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NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

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# CE6101 THEORY OF ELASTICITY AND PLASTICITY

**Note:** (a) *Answer* **all** *questions* (b) Suitably assume any missing data (c) Read questions carefully before answering

1. The stress tensor at a point with respect to *xyz* coordinates is given by

$$\sigma = \begin{bmatrix} 40 & 20 & 30 \\ 20 & -60 & 0 \\ 30 & 0 & -70 \end{bmatrix} \text{MPa.}$$

   Find (a) the normal and shear stresses on a plane with outward normal **n** = [0.4,–0.6,–0.69282], (b) the stress invariants, and (c) the principal stresses at the point.

2. The stress distribution in an elastic body is described by the following stress components: $\sigma_x = 4x^2y + 3y^2z$; $\sigma_y = 10(x^2y^2 + yz^2 + 5xyz)$; $\sigma_z = 2x^2(x+y+z) - 3y^2(x+2z)$; $\tau_{xy} = 5(x^2 + y^2)$; $\tau_{yz} = 8(x^2 + y^2 + z^2)$; and $\tau_{zx} = 5(x^2 - z^2)$. What *body force* field will hold it in equilibrium?

3. Given the isotropic strain-stress relations:

$$\varepsilon_{ij} = \frac{-\nu}{E}\sigma_{kk}\delta_{ij} + \frac{1+\nu}{E}\sigma_{ij}$$

   Derive the corresponding stress-strain relations.

4. If **A** and **B** are square matrices and **X** and **Y** are vectors, *represent* the following concisely using the index notation: (i) (**A**+**B**)**X**, (ii) $\mathbf{X}^T$(**A**+**B**)**X** (iii) (**X**×**Y**)•(**Y**×**X**)

5. If **A**, **B**, **C** are square matrices and **X**, **Y**, **Z** are vectors, *represent* the following in index notation:
   (a) (**A** + **B**) **X** = **C** **Z** = **Y**, (b) **X**•**Y**×**Z** (**X** *dot* **Y** *cross* **Z**)

6. (a) Consider two Cartesian coordinate systems $x_i$ and $x_i'$. The direction cosines of $x_1'$ and $x_2'$ with respect to $x_i$ are $\mathbf{n}_1$ = [0.6124, –0.5, –0.6124] and $\mathbf{n}_2$ = [0.3536, 0.866, –0.3536] respectively. *Find* the components of the transformation tensor $a_{ij}$. [3]
   (b) If the displacement vector at a point with respect to $x_i$ is: **u** = [0.215, –0.512, 0.696] mm. Find its components with respect to $x_i'$. Check your answer by comparing the magnitudes of the two vectors.
   (c) If the stress components at a point with respect to $x_i$ are: $\sigma_x$ = 40MPa, $\sigma_y$ = –60MPa, $\sigma_z$ = 20MPa, $\tau_{xy}$ = 25MPa, $\tau_{yz}$ = 5MPa, $\tau_{zx}$ = –15MPa, determine the stress components $\sigma_{ij}'$ with respect to $x_i'$. Check the answer by comparing the stress invariants.

7. If $B_i$ is a vector and $C_{ij}$ a second order tensor, show that (i) $B_i\, C_{jk}$ is a third order tensor and (ii) $B_i\, C_{ij}$ is a vector.

8. If $B_{ij}$ and $C_{ij}$ are second order tensors show that (i) $B_{ij}\, C_{ki}$ is a second order tensor and (ii) $B_{ij}\, C_{ji}$ is a scalar.

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