$

$$\therefore H = \frac{wl^2}{8h}$$

$$\therefore y = \frac{4hx^2}{l^2} \quad \boxed{\text{Parabola}}$$

Now, tension in the cable is $T = H/\cos\theta$

T_{max} corresponds to maximum θ.

$$\tan\theta = \frac{dy}{dx} = \frac{w}{H}x; \qquad \theta_{\max} = \tan^{-1}\left(\frac{wl}{2H}\right)$$

$$T_{\max} = \frac{H}{\cos\left(\tan^{-1}\frac{wl}{2H}\right)}$$

which can be simplified as

$$T_{\max} = \frac{wl}{2}\sqrt{1+\left(\frac{l}{4h}\right)^2}$$

Length of the cable:

$$L = 2\int_0^{l/2} ds = 2\int_0^{l/2} \sqrt{dx^2 + dy^2} = 2\int_0^{l/2} dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$= 2\int_0^{l/2} \left[1+\left(\frac{8hx}{l^2}\right)^2\right]^{1/2} dx$$

$$= \frac{l}{2}\left[1+\left(\frac{4h}{l}\right)^2\right]^{1/2} + \frac{l^2}{8h}\sinh^{-1}\frac{4h}{l}$$

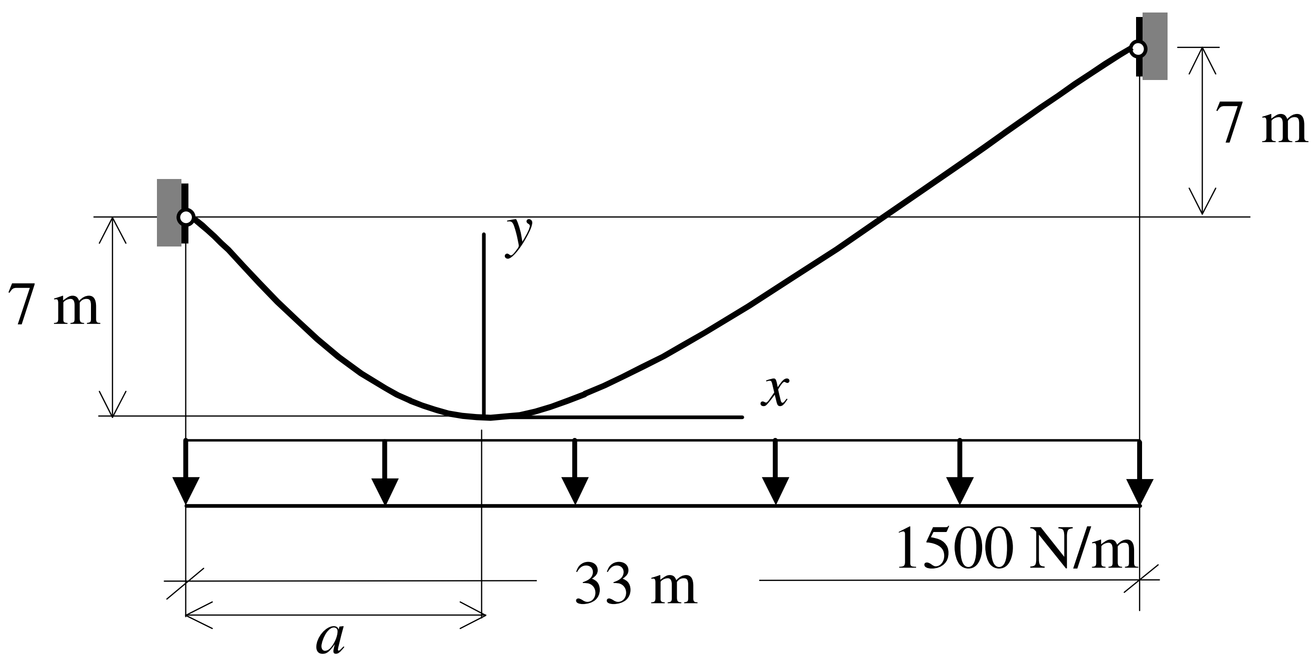
Alternatively,

$$L = 2\int_0^{l/2} [1+\frac{1}{2}\left(\frac{dy}{dx}\right)^2 - \frac{1}{8}\left(\frac{dy}{dx}\right)^4 + \ldots]\, dx$$

$$= l\left[1+\frac{8}{3}\left(\frac{h}{l}\right)^2 - \frac{32}{5}\left(\frac{h}{l}\right)^4 + \ldots\right]$$

This series converges for cables with small slopes (<45°); use first few terms only.

Example: The left side of a cable is mounted at an elevation of 7 m below the right side. The sag, measured from left support, is 7 m. Find the maximum tension if the cable has uniform loading in the vertical direction of 1500 N/m.



Choose the origin as shown above.

$$y = \frac{1}{H}\int wx\,dx + C_1 x + C_2$$

At $x = 0$, $y = 0$ and $y' = 0$; hence,

$$y = \frac{wx^2}{2H}$$

At $x = -a$, $y = 7$ m; hence

$$7 = \frac{wa^2}{2H}; \quad \therefore a = \sqrt{\frac{14H}{w}}$$

At $x = 33 - a$, $y = 14$; hence

$$14 = \frac{w(33-a)^2}{2H}; \quad \therefore \boxed{H = 20018.878 \text{ N}}$$

And, $a = 13.669$ m

T_{max} occurs at $x = 33 - a$. $T = H/\cos\theta$

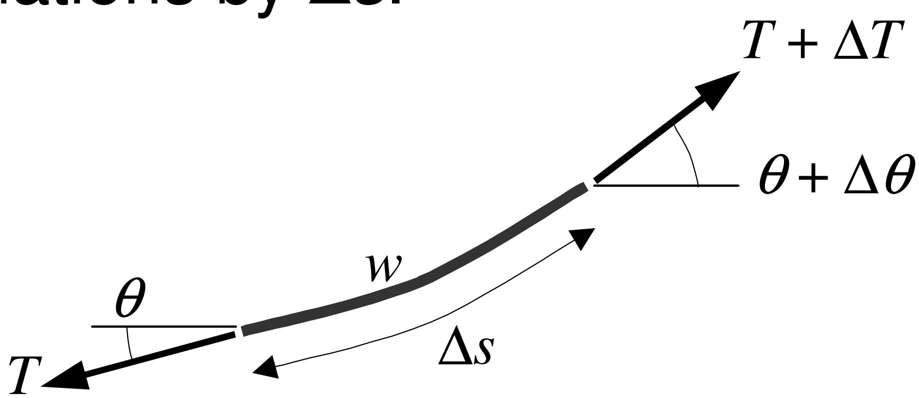
$$\tan\theta = \frac{dy}{dx} = \frac{wx}{H} = \frac{w(33-a)}{H}; \therefore \theta = 55.38^0$$

$$\therefore T_{\max} = \frac{H}{\cos\theta} = 35{,}235.612 \text{ N}$$

A Coplanar Cable with Loading being the self weight of the cable (Catenary):

Now, the loading is a function of *s*, the position along the cable.

We can replace Δx in the earlier equations by Δs.



$$\frac{d}{ds}\left(T\cos\theta\right) = 0$$

$$\underline{\underline{T\cos\theta = H}} = \text{a constant} \qquad \boxed{\mathbf{A}}$$

$$\frac{d}{ds}\left(T\sin\theta\right) = w(s)$$

$$\underline{\underline{T\sin\theta = \int w(s)\,ds + C_1'}} \qquad \boxed{\mathbf{B}}$$

B ÷ **A**, we get

$$\tan\theta = \frac{dy}{dx} = \frac{1}{H}\int w(s)\,ds + C_1 \qquad \boxed{\mathbf{C}}$$

We cannot integrate the above directly to obtain *y*. We have,

$$ds^2 = dx^2 + dy^2$$

$$dy = \sqrt{ds^2 - dx^2}; \quad \therefore \quad \frac{dy}{dx} = \sqrt{\left(\frac{ds}{dx}\right)^2 - 1}$$

From [C] we have

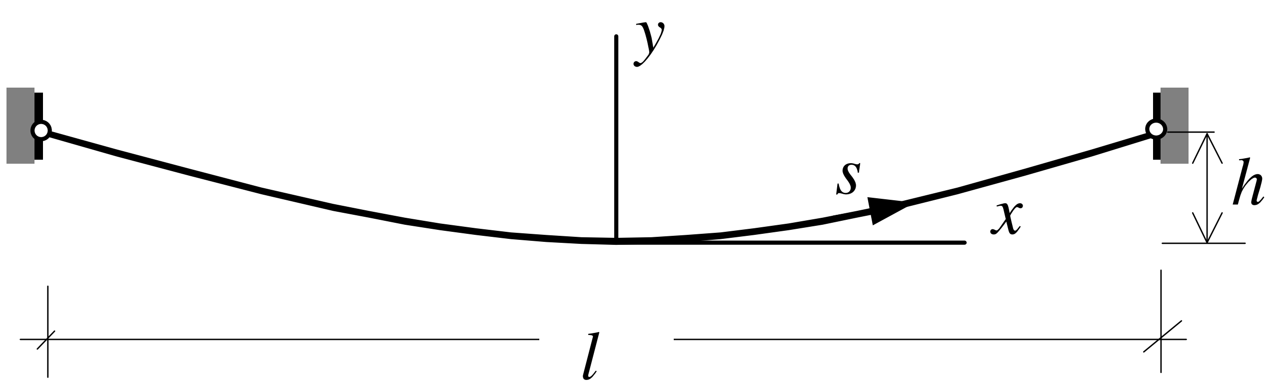
$$\left[\left(\frac{ds}{dx}\right)^2 - 1\right]^{1/2} = \frac{1}{H}\int w(s)\,ds + C_1$$

$$\therefore \frac{ds}{dx} = \left\{1 + \left[\frac{1}{H}\int w(s)\,ds + C_1\right]^2\right\}^{1/2}$$

Separating the variables and integrating, we obtain

$$x = \int \frac{ds}{\left\{1 + \left[\frac{1}{H}\int w(s)\,ds + C_1\right]^2\right\}^{1/2}} + C_2$$

<u>Example</u>: Determine the shape of the cable shown below when loaded by self weight only.



Let *w* be the weight per unit length of the cable.

$$\frac{dy}{dx} = \frac{1}{H}\int w(s)\,ds + C_1 = \frac{ws}{H} + C_1 \qquad \boxed{\mathbf{a}}$$

With the coordinates as shown in figure, at $s = 0$, $dy/dx = 0$.

Hence $C_1 = 0$.

