Monsoon Semester 2022

Mid-Sem Exam, 11 Oct 2022

CE6101D Theory of Elasticity and Plasticity

Duration: 2 hours   Maximum Marks: [30]

Note: Answer all questions; Provide neat sketches; Assume missing data; Read questions carefully

1. Derive the *force* and *moment* equations of equilibrium of a body subjected to body forces and surface forces.   [4]

2. At a point in a body, the stress tensor with respect to *xyz* coordinates is given by

$$\sigma = \begin{bmatrix} 30 & -10 & 20 \\ -10 & 40 & 15 \\ 20 & 15 & 50 \end{bmatrix} \text{MPa.}$$

   Find: (a) The *normal* and *shear stresses* at the point on a plane whose outward normal is **n** = 0.4 **i** + 0.6 **j** – 0.69282 **k**; and (b) The *stress invariants* and the *principal stresses* at the point.   [6]

3. The stress distribution in an elastic body in MPa is described by the following stress components

$$\sigma_y = 3yz + 2y^2; \ \sigma_z = 10(z^2 - yz); \ \tau_{yz} = 3(y^2 + z - 10); \text{ and } \sigma_x = \tau_{xz} = \tau_{xy} = 0.$$

   Evaluate the distribution of body forces **b** necessary to keep the above stress field in equilibrium. (Hint: Use the equations of equilibrium)   [3]

4. Derive the strain-displacement relations for the normal strain ε_z and shearing strain γ_{yz}.   [3]

5. (a) For a homogeneous isotropic linearly elastic body write the following in index notation: (i) the equations of equilibrium, (ii) stress-strain relations and (ii) strain-displacement relations.

   (b) Combine them to obtain the equations of equilibrium in terms of displacements.   [4]

6. The *compatibility equation* is given by

$$\varepsilon_{ij,kl} + \varepsilon_{kl,ij} - \varepsilon_{ik,jl} - \varepsilon_{jl,ik} = 0.$$

   Use it to obtain the corresponding *six* compatibility conditions in longhand notation.   [3]

7. (a) Define an *isotropic tensor*? (b) Write the transformation rule for a fourth order tensor.

   (c) Use it to show that the fourth order tensor $D_{ijkl} = \alpha \ \delta_{ij} \ \delta_{kl} + \beta \ \delta_{ik} \ \delta_{jl} + \gamma \ \delta_{il} \ \delta_{jk}$ is isotropic. [4]

8. If **A** and **B** are square matrices (of size 3) and **X**, **Y**, **Z** are vectors, *denote* the following using index notation: (i) **X·Y**, (ii) **X×Y**, (iii) **X×Y·Z**, (iv) $\mathbf{X}^T(\mathbf{A} + \mathbf{B})\mathbf{X}$ and (v) (**X×Y**)·(**Y×Z**).   [5]

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