PLL and Grid Synchronization

by

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SUMMARY OF PRESENTATION

- Unit vectors and its significance
- Basics of transformations

Session 1: Unit Vectors For 3\(\phi\) Balanced/Unbalanced Grid
  - Selection of reference variable for control
  - Selection of output parameter for PI-controller
  - Design of PI-controller constants
  - System bandwidth and method to select bandwidth
  - Unit vector under unbalanced grid condition
  - Test results
  - A Simple method for unit vector construction for balanced grid

Session 2: Unit Vectors For 1\(\phi\) Grid
  - Constructing two unit magnitude 90\(^0\) displaced components
  - Mitigating the effect of grid frequency variation
    - Approximation method
    - Rigorous method
  - Test results
Unit Vectors

What is unit vector?
- Two unity magnitude fundamental sinusoidal quantities, which are displaced by 90° from each other
- One of the unit vector is in phase with grid voltage
- This should be free from harmonics
- Phase angle error from grid voltage should be minimum

Significance of unit vector?
- STATCOM is an independent voltage source
- Unit vector helps to synchronize STATCOM voltage and Grid voltage
- Also unit vector is used to extract the active and reactive power component separately
- It is also used to separate the individual harmonics in case of active power filters
Transformations

- $3\phi$ to $\alpha$-$\beta$ transformations

$$X_\alpha = X_R - \frac{1}{2}(X_Y + X_B)$$

$$X_\beta = \frac{\sqrt{3}}{2}(X_Y - X_B)$$

- $\alpha$-$\beta$ to D-Q transformations

$$\begin{bmatrix} X_d \\ X_q \end{bmatrix} = \begin{bmatrix} U_1 & U_2 \\ -U_2 & U_1 \end{bmatrix} \begin{bmatrix} X_\alpha \\ X_\beta \end{bmatrix}$$

- $U_1$ and $U_2$ unity magnitude components, where $U_2$ lag $U_1$ by $90^\circ$

- For grid connected system $U_1$ is aligned along R-phase grid voltage
SESSION 1

UNIT VECTORS FOR THREE PHASE BALANCED/UNBALANCED GRID
Unit vector for Balanced/Unbalanced Grid condition

Let the grid voltages are,

\[ V_R = V_g \sin \omega_g t \]
\[ V_Y = V_g \sin(\omega_g t - 120) \]
\[ V_B = V_g \sin(\omega_g t + 120) \]

Objective,

\[ U_1 = \sin \omega_g t \]
\[ U_2 = -\cos \omega_g t \]

Apply three phase to two phase (\(\alpha-\beta\)) transformation,

\[ V_\alpha = V_R - \frac{1}{2} (V_Y + V_B) = \frac{3}{2} V_g \sin \omega_g t \]
\[ V_\beta = \frac{\sqrt{3}}{2} (V_Y - V_B) = -\frac{3}{2} V_g \cos \omega_g t \]
Unit vector for Balanced/Unbalanced Grid condition

- If \( U_1 \) and \( U_2 \) are known then \( V_\alpha \) and \( V_\beta \) can be transformed to D-Q axis

- Assume \( U_1 \) and \( U_2 \) are known, then
  
  \[
  \begin{bmatrix}
  V_d \\
  V_q 
  \end{bmatrix} = \begin{bmatrix}
  U_1 & U_2 \\
  -U_2 & U_1 
  \end{bmatrix} \begin{bmatrix}
  V_\alpha \\
  V_\beta 
  \end{bmatrix}
  \]

- Let \( U_1 \) is not synchronized to \( V_R \) and its frequency is \( \omega'_g \), then,
  
  \[
  \begin{bmatrix}
  V_d \\
  V_q 
  \end{bmatrix} = \begin{bmatrix}
  \sin \omega'_g t & -\cos \omega'_g t \\
  \cos \omega'_g t & \sin \omega'_g t 
  \end{bmatrix} \begin{bmatrix}
  \frac{3}{2} V_g \sin \omega_g t \\
  -\frac{3}{2} V_g \cos \omega_g t 
  \end{bmatrix} = \begin{bmatrix}
  \frac{3}{2} V_g \cos((\omega_g - \omega'_g) t) \\
  \frac{3}{2} V_g \sin((\omega_g - \omega'_g) t) 
  \end{bmatrix}
  \]