Now, $$x = \int \frac{ds}{\left\{1+\left(\frac{ws}{H}\right)^2\right\}^{1/2}} + C_2$$

Integrating, we obtain

$$x = \frac{H}{w}\sinh^{-1}\frac{ws}{H} + C_2$$

At $x = 0$, $s = 0$. Hence $C_2 = 0$.

$$\therefore s = \frac{H}{w}\sinh\frac{wx}{H}$$

From Eq [a], we get

$$\frac{dy}{dx} = \sinh\frac{wx}{H}$$

$$\therefore\ y = \frac{H}{w}\cosh\frac{wx}{H} + C_3$$

At $x = 0$, $y = 0$. Hence $C_3 = -H/w$.

$$\therefore\ y = \frac{H}{w}\left(\cosh\frac{wx}{H} - 1\right)$$

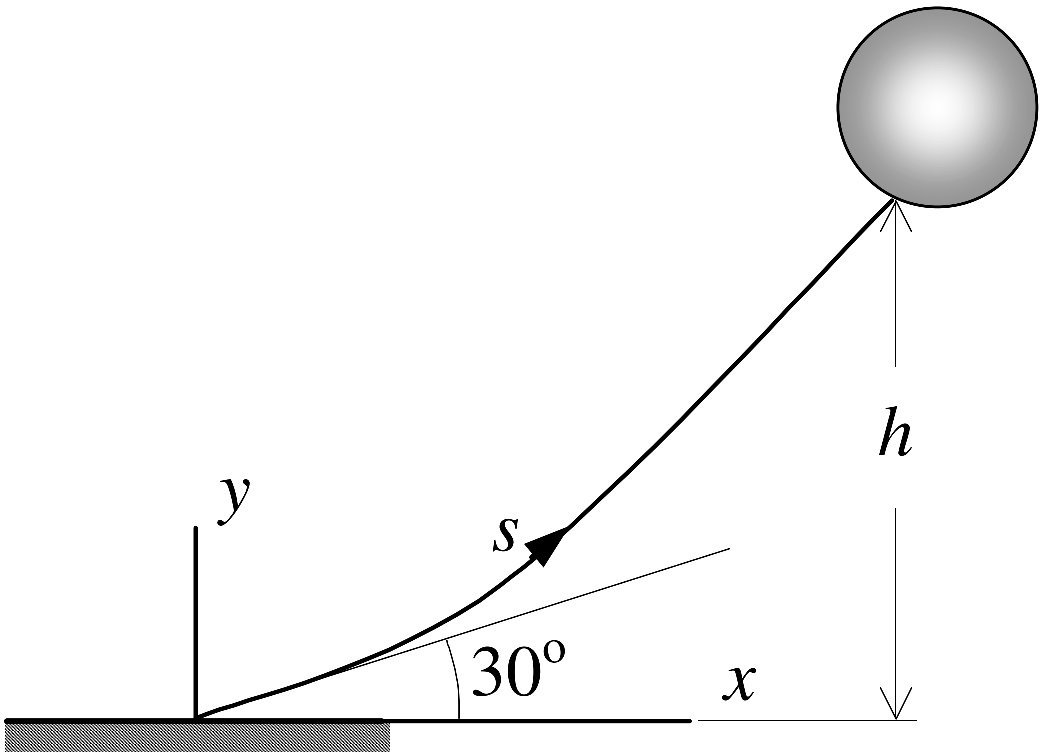
This curve is called a catenary (Latin *catena* means chain).

At $x = l/2$, $y = h$.

$$h = \frac{H}{w}\left(\cosh\frac{wl}{2H} - 1\right)$$

This equation can be solved using numerical methods to obtain *H.* Thus, T_{max} can be determined.

Example: A large balloon has a buoyant force of 100 lb. It is held by a 150-ft cable whose weight is 0.5 lb/ft. What is the height *h* of the balloon above the ground when a steady wind causes it to assume the position shown? What is the maximum tension in the cable?



$$\frac{dy}{dx} = \frac{1}{H} ws + C_1 \qquad \textbf{(a)}$$

At $s = 0$, $dy/dx = \tan 30 = 1/\sqrt{3}$;

Hence $\underline{\underline{C_1 = 1/\sqrt{3}}}$

$$x = \int \frac{ds}{\left\{1 + \left[\frac{ws}{H} + \frac{1}{\sqrt{3}}\right]^2\right\}^{1/2}} + C_2$$

$$\therefore x = \frac{H}{w} \sinh^{-1}\left(\frac{ws}{H} + \frac{1}{\sqrt{3}}\right) + C_2 \qquad \textbf{(b)}$$

$$\therefore s = \frac{H}{w}\left\{\sinh\left[\frac{(x - C_2)w}{H}\right] - \frac{1}{\sqrt{3}}\right\} \qquad \textbf{(c)}$$

From Eq. (a), we have

$$\frac{dy}{dx} = \sinh\left[\frac{(x - C_2)w}{H}\right] \qquad \textbf{(d)}$$

Integrating the above, we have

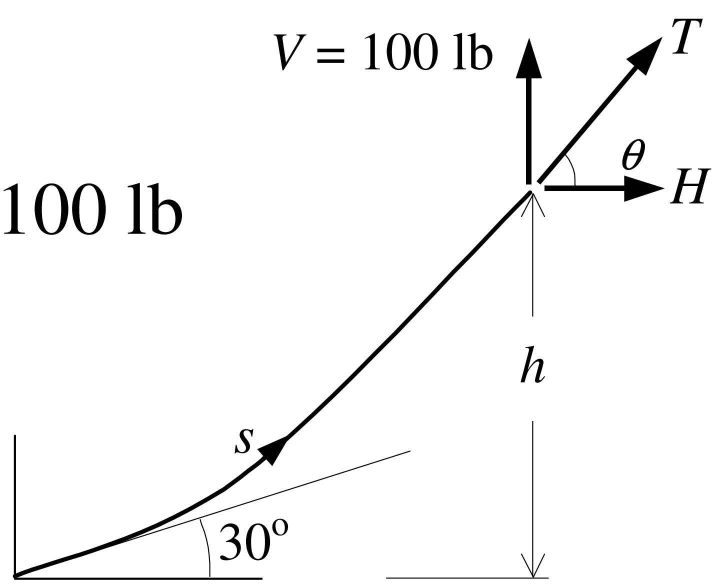
$$y = \frac{H}{w}\cosh\left[\frac{(x - C_2)w}{H}\right] + C_3 \qquad \textbf{(e)}$$

At s = 150, from Eq. (a) we get

$$\frac{dy}{dx} = \frac{ws}{H} + \frac{1}{\sqrt{3}} = \frac{0.5 \times 150}{H} + \frac{1}{\sqrt{3}}$$

$$= \tan\theta = \frac{100}{H} \qquad \textbf{(f)}$$

$$V = T\sin\theta = 100 \text{ lb}$$



At $x = 0$, $s = 0$, we get from Eq. (b)

$$0 = \frac{H}{w}\sinh^{-1}\left(\frac{1}{\sqrt{3}}\right) + C_2$$

$$\therefore C_2 = -\frac{H}{w}\sinh^{-1}\left(\frac{1}{\sqrt{3}}\right) \qquad \textbf{(g)}$$

From Eq. (f), we can write

$$\frac{H}{\sqrt{3}} = 100 - 0.5 \times 100 = 25;$$

$$\therefore \underline{\underline{H = 25\sqrt{3}}}$$

$$\tan\theta_{\max} = \frac{75}{25\sqrt{3}} + \frac{1}{\sqrt{3}}; \quad \therefore \underline{\underline{\theta_{\max} = 66.587^0}}$$

$$T_{\max} = \frac{H}{\cos\theta_{\max}} = 108.972\,\text{lb}$$

From Eq. (g), $C_2 = -\frac{25\sqrt{3}}{0.5}\sinh^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Friction Forces

Introduction

Friction is the force distribution at the contact surface between two bodies that impedes sliding motion of one body relative to the other.

This force distribution is tangential to the contact surface and acts opposite to the direction of any possible movement.

Frictional forces are associated with energy dissipation

They are at times undesirable, and at times beneficial!

Coulomb friction occurs between bodies with *dry* contact.

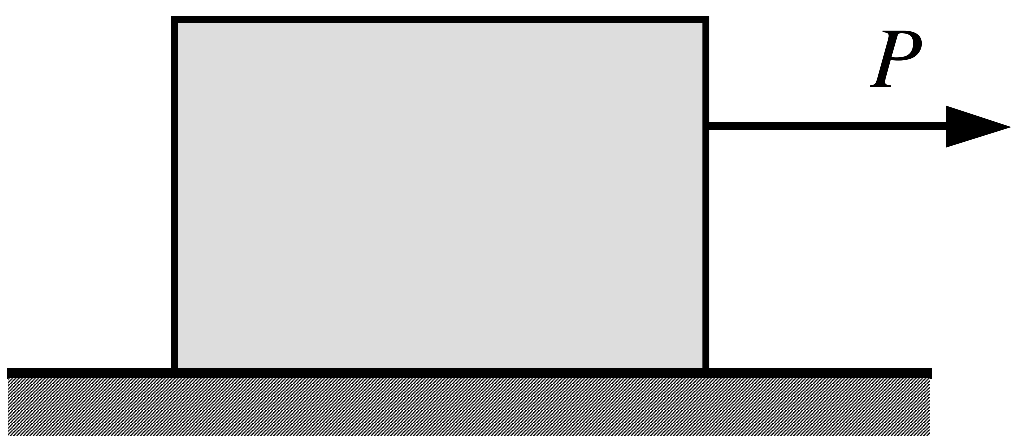
Lubrication problems are completely different, and come under the purview of fluid mechanics.

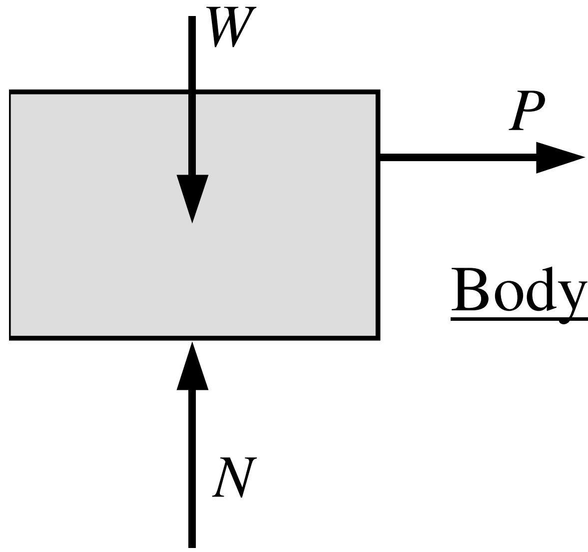
Major cause of dry friction is associated with microscopic roughness of the surface of contact.

Interlocking microscopic protuberances oppose the relative motion between the surfaces.

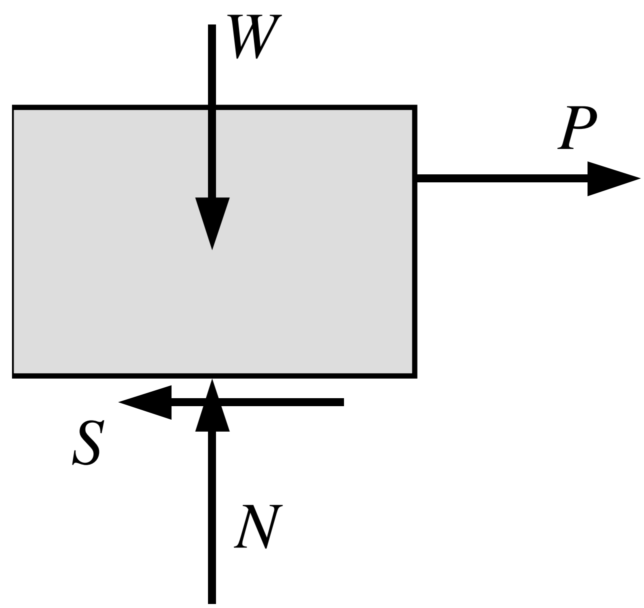
During sliding, some of these protuberances are sheared off and some are melted—which explains the high rate of “wear” during dry friction.

A “smooth” surface of contact can only support a normal force; a “rough” one in addition supports a tangential force.





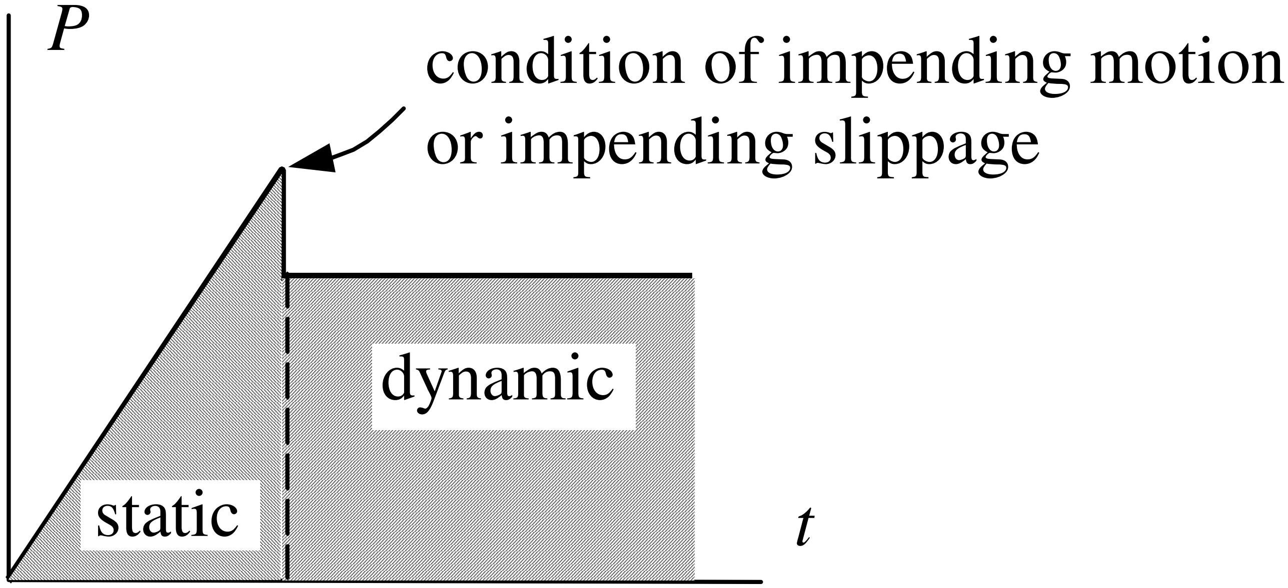
Body on a smooth surface



Body on a rough surface

Laws of Coulomb Friction

When we slide a block of wood over a surface, we exert continuously increasing load which is completely resisted by friction until the object starts moving, usually with a lurch.



Idealised plot of applied force P

Coulomb in 1781 presented certain conclusions after carrying out experiments on blocks which tend to move or are moving without rotation.

These conclusions are applicable at the *condition of impending slippage* or once *slippage* has begun.

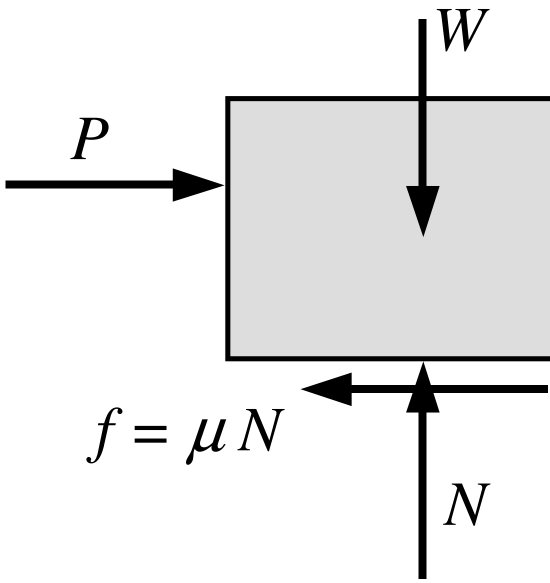
These are known as *Coulomb's laws of friction.*

The laws of Coulomb friction are:

1. The total force of friction is independent of area of contact.
2. For low relative velocities between sliding objects, the frictional force is independent of velocity. However, the sliding frictional force is less than the frictional force corresponding to impending slippage.
3. The total frictional force is proportional to the normal force transmitted across the surface of contact.

Static Coefficient of Friction	
Steel on cast iron	0.4
Copper on steel	0.36
Hard steel on hard steel	0.42
Mild steel on mild steel	0.57
Rope on wood	0.7
Wood on wood	0.20 – 0.75

Thus, the frictional force *f* is proportional to the normal reaction. That is

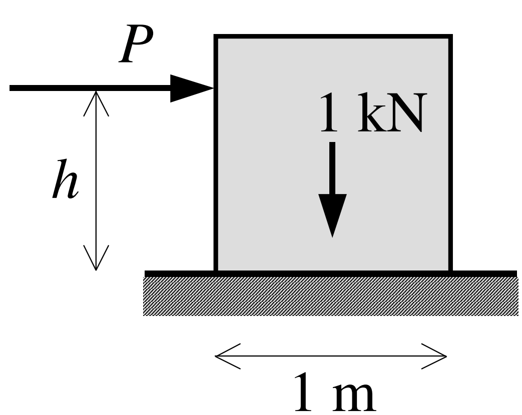


$$f \propto N$$

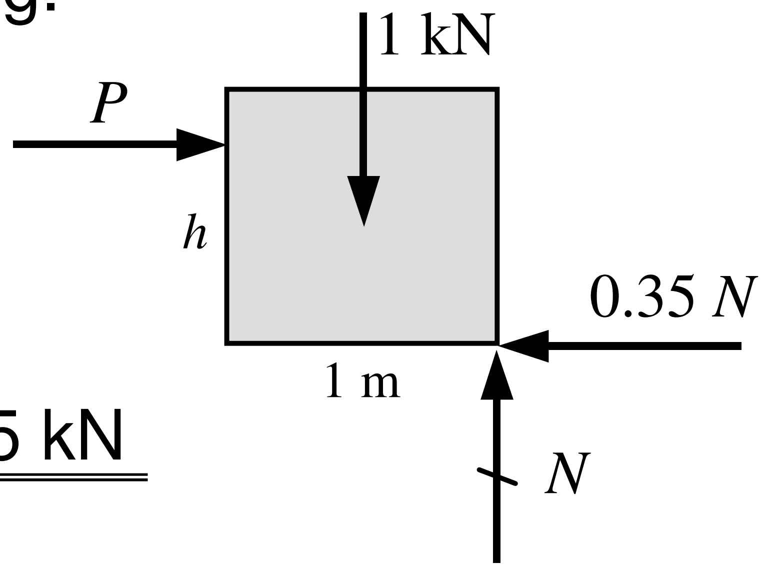
$$f = \mu N$$

Note: Direction of *f* is such that it opposes the motion.

Example 1: A heavy box weighing 1 kN rests on a floor. The static coefficient of friction between the surfaces is 0.35. What is the largest force *P*, and what is the highest position *h* for applying this force that will not allow the box either to *slip* or to *tip*?



Consider the free body diagram shown below corresponding to the condition of impending tipping.



$\Sigma F_y = 0$:

$\underline{\underline{N = 1\text{ kN}}}$

$\Sigma F_x = 0$:

$P = 0.35N = \underline{\underline{0.35\text{ kN}}}$

$\Sigma M_A = 0$:

$Ph = 1 \times 0.5;\ \therefore\ h = 0.5/0.35 = \underline{\underline{1.4\text{ m}}}$

$h < h_{max}$ to avoid tipping.

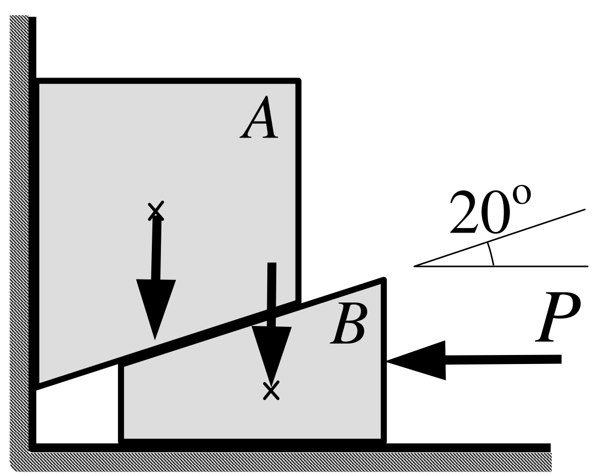
Example 2: Will P hold block A in equilibrium, move block A up or is it too small to prevent A from coming down?

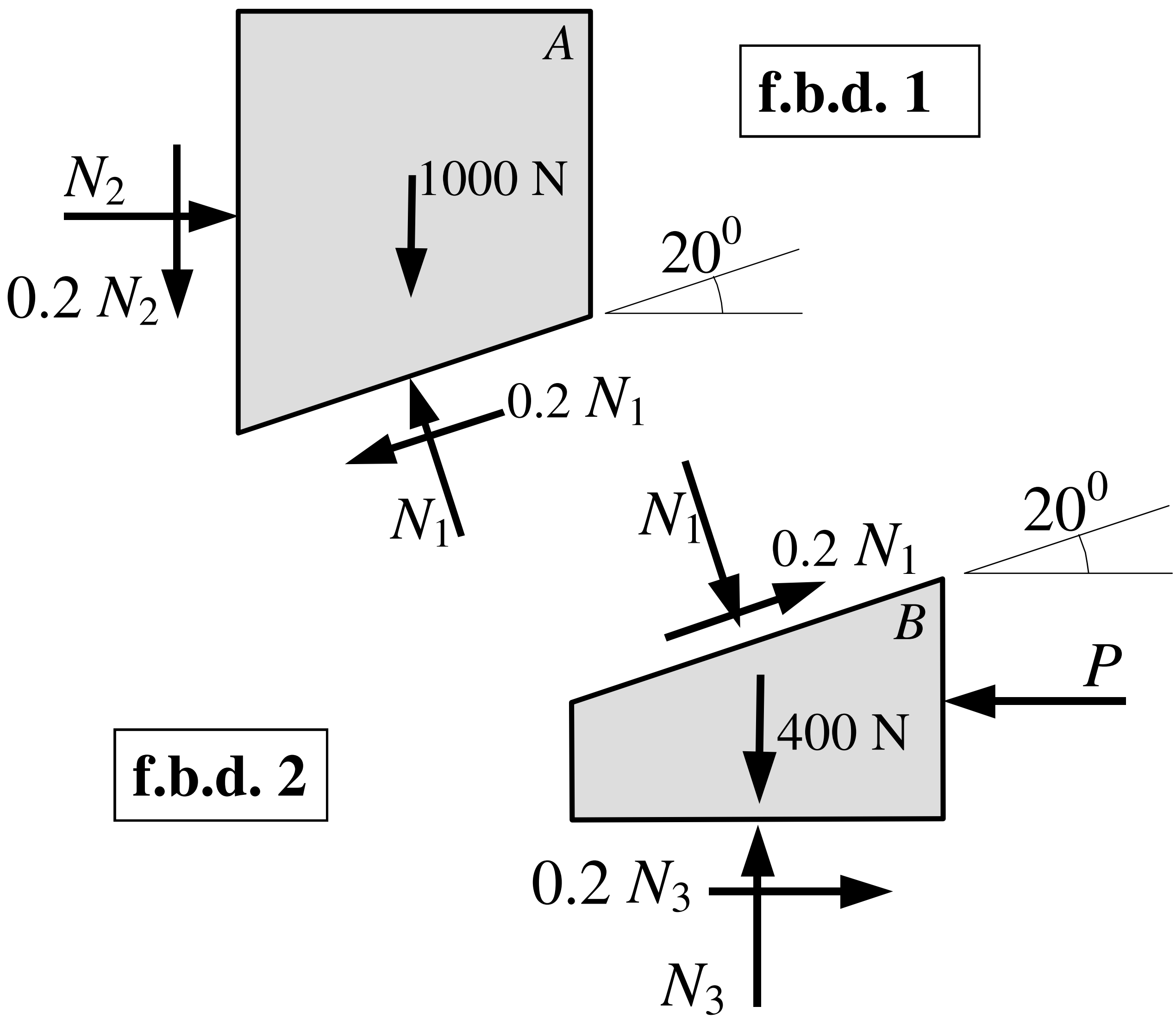
$W_A = 1000$ N

$W_B = 400$ N

$P = 250$ N

$\mu = 0.2$ for all contact surfaces.