- When \( U_1 \) gets synchronized with \( V_R \), then \( \omega_g = \omega'_g \) and,
  
  \[
  \begin{bmatrix}
  V_d \\
  V_q 
  \end{bmatrix} = \begin{bmatrix}
  \frac{3}{2} V_g \\
  0 
  \end{bmatrix}
  \]
Unit vector for Balanced/Unbalanced Grid condition

- If $V_q = 0$ is ensured, then $U_1$ will be synchronized to $V_R$
- To ensure $V_q = 0$, feedback with simple PI-controller can be used
- PI-controller will be sufficient,
  - Since $V_d$ and $V_q$ are d.c. quantities and variation in $e(t)$ with time is minimal
Unit vector for Balanced/Unbalanced Grid condition

- What should be the output parameter of PI-controller?
  - PI-controller performs well when the selected output parameter magnitude swing is minimal
  - Our objective is to obtain $U_1$ and $U_2$, but they are sinusoidally varying quantities and results in large swing in magnitude
  - The angle of $U_1$ and $U_2$ are also will be varying with time in large extend
  - Instead the frequency of $U_1$ and $U_2$ can be thought of selecting as output parameter of PI-controller

$$U_1 = \sin \omega t, \quad U_2 = -\cos \omega t$$
Unit vector for Balanced/Unbalanced Grid condition

What should be the output parameter of PI-controller?

- If the grid frequency variation is minimal, then
- PI-controller performance can be further improved by selecting the output of PI-controller as $\Delta \omega$ (variation in grid frequency) instead of absolute grid frequency, $\omega$.

$\omega_{\text{ref}}$ is the nominal grid frequency.

$\omega_{\text{ref}}$ is the nominal grid frequency.
Unit vector for Balanced/Unbalanced Grid condition

- Design of PI-controller constants?
  - At time t=0, let grid and unit vectors (U₁ and U₂) frequency be \( \omega_g \)
  - At time t=0\(^+\), grid frequency changed from \( \omega_g \) to \( \omega'_g \) and unit vector frequency remains at \( \omega_g \)
  - 3\( \phi \) to \( \alpha-\beta \) transformation of grid voltage is,

\[
V_\alpha = \frac{3}{2} V_g \sin \omega'_g t, \quad V_\beta = -\frac{3}{2} V_g \cos \omega'_g t
\]

- \( \alpha-\beta \) to D-Q transformation gives,

\[
\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin \omega_g t & -\cos \omega_g t \\ \cos \omega_g t & \sin \omega_g t \end{bmatrix} \begin{bmatrix} \frac{3}{2} V_g \sin \omega'_g t \\ \frac{3}{2} V_g \cos \omega'_g t \end{bmatrix} = \begin{bmatrix} \frac{3}{2} V_g \cos((\omega'_g - \omega_g) t) \\ \frac{3}{2} V_g \sin((\omega'_g - \omega_g) t) \end{bmatrix}
\]
Unit vector for Balanced/Unbalanced Grid condition

- Design of PI-controller constants?
  - Assuming grid frequency variation is minimal, then
    \[
    \sin((\omega'_g - \omega_g)t) \approx (\omega'_g - \omega_g)t
    \]
  - Hence \( V_q \) is,
    \[
    V_q = \frac{3}{2} V_g (\omega'_g - \omega_g)t
    \]

\[
\begin{bmatrix}
V_d \\
V_q
\end{bmatrix} = \begin{bmatrix}
\frac{3}{2} V_g \cos((\omega'_g - \omega_g)t) \\
\frac{3}{2} V_g \sin((\omega'_g - \omega_g)t)
\end{bmatrix}
\]
Unit vector for Balanced/Unbalanced Grid condition

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Unit vector for Balanced/Unbalanced Grid condition

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    \[ \sin((\omega'_g - \omega_g)t) \approx (\omega'_g - \omega_g)t \]
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    V_q = \frac{3}{2} V_g (\omega'_g - \omega_g)t
    \]
Unit vector for Balanced/Unbalanced Grid condition