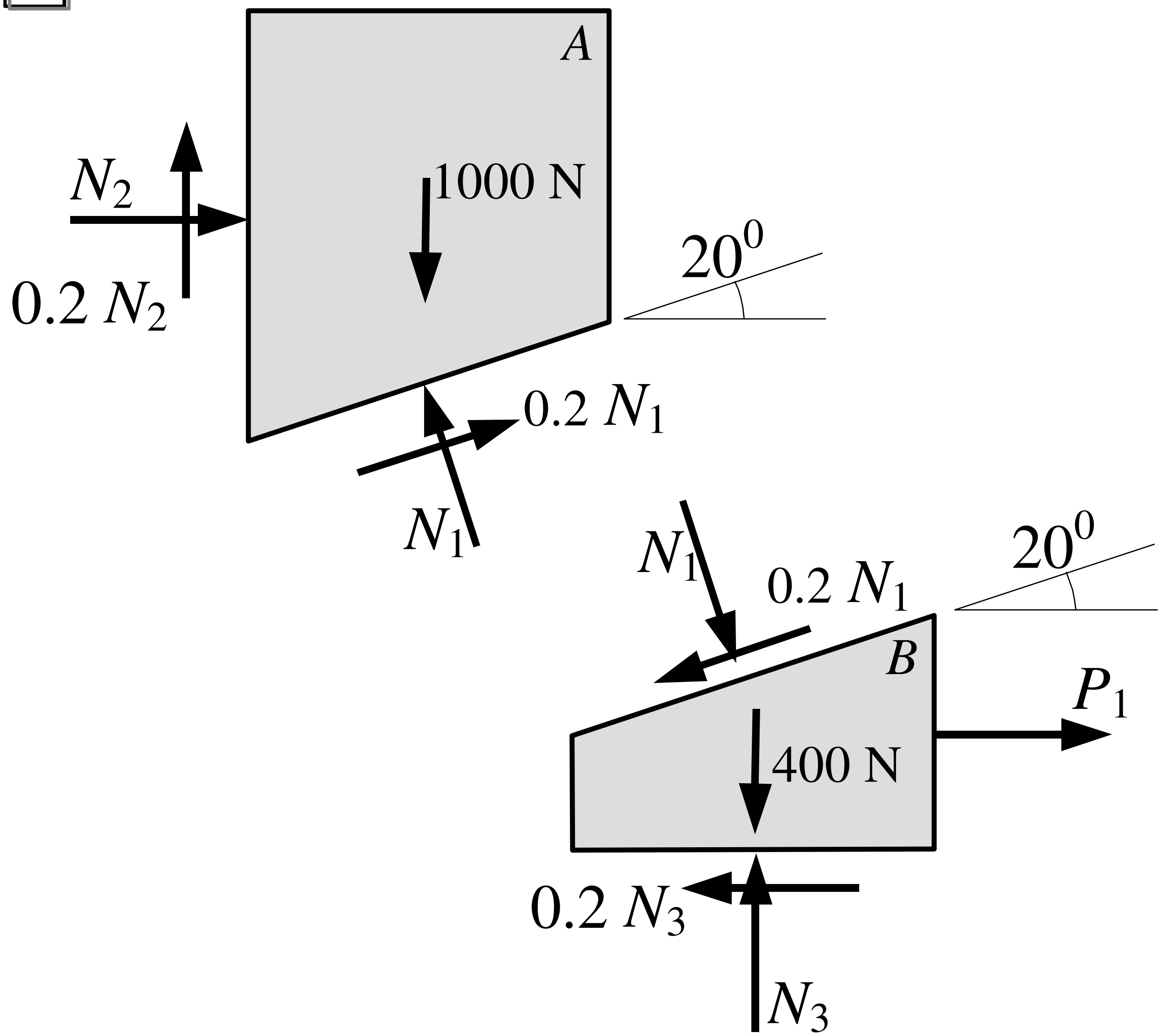




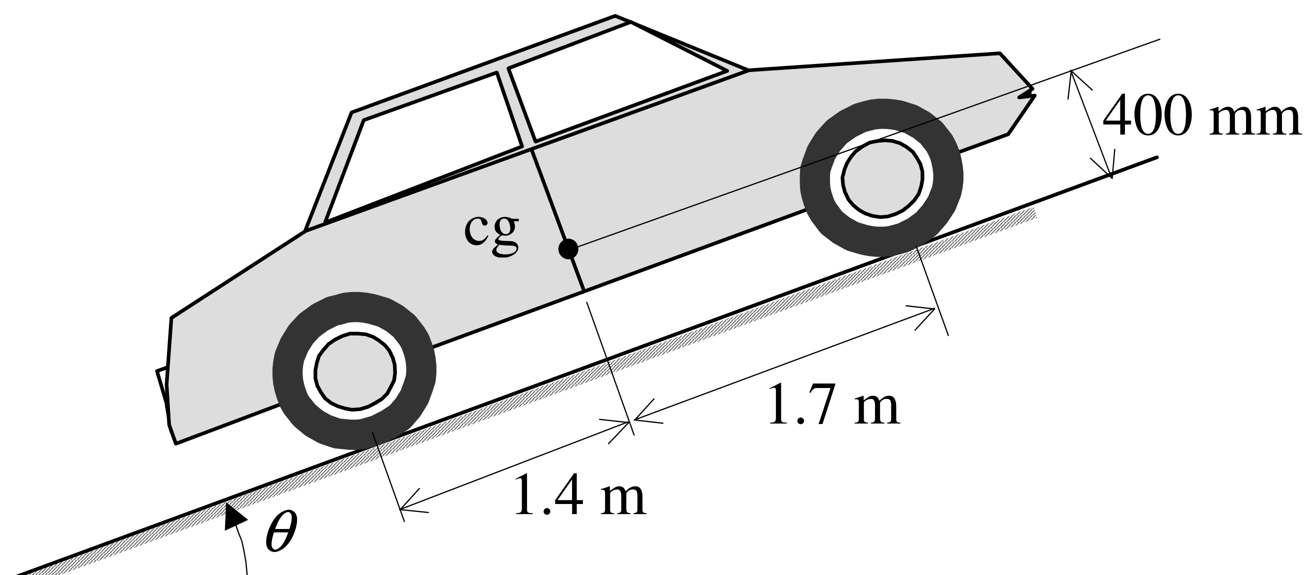
Find N_1 and N_2 from f.b.d 1.

f.b.d 2 corresponds to *“B”* moving to left. Find P.

If *“B”* moves to the right, determine P_1 (reverse directions of P and the frictional forces).

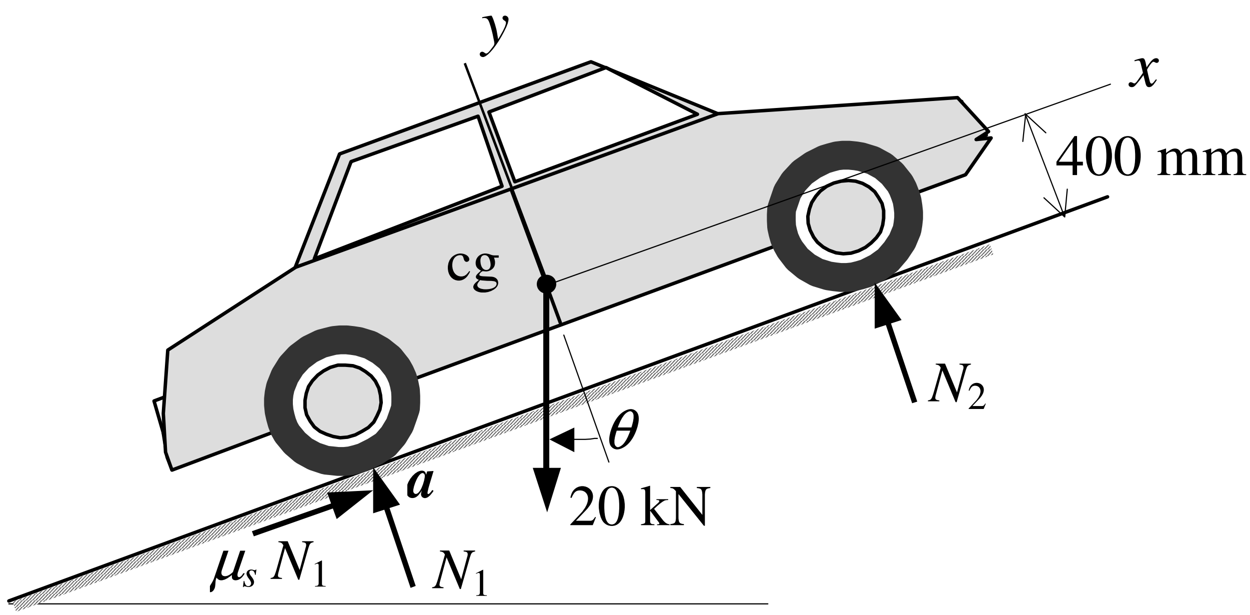

A
N_2
1000 N
20^0
$0.2\ N_2$
$0.2\ N_1$
N_1
N_1
$0.2\ N_1$
20^0
B
P_1
400 N
$0.2\ N_3$
N_3

Example: **(a)** An automobile is shown in the figure below on a roadway inclined at an angle θ with the horizontal. If the coefficient of static and dynamic friction between the tires and the road are 0.6 and 0.5 respectively, what is the maximum inclination θ_{max} that the car can climb at uniform speed? It has rear-wheel drive and has a total loaded weight of 20 kN. The centre of gravity for this loaded condition has been shown in the diagram.



We assume that the drive wheels do not spin (i.e., there is no slip between the wheels and the road surface).

The maximum friction force possible is μ_s times the normal force as shown below:



$$\Sigma F_x = 0: \quad 0.6\, N_1 = 20 \sin \theta_{max} \qquad \textbf{(i)}$$

$$\Sigma F_y = 0: \quad N_1 + N_2 = 20 \cos \theta_{max} \qquad \textbf{(ii)}$$

$$\Sigma M_a = 0:$$

$$3.1\, N_2 = 20 \cos \theta_{max} \times 1.4 - 20 \sin \theta_{max} \times 0.4 \qquad \textbf{(iii)}$$

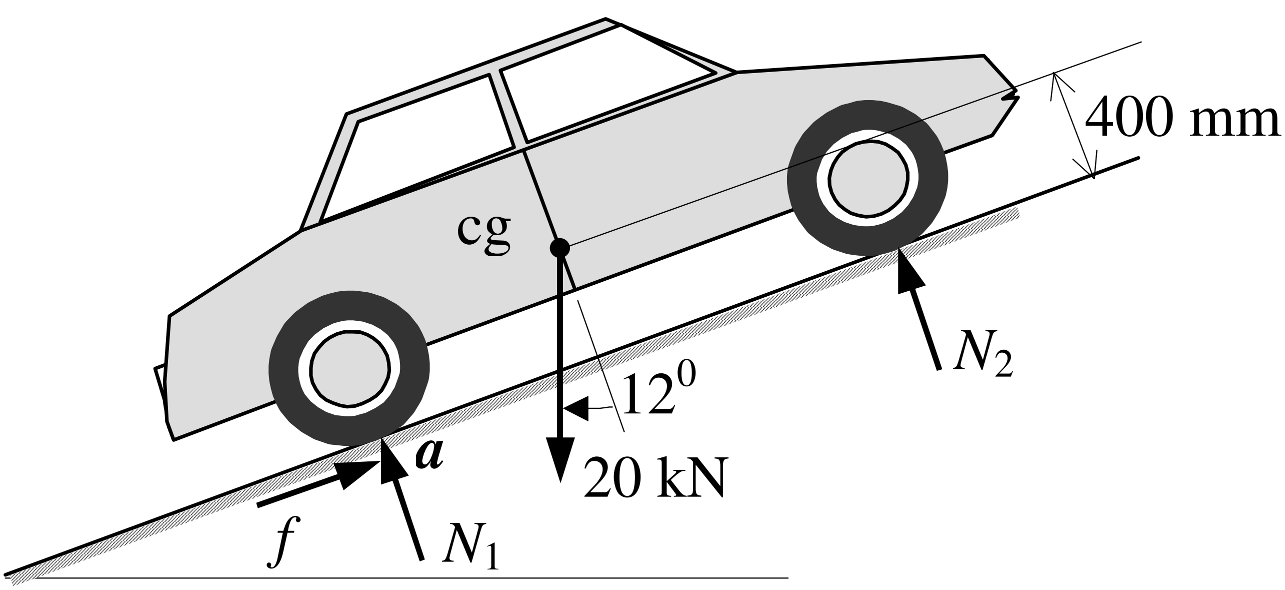
Eliminate N_1 from (i); then eliminate N_2 from (ii); put these in (iii) and simplify to get $\tan\,\theta_{\max} = 0.3566.$

Hence, $\underline{\underline{\theta_{\max} = 19.628^0}}$

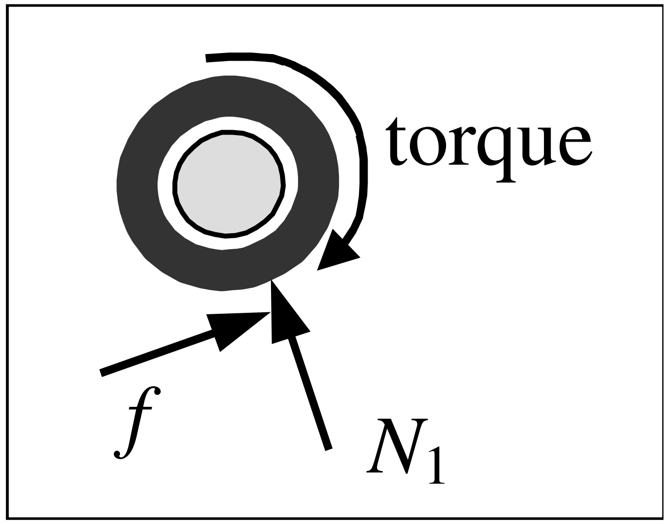
If the drive wheels spin, we need to use μ_d in the place of μ_s.

We will get a smaller value for $\theta_{\max}$ in that case ($\theta_{\max} = 15.33^0$).

(b) Using the above data, compute the torque needed by the drive wheels to move the car at a uniform speed up an incline with $\theta = 12^0$. Wheel dia = 450 mm.



In this case, the friction force *f* has to be determined from Newton's laws (and not by Coulomb's) as there is no impending slippage between the wheel and the road.



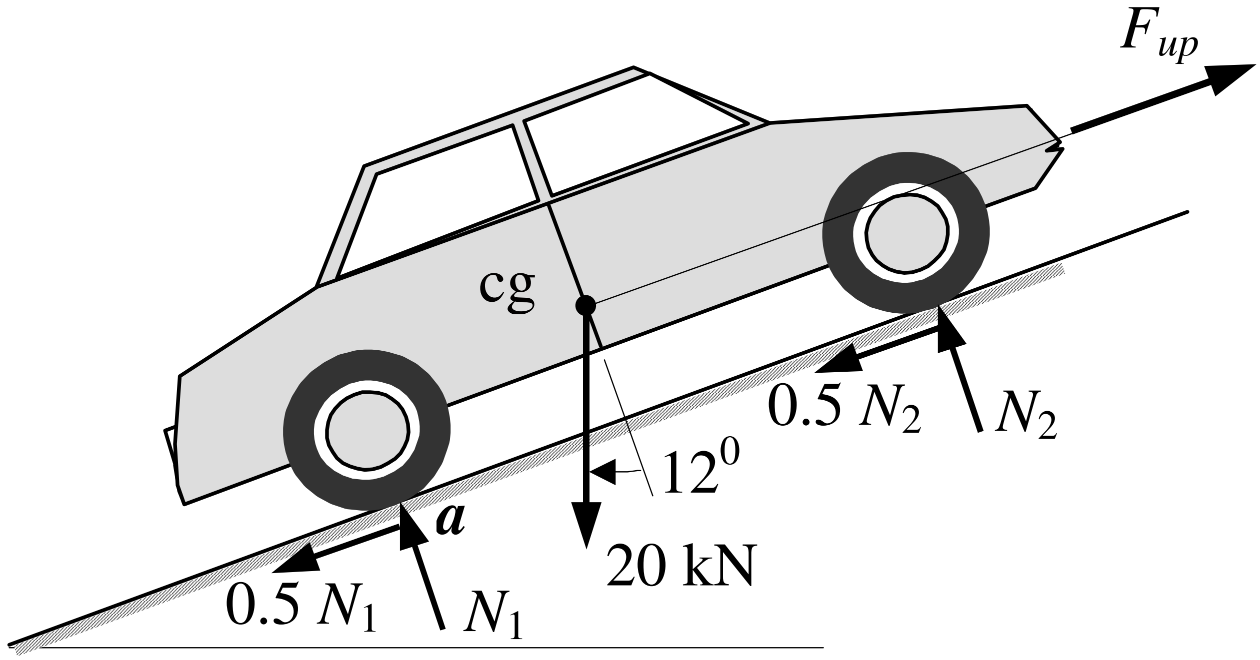
$\Sigma F_x = 0$: $f = 20 \sin 12^0$

Hence, $f = 4.158$ kN.

Now, looking at the free body diagram of the wheel alone, we get

torque $f \times r = 4.158 \times .45 = \underline{\underline{1.87 \text{ kNm}}}$

(c) What force is needed to tow the car up the incline when it is parked with brakes locked on all the four wheels with $\theta = 12^0$? What force is necessary to tow it down the incline?

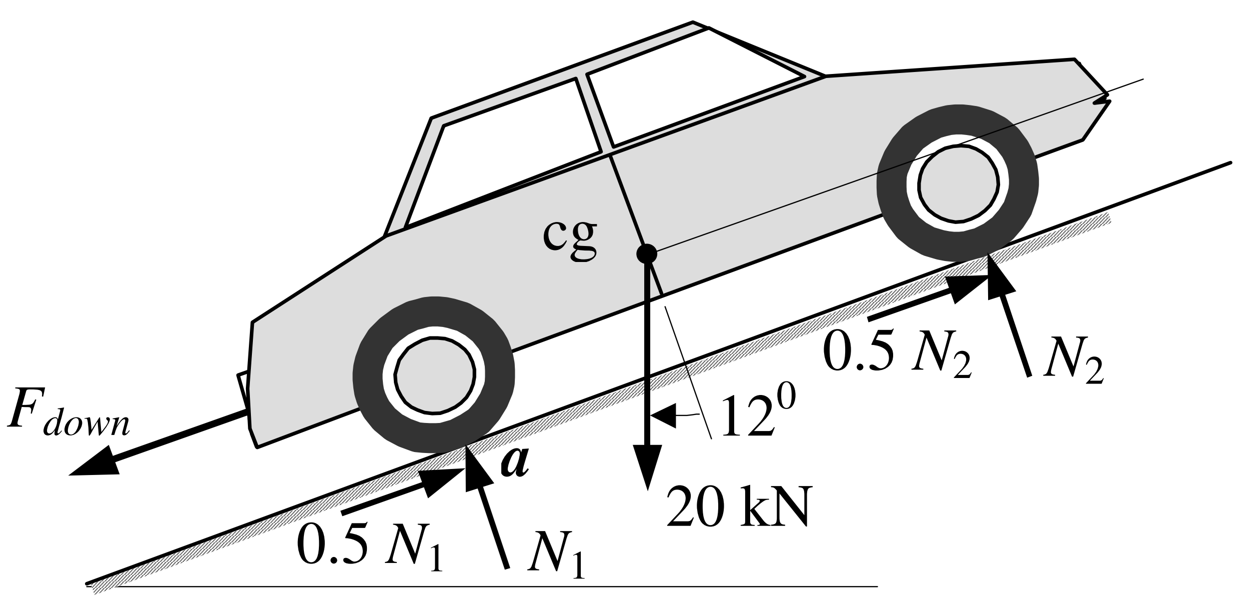


$$N_1 + N_2 = 20 \cos 12^0$$

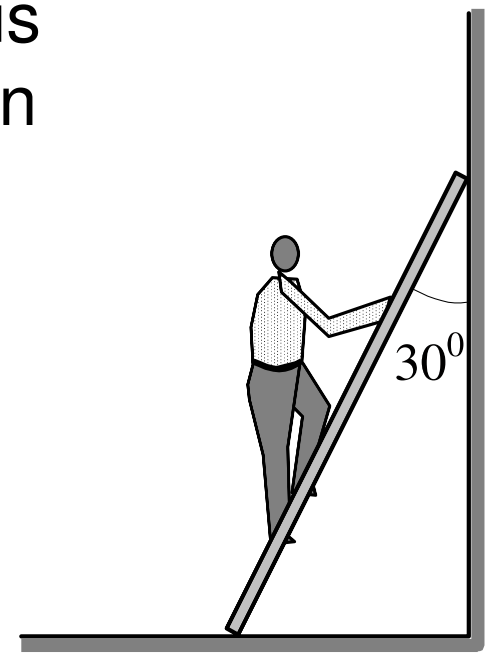
$$F_{up} = 0.5(N_1 + N_2) + 20 \sin 12^0$$

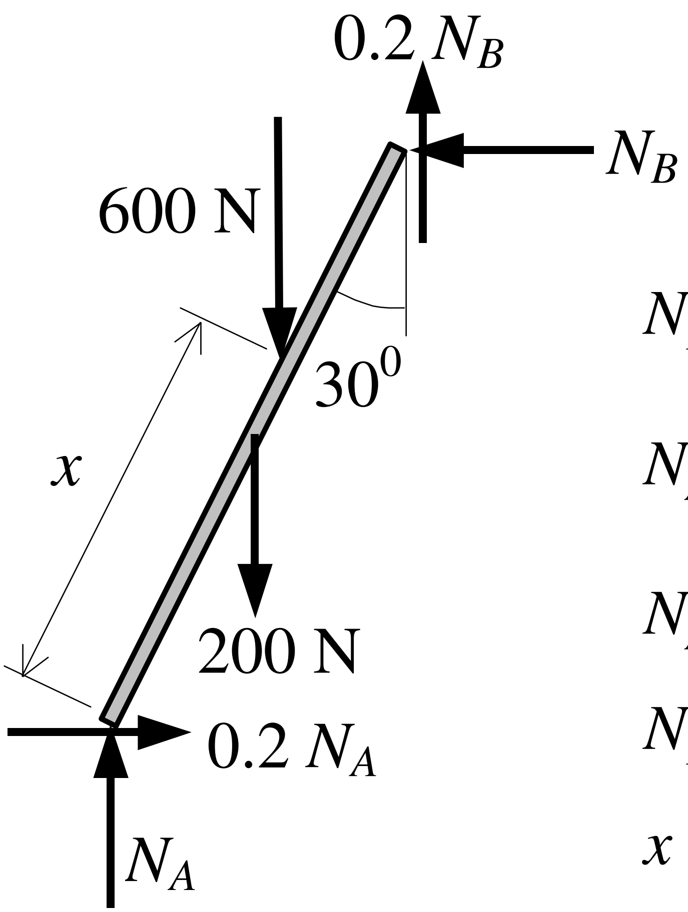
$$= \underline{\underline{13.94 \text{ kN}}}$$

Similarly calculate F_{down}. (= 5.623 kN)



Exercise: A ladder 4 m long and 200 N weight is placed as shown. A man weighing 600 N climbs up the ladder. At what position will he induce slipping? Coefficient of static friction for all contacts is 0.2.