- Design of PI-controller constants?
  - The block diagram can be further reduced by removing the quantities not taking part in transients
  - \( \omega_{\text{ref}} \) is constant and no effect on transients
  - Once grid voltage is changed from \( \omega_g \) to \( \omega'_g \), \( \omega'_g \) can be assumed to be constant
  - Simplified block diagram is,

\[
\begin{align*}
V_{q\text{ref}} = 0 & \quad \rightarrow \quad k_p + \frac{k_i}{s} \rightarrow \frac{1}{s} \\
\frac{3}{2}V_g & \quad \rightarrow \quad \frac{3}{2}V_g \\
\end{align*}
\]

\[
\begin{align*}
e(t) & \quad \rightarrow \quad k_p + \frac{k_i}{s} \rightarrow \Delta \omega \\
\frac{3}{2}V_g & \quad \rightarrow \quad \omega_g t \\
\end{align*}
\]
Unit vector for Balanced/Unbalanced Grid condition

- Design of PI-controller constants?
  - Open loop transfer function of the system is given by,

\[
G(s)H(s) = \frac{3}{2} V_g \left( \frac{sk_p + k_i}{s} \right) \left( \frac{1}{s} \right)
\]

\[
G(s)H(s) = \frac{3}{2} V_g \left( \frac{s}{k_i/k_p + 1} \right) \left( s/\sqrt{k_i} \right)^2
\]
Unit vector for Balanced/Unbalanced Grid condition

- Design of PI-controller constants?
  - Find asymptotic bode plot of the system,
  - For zero:
    - Corner frequency, \( \omega_z = k_i/k_p \)
    - Magnitude plot: +20dB/decade
    - Phase plot: +45\(^0\)/decade
  - For poles:
    - Corner frequency, \( \omega_p = \sqrt{k_i} \)
    - Magnitude plot: -40dB/decade
    - Phase plot: -180\(^0\)

\[
G(s)H(s) = \frac{3}{2} \frac{V_g}{\left( \frac{s}{k_i/k_p} + 1 \right)^2}
\]
Unit vector for Balanced/Unbalanced Grid condition

- Design of PI-controller constants?
  - Closed loop system will be stable,
    - If the open loop gain crosses 0dB (unity gain) with -20dB/decade and ensuring at least -135° Phase margin
  - If \( \omega_z > \omega_p \), leads to a phase close to -180° makes system prone to unstable
  - Hence to make system more stable, keep \( \omega_z \leq \omega_p \)
  - Since phase plot of zero has a slope of +45°/decade, for \( \omega_z = \omega_p \), the phase is -135° leads to phase margin of 45°
Unit vector for Balanced/Unbalanced Grid condition

- Design of PI-controller constants?
  - $\omega_z = \omega_p$ implies,
    
    $$\frac{k_i}{k_p} = \sqrt{k_i}$$
    $$k_p = \sqrt{k_i}$$

- One more component present in the open loop transfer function is,
  
  $$\frac{3}{2} V_g$$

- This will not affect the phase plot, but gain cross over frequency will be pushed further away increasing the bandwidth of the system
Unit vector for Balanced/Unbalanced Grid condition

- Find system bandwidth?
  - Defined as the frequency at which the closed-loop magnitude is equal to -3 dB
  - For phase of -120°, the open loop bandwidth and closed loop bandwidth are found to be closer
  - Since gain at \( \sqrt{k_i} \) is 0dB, the gain cross over frequency of open loop system is,

\[
\omega_g = \sqrt{k_i}
\]
Unit vector for Balanced/Unbalanced Grid condition

- Find system bandwidth?
  - The modified crossover frequency considering $3/2V_g$ is,
  - Gain at $\omega_g = \sqrt{k_i}$ is,
    
    \[20 \log\left(\frac{3}{2} V_g\right)\]

- Gain at new crossover frequency ($\omega'_g$) is 0dB
- Slope during this time is -20dB/decade

\[
0 - 20 \log\left(\frac{3}{2} V_g\right) \leq \log \omega'_g - \log(\sqrt{k_i}) = -20
\]
Unit vector for Balanced/Unbalanced Grid condition

- Find system bandwidth?
  - The modified crossover frequency considering $3/2V_g$ is,
    \[ \omega'_g = \frac{3}{2} V_g \sqrt{k_i} \]
  - Closed loop bandwidth (appx.) is,
    \[ BW = \frac{3}{2} V_g k_p \]
  - Once the bandwidth is fixed proportional constant ($k_p$) can be found

\[ k_p = \sqrt{k_i} \]
Unit vector for Balanced/Unbalanced Grid condition

- How to fix the bandwidth?
  - Bandwidth is decided by the harmonics present in $V_d$ and $V_q$ components as well as the response time requirement in transient conditions.
  - For a balanced three wire system the minimum harmonics expected on $V_d$ and $V_q$ are 300Hz (transformation of $5^{th}$ and $7^{th}$ harmonics).
    - Here bandwidth of 30Hz to 60Hz will be sufficient for proper attenuation of harmonics in grid voltages.

- In Summary:
  - Based on the application and transient requirement fix the bandwidth.
  - If grid voltage peak is known, proportional constant ($k_p$) can be found.
    \[
    k_p = \frac{BW}{\sqrt{\frac{3}{2} V_g}}
    \]
  - Once $k_p$ is known integral constant $k_i$ can be computed.
    \[
    k_i = k_p^2
    \]
Unit vector for Balanced/Unbalanced Grid condition

- Unit vector under unbalanced grid condition?
  - Under unbalanced grid conditions, the grid voltage contains fundamental positive sequence as well as fundamental negative sequence components.
  - Let us construct the unit vector such that it is synchronized with fundamental positive sequence component.
  - As earlier fundamental positive sequence present in the grid voltage when transformed to D-Q reference frame reflected as d.c. component and fundamental negative sequence present in the grid voltage reflected as 100Hz component.
  - Since we required only d.c. component in $V_q$, 100Hz component can be easily removed by using simple low pass filter of corner frequency around 10Hz.
Unit vector for Balanced/Unbalanced Grid condition

- Unit vector under unbalanced grid condition?
  - \( \omega_c \) is the corner frequency of low pass filter
  - Low pass filter will not affect the information of fundamental positive sequence, as it is a d.c. quantity
Unit vector for Balanced/Unbalanced Grid condition

- **Test results**
  - Transient performance analysis carried out by simulation using PSIM simulation package

50% step voltage sag appeared in $V_R$ and $V_Y$

Voltage restored
Unit vector for Balanced/Unbalanced Grid condition

- Test results
  - Transient performance analysis carried out by simulation using PSIM simulation package

Step change in frequency (from 50Hz to 30Hz) appeared in \( V_R, V_Y \), and \( V_B \)
Unit vector for Balanced/Unbalanced Grid condition

- Test results
  - Unit vector implemented for unbalanced grid condition in TI’s DSP TMS320F2812

Balanced grid voltage: $V_R$ and $V_B$ are nominal, $V_Y=0$

Unit vector and angle $\omega t$
A Simple method for Unit vector in Balanced Grid condition

- A simple method exists based on trigonometry to compute unit vector for balanced grid condition
- Transform three phase grid voltage to $\alpha$-$\beta$ co-ordinate

$$V_\alpha(t) = \frac{3}{2}V_g \sin \omega_g t, \quad V_\beta(t) = -\frac{3}{2}V_g \cos \omega_g t$$

- The output of Low Pass Filter (LPF) is given by,

$$V'_\alpha(s) = \frac{s}{s + \omega_c} V_\alpha(s), \quad V'_\beta(s) = \frac{s}{s + \omega_c} V_\beta(s)$$
A Simple method for Unit vector in Balanced Grid condition

- Output of LPF in time domain is,

\[
V'_\alpha(t) = \frac{3}{2} \frac{V_g \omega_c}{\sqrt{\omega_g^2 + \omega_c^2}} \sin(\omega_g t - \phi), \quad V'_\beta(t) = -\frac{3}{2} \frac{V_g \omega_c}{\sqrt{\omega_g^2 + \omega_c^2}} \cos(\omega_g t - \phi)
\]

where, \( \tan \phi = \frac{\omega_g}{\omega_c} \)

- \( V'_\alpha(t) \) and \( V'_\beta(t) \) are always 90° displaced irrespective of grid frequency and corner frequency of LPF