$$N_B = 0.2\,N_A$$

$$N_A + 0.2N_B = 800\text{ N}$$

$$N_A = 769.231\text{ N}$$

$$N_B = 153.846\text{ N}$$

$$x =$$

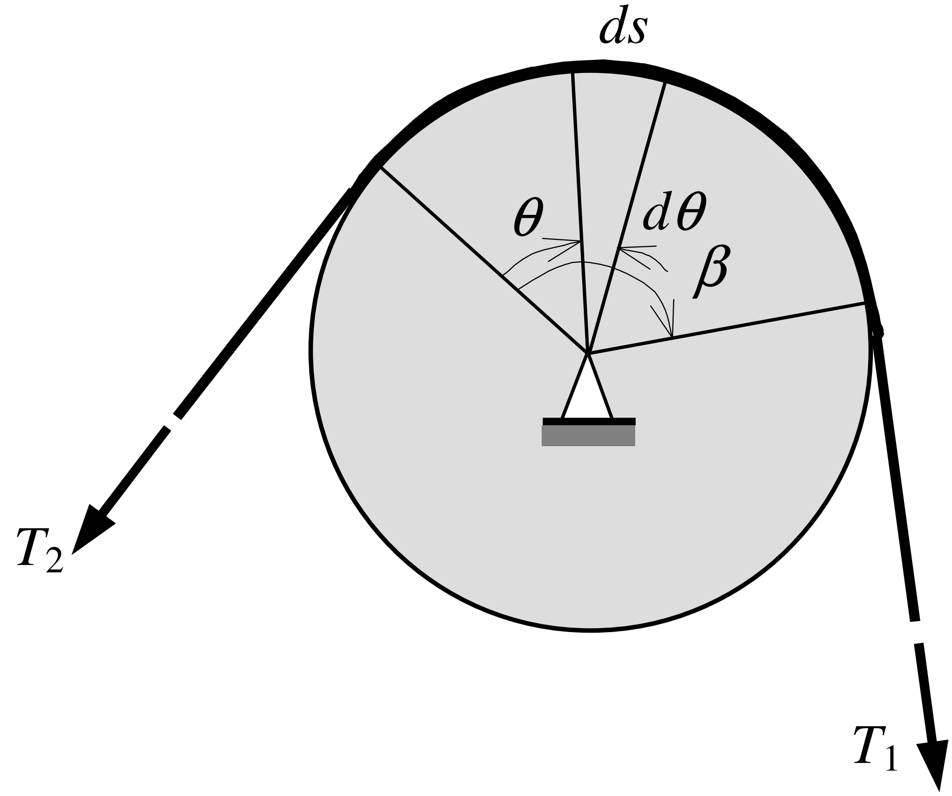
Belt Friction

Consider the flexible belt shown in figure wrapped around a portion of a drum.

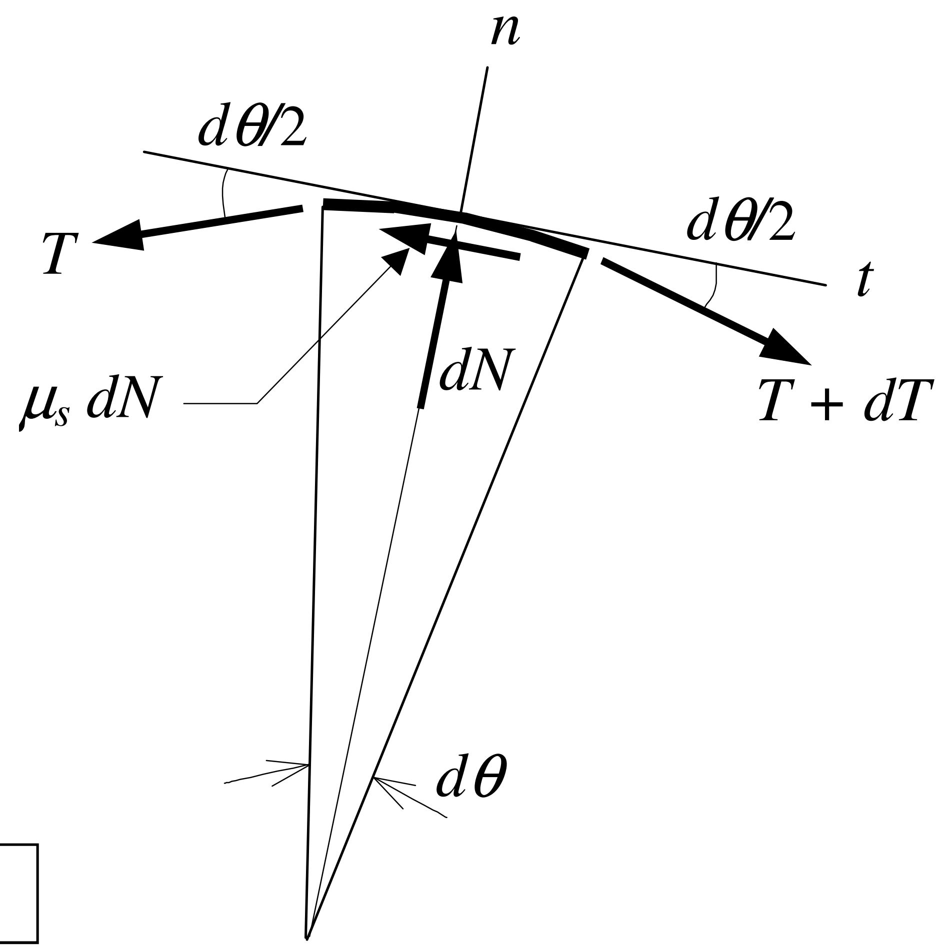
The *angle of wrap* is β.

Assume that the drum is stationary and the belt is just about to move (impending motion).

Also, $T_1 > T_2$ (and hence the belt moves clockwise with respect to the drum).



Consider an infinitesimal segment of the belt as a free body diagram.



$\boxed{\Sigma F_t = 0:}$

$$-T\cos\frac{d\theta}{2} + (T + dT)\cos\frac{d\theta}{2} - \mu_s dN = 0$$

or $\qquad \underline{\underline{dT = \mu_s\, dN}} \qquad$ **(i)**

$\boxed{\Sigma F_n = 0:}$

$$-T\sin\frac{d\theta}{2} - (T + dT)\sin\frac{d\theta}{2} + dN = 0$$

or $\qquad \underline{\underline{T\, d\theta = dN}} \qquad$ **(ii)**

$$\frac{dT}{T} = \mu_s d\theta$$

Integrating the above, we obtain

$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^{\beta} \mu_s d\theta$$

That is

$$\ln \frac{T_1}{T_2} = \mu_s \beta$$

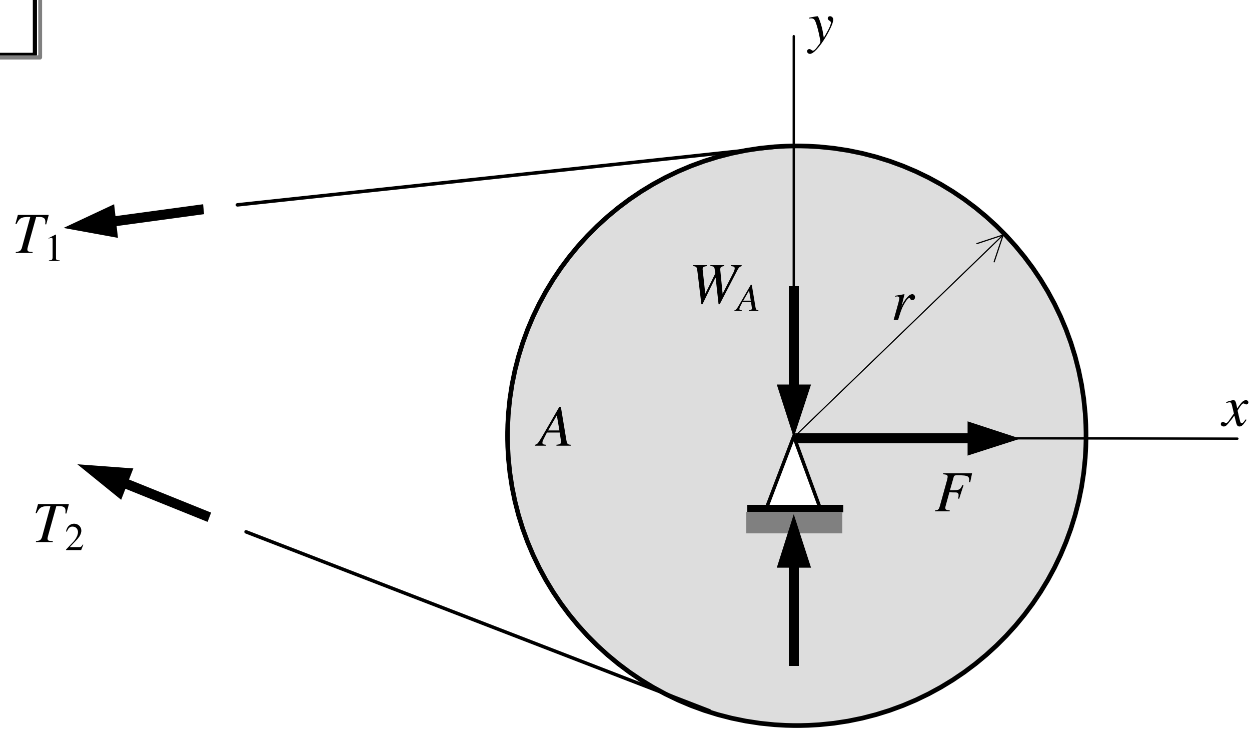
Or

$$\boxed{\frac{T_1}{T_2} = e^{\mu_s \beta}}$$

Thus, the ratio of tensions depend only on the angle of wrap β.

Note: (i) β should be in radians.

(ii) Appropriate μ should be used.



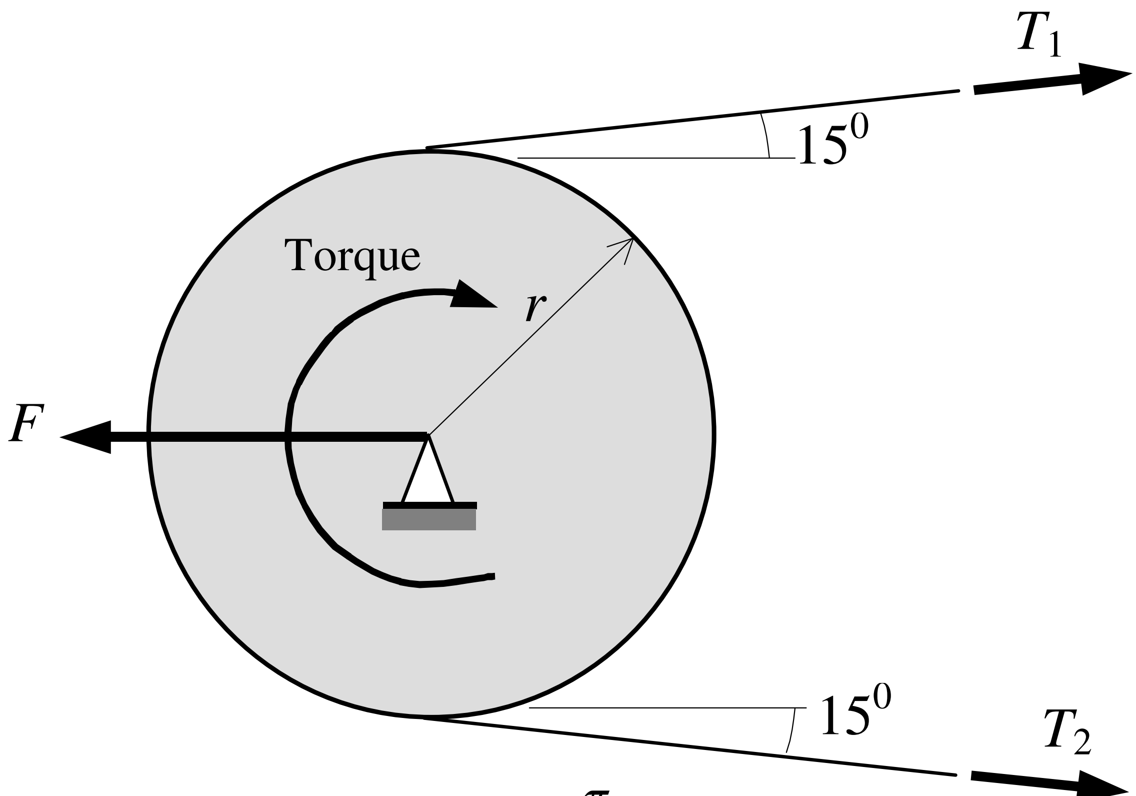
If the drum *A* is forced to the right, as long as the angle of wrap β remains unchanged, the ratio of T_1/T_2 for impending or actual constant speed slippage will remain unaffected.