- At the zero crossing of \( V'_\alpha(t) \), \( V'_\beta(t) \) will be in peak and vice versa

- Unit magnitude of \( V'_\alpha(t) \) and \( V'_\beta(t) \) can be obtained by dividing the term by its own magnitude,
A Simple method for
Unit vector in Balanced Grid condition

- Block diagram of unit vector construction

\[ U_1'(t) = \sin(\omega_g t - \phi) \]

\[ U_2'(t) = -\cos(\omega_g t - \phi) \]
A Simple method for Unit vector in Balanced Grid condition

- Let us perform the following operations,

$$F_1(t) = V'_\alpha(t) - V'_\beta(t)$$

$$= \frac{3}{\sqrt{2}} \frac{V_g \omega_c}{\sqrt{\omega_c^2 + \omega_g^2}} \sin(\omega_g t + \rho)$$

$$F_2(t) = V'_\alpha(t) + V'_\beta(t)$$

$$= -\frac{3}{\sqrt{2}} \frac{V_g \omega_c}{\sqrt{\omega_c^2 + \omega_g^2}} \cos(\omega_g t + \rho)$$

where, \( \tan \rho = \frac{\omega_c - \omega_g}{\omega_c + \omega_g} \)

- Phase angle (\( \rho \)) is minimum when \( \omega_c = 2\pi 50 \)
- For \( \omega_c = 2\pi 50 \), \( \rho \) is zero when grid frequency is 50Hz
A Simple method for Unit vector in Balanced Grid condition

- Complete block diagram of unit vector construction

where, $\omega_c = 2\pi 50$

- This method will introduce a small phase angle error when grid frequency varies
- Each harmonics is reduced by $\sqrt{\frac{2}{1 + h^2}}$ when compared to fundamental
A Simple method for
Unit vector in Balanced Grid condition

- Test results
SESSION 2

UNIT VECTORS FOR SINGLE PHASE GRID
Overview of the presentation

- Constructing two unit magnitude $90^0$ displaced components
- Mitigating the effect of grid frequency variation
  - Approximation method
  - Rigorous method
Overview of the presentation

- Constructing two unit magnitude 90° displaced components
- Mitigating the effect of grid frequency variation
  - Approximation method
  - Rigorous method
Innovative method for constructing unit vector

- Let R-phase grid voltage be, \( V_R = V_g \sin(\omega_s t) \)
- Let the above voltage is passed through a LPF of corner frequency, \( \omega_c \)
- Using Laplace analysis

\[
F_R(s) = \frac{V_g \omega_c \omega_s}{\omega_s^2 + \omega_c^2} \left( \frac{1}{s + \omega_c} + \left( \frac{\omega_c}{\omega_s} \right) \frac{\omega_s}{s^2 + \omega_s^2} - \frac{s}{s^2 + \omega_s^2} \right)
\]

- The steady state output, \( F_R(t) \) is given by

\[
F_R(t)|_{\text{steady}} = \frac{V_g \omega_c}{\sqrt{\omega_s^2 + \omega_c^2}} \sin(\omega_s t - \phi) \quad \text{and} \quad \tan \phi = \frac{\omega_s}{\omega_c}
\]

- Transient term of \( F_R(t) \) is given by

\[
F_R(t)|_{\text{transient}} = \frac{V_g \omega_c \omega_s}{\omega_c^2 + \omega_s^2} e^{-\omega_c t}
\]
constructing unit vector (contd.)

- Let the $F_R(t)$ is again pass through a LPF of same corner frequency, $\omega_c$

- Using Laplace analysis

$$F_R'(s) = V_g \omega_c^2 \omega_s \frac{1}{\omega_s^2 + \omega_c^2} \left( 2 \omega_c \frac{1}{s + \omega_c} + \left( \omega_s^2 + \omega_c^2 \right) \frac{1}{(s + \omega_c)^2} - 2 \omega_c \frac{s}{s^2 + \omega_s^2} + \left( \omega_c^2 - \omega_s^2 \right) \frac{1}{s^2 + \omega_s^2} \right)$$