The torque developed by the belt on the drum as a result of friction *is affected* by the force *F.*

The torque is obtained as

$$\text{torque} = T_1\,r - T_2\,r = (T_1 - T_2)\,r$$

Example: A drum having a radius of 400 mm as shown in the figure below needs a torque of 500 Nm to get it to start rotating. If μ_s = 0.25, determine the minimum axial force F needed to create sufficient tension to start the rotation of the drum.



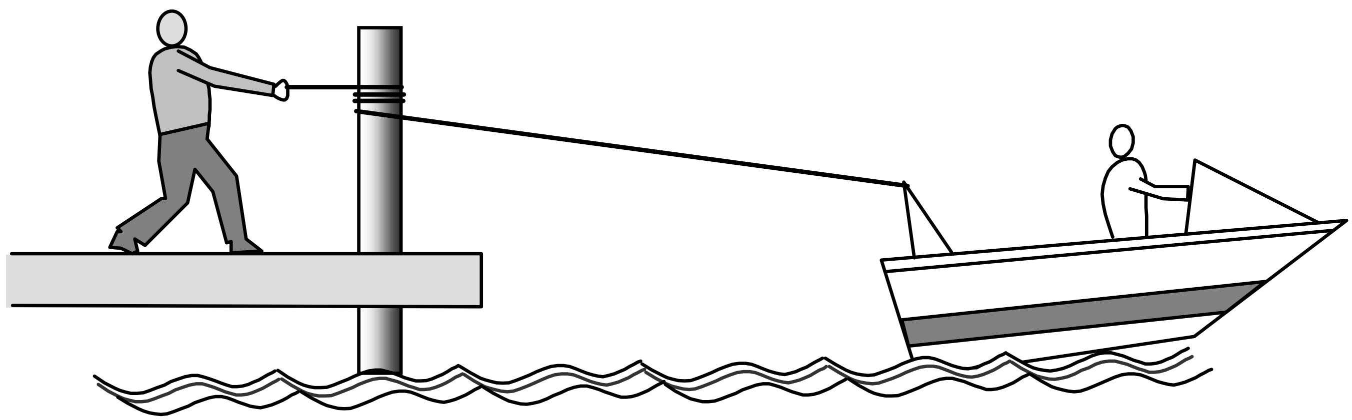
$$\frac{T_1}{T_2} = e^{\mu_s \beta} = e^{0.25 \times 150 \times \frac{\pi}{180}}$$

Torque = 500 Nm = $(T_1 - T_2)\, r$

Hence get T_1 and T_2; then

$F = (T_1 + T_2) \cos 15^0$

Example: The seaman pulls with 100 N force and wants to stop the motor boat from moving away from the dock. How few wraps *n* of the rope must he make around the post if the motor boat develops a thrust of 3500 N. (μ_s = 0.2 between rope and the post.)

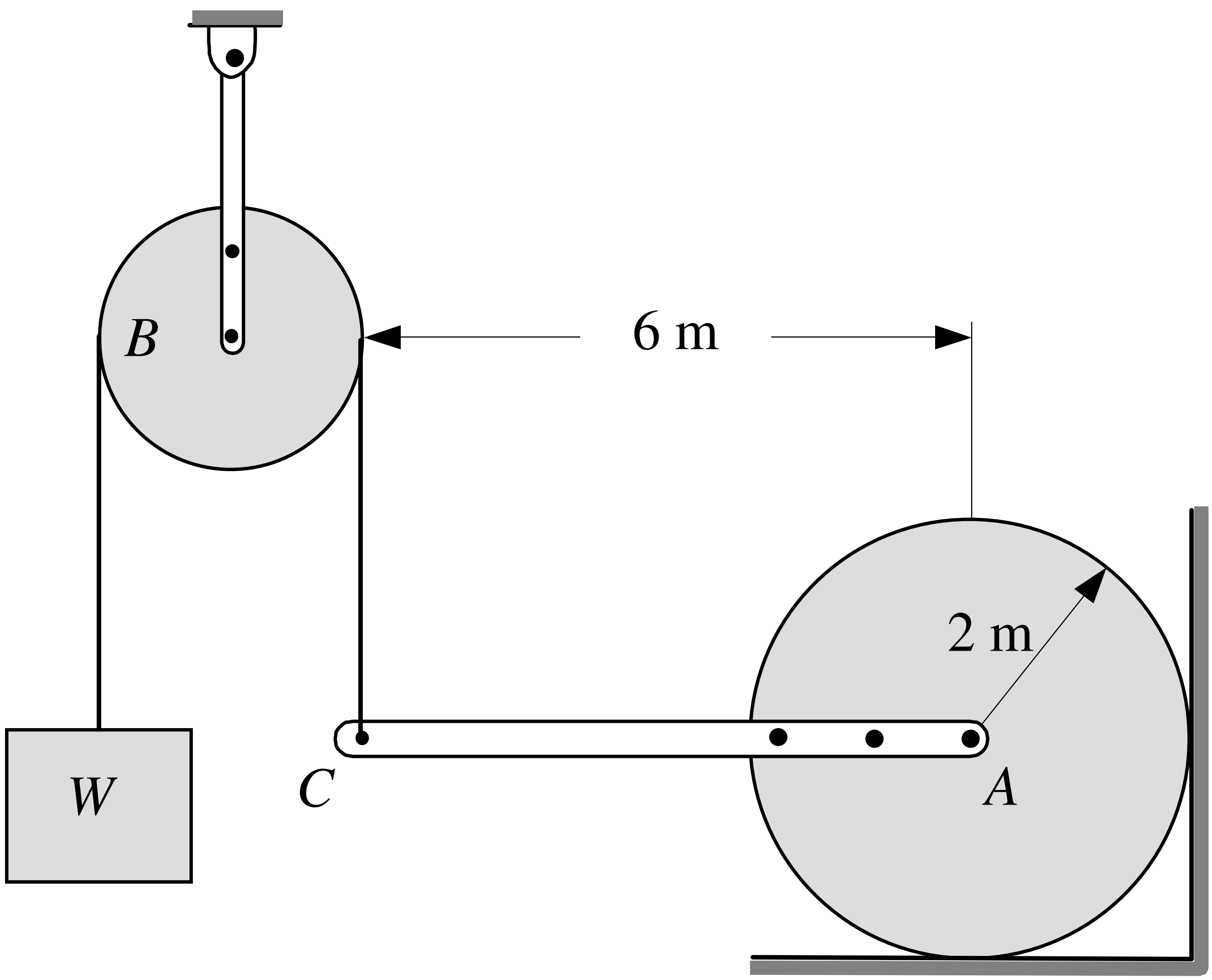


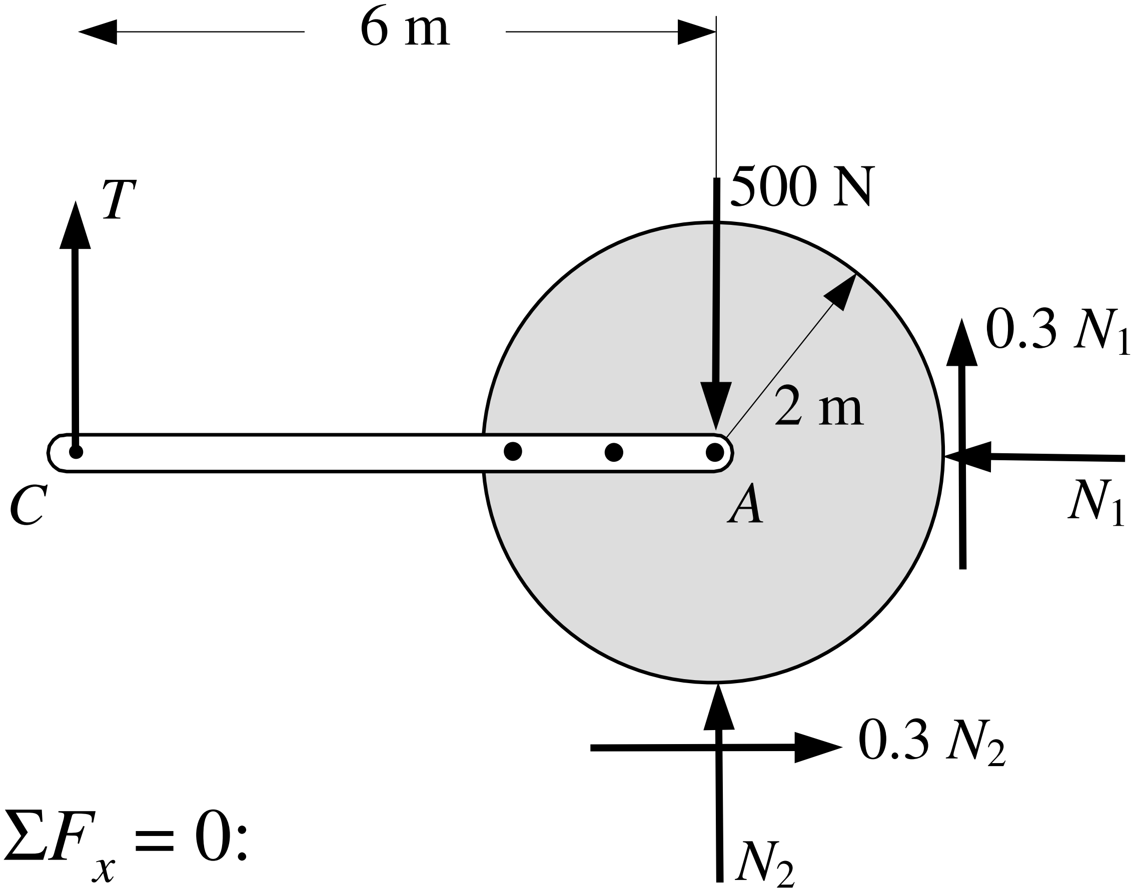
$$\frac{T_1}{T_2} = e^{\mu_s \beta}$$

T_1 = 3500 N; T_2 = 100 N

Hence β = 17.776^0 = 2.83 wraps.

Example: What is the maximum weight W that can be supported by the system in the position shown? Pulley B cannot turn. Bar AC is fixed to cylinder A, which weighs 500 N. $\mu_s = 0.3$ for all contact surfaces.





$\Sigma F_x = 0$:

$0.3\ N_2 = N_1$

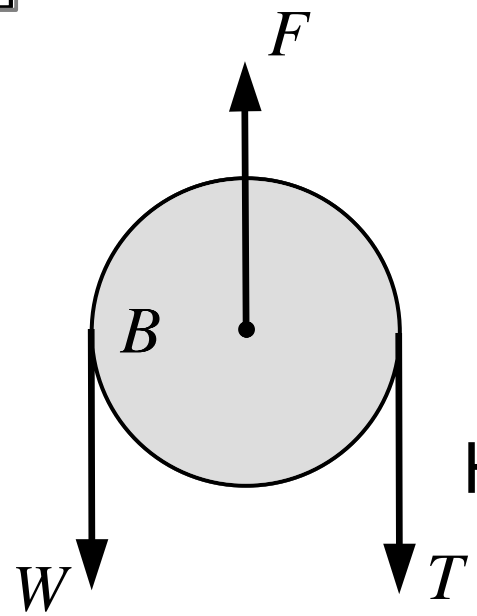
$\Sigma F_y = 0$: $0.3\ N_1 + N_2 + T = 500$

$\therefore\ N_2 = (500 - T)/1.09$

$\Sigma M_a = 0$: $6\ T = (0.3\ N_1 + 0.3\ N_2)$

$= 0.78\ N_2$

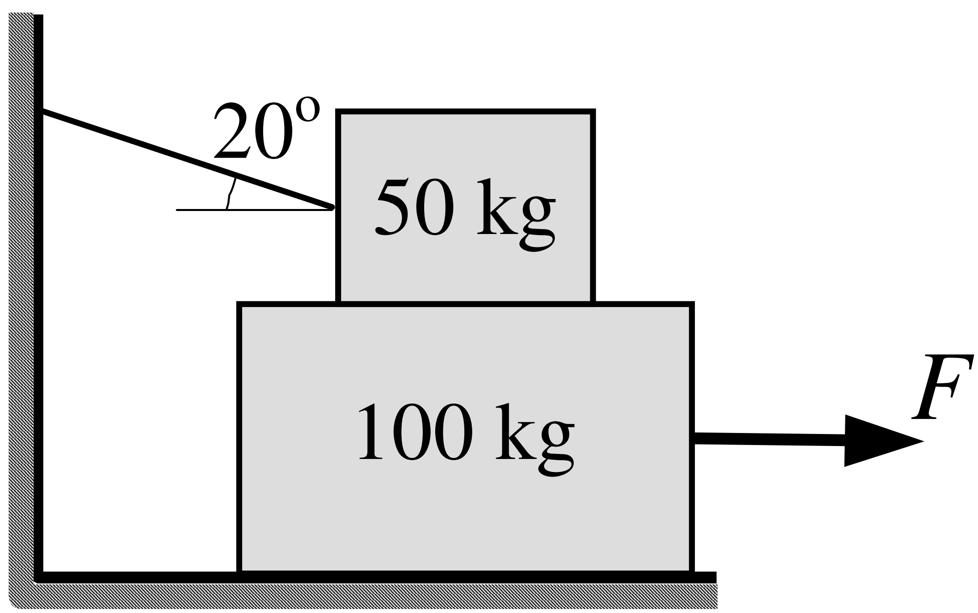
$\therefore\ \underline{\underline{T = 53.278\text{ N}}}$



$$\frac{W}{T} = e^{0.3\pi}$$

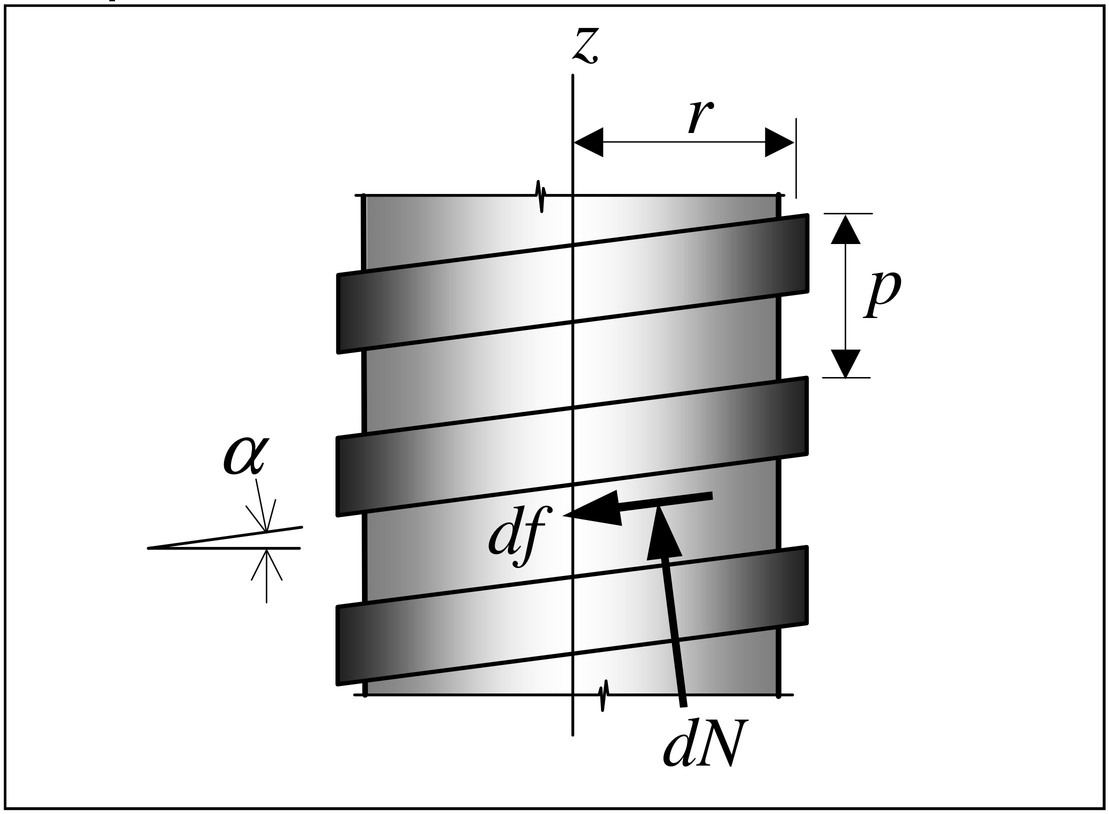
Hence, $\underline{\underline{W = 136.731 \text{ N}}}$

<u>Example</u>: Find the force F needed to start the block of mass 100 kg moving to the right if $\mu_s = 0.35$.



The Square Screw Thread

Consider the action of a nut on a screw with square screw thread.



r is the mean radius from the centre-line of the screw thread;

p is the *pitch* (distance along screw between adjacent threads);

L is the *lead* (distance the nut will advance in the direction of axis in one revolution).

And for an *n*-threaded screw, $L = np$

Forces are transmitted from screw to nut over several revolutions of the thread.

Hence, we have a distribution of normal and friction forces.

Due to the narrow width, we may assume that the distribution is confined to the centre-line, thus forming a line load distribution over several turns of the thread.

The local slope is given by

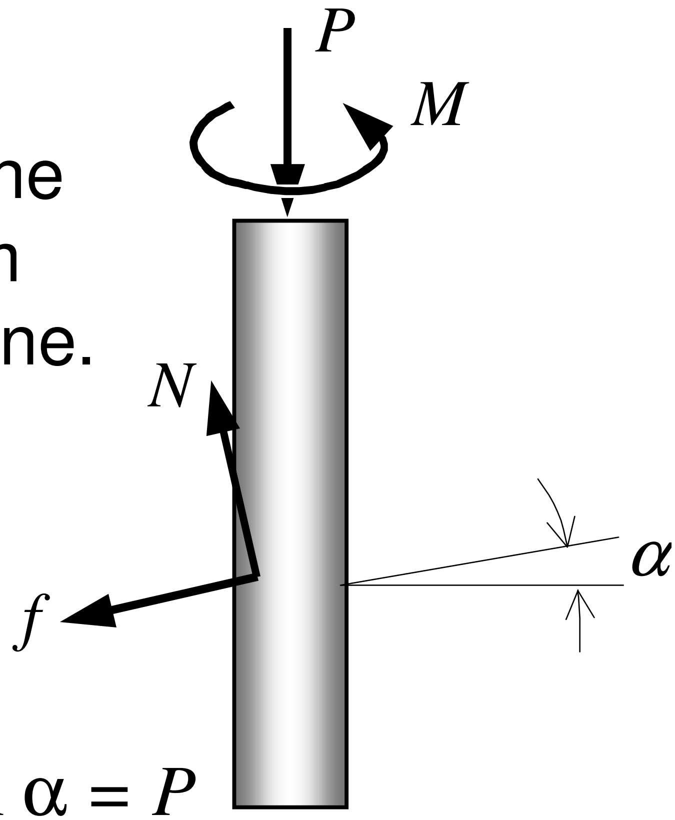
$$\tan\alpha = \frac{L}{2\pi r} = \frac{np}{2\pi r}$$

Infinitesimal normal and friction forces *dN* and *dS* have been indicated in figure.

All elements of the distributed forces have the same inclination with respect to the *z*-direction.

Hence, the force distribution could be replaced by a single normal force *N* and a single friction force *f* at the inclinations shown in figure below.

Note also that elements of the distribution have the same moment arm about the centre-line.



$$\Sigma F_z = 0:$$
$$N\cos\alpha - \mu_s N\sin\alpha = P$$

$$\Sigma M_z = 0:$$
$$M_z = r\mu_s N\cos\alpha + rN\sin\alpha$$

These equations are used to eliminate the force *N* and to get a relation between *P* and M_z.

Thus we get

$$M_z = \frac{P\,r(\mu_s \cos\alpha + \sin\alpha)}{\cos\alpha - \mu_s \sin\alpha}$$

Is the screw *self-locking* is an important question. That is, having raised the load, if the torque M_z is released will the load *P* stay at the raised position or will it unwind under the action of the load?

For this, go back to the equations of equilibrium.

Set $M_z = 0$, and simultaneously reverse the direction of friction forces.

Eliminating *N*, we get as the condition for impending unwinding of the screw as:

$$\frac{P\,r(-\mu_s \cos\alpha + \sin\alpha)}{\cos\alpha + \mu_s \sin\alpha} = 0$$

which requires that

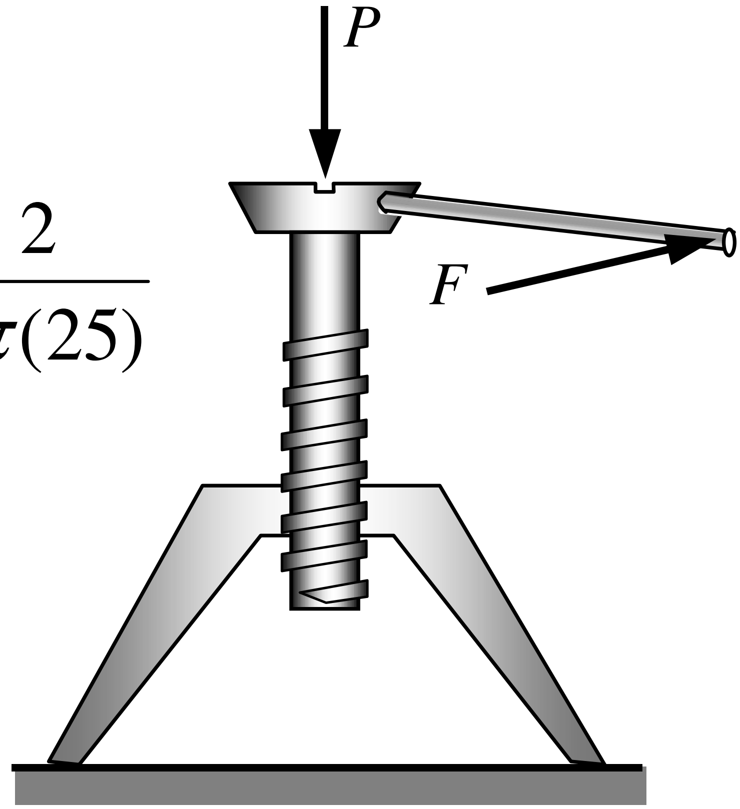
$$-\mu_s \cos\alpha + \sin\alpha = 0$$

That is, $\underline{\underline{\mu_s = \tan\alpha}}$

Thus, if the coefficient of friction μ_s equals or exceeds tan α, the screw is of self-locking type; else it will unwind under the load.

<u>Example</u>: A single threaded jackscrew has a pitch of $p = 2$ mm and a mean diameter of 50 mm. What torque M is needed to lift a load $P = 5000$ N up? If the torque is released, will the load stay in the raised position? $\mu_s = 0.25$.

$$\tan\alpha = \frac{p}{2\pi r} = \frac{2}{2\pi(25)}$$



Hence, $\alpha = 0.729^0$

$$M_z = \frac{P\,r(\mu_s \cos\alpha + \sin\alpha)}{\cos\alpha - \mu_s \sin\alpha}$$

$$= \frac{5000\times 25\times(0.25\cos\alpha + \sin\alpha)}{\cos\alpha - 0.25\sin\alpha}$$

$$= 32946.42\,\text{Nmm}$$

It is self-locking as $\mu_s > \tan\ \alpha$.