- The steady state output, $F_R'(t)$ is given by

$$F_R'(t) = - \frac{V_g \omega_c^2}{\omega_s^2 + \omega_c^2} \cos(\omega_st + \psi) \quad \text{and} \quad \cos \psi = \left( \frac{2 \omega_c \omega_s}{\omega_s^2 + \omega_c^2} \right), \quad \sin \psi = \frac{\left( \omega_c^2 - \omega_s^2 \right)}{\left( \omega_s^2 + \omega_c^2 \right)}$$

- Transient term of $F_R'(t)$ is given by,

$$F_R'(t)|_{\text{transient}} = \frac{V_g \omega_c^2 \omega_s}{\left( \omega_c^2 + \omega_s^2 \right)^2} \left( 2 \omega_c + \left( \omega_c^2 + \omega_s^2 \right) t \right) e^{-\omega_c t}$$
constructing unit vector (contd.)

- Subtract $F_R'(t)$ from $F_R(t)$, let the result be $F_R''(t)$

- The steady state output, $F_R''(t)$ is given by

$$F_R''(t) = \frac{V_g \omega_c \omega_s}{\omega_s^2 + \omega_c^2} \sin(\omega_s t + \psi) \quad \text{and} \quad \cos \psi = \frac{2 \omega_c \omega_s}{\omega_s^2 + \omega_c^2}, \quad \sin \psi = \frac{\omega_c^2 - \omega_s^2}{\omega_s^2 + \omega_c^2}$$

- Transient term of $F_R''(t)$ is given by,

$$F_R''(t)|_{\text{transient}} = \frac{V_g \omega_c \omega_s}{\omega_c^2 + \omega_s^2} \left( \frac{\omega_s^2 - \omega_c^2}{\omega_c^2 + \omega_s^2} \right) + \omega_c t \ e^{-\omega_c t}$$
constructing unit vector (contd.)

- Comparing steady state term of $F_R'(t)$ and $F_R''(t)$

$F_R'(t) = -\frac{V_g \omega_c^2}{(\omega_s^2 + \omega_c^2)} \cos(\omega_s t + \psi)$

$F_R''(t) = \frac{V_g \omega_c \omega_s}{(\omega_s^2 + \omega_c^2)} \sin(\omega_s t + \psi)$

- $F_R'(t)$ and $F_R''(t)$ are always 90° displaced irrespective of grid frequency and corner frequency of LPF

- At the zero crossing of $F_R''(t)$, $F_R'(t)$ will be in peak and vice versa

- Unit magnitude of $F_R'(t)$ and $F_R''(t)$ can be obtained by dividing the term by its own magnitude
constructing unit vector (contd.)

Inference from the above result
- \( F_2(t) \) is phase shifted from grid voltage by an angle \( \psi \)

- \( V_R = V_g \sin(\omega_S t) \)
constructing unit vector (contd.)

- Inference from the above result (contd.)
  - If corner frequency of LPF ($\omega_c$) is set equal to grid frequency ($\omega_s$), i.e. $\omega_c = \omega_s$:
    - phase shift $\psi = 0$
    - $F_2(t)$ is in phase with grid voltage and $F_1(t)$ is lagging the grid voltage by $90^0$

- Fix the corner frequency of LPF ($\omega_c$) is equal to $\omega_s = 2\pi 50$ rad/sec, where grid frequency $f_s = 50$Hz

- Let Grid frequency varies a maximum of $\pm 10\%$ (45Hz to 55Hz)

- Phase shift, $\psi = 6^0$ for -10\% of grid frequency variation

- Loose the proper synchronization
constructing unit vector (contd.)

- Phase error with variation in grid frequency

![Graph showing phase error with variation in grid frequency](image)

- 1ph without comp

- % ge of grid frequency variation

- Phase error

- % ge of grid frequency variation
Overview of the presentation

- Constructing two unit magnitude 900 displaced components
- Mitigating the effect of grid frequency variation
  - Approximation method
  - Rigorous method
constructing unit vector (contd.)

- **Mitigating the effect of grid frequency variation**
  - Give a $\Delta \omega$ variation for the grid frequency $\omega_s$ in the peak of $F''_R(t)$
  - Substitute $\omega_c = \omega_s$ and solving will give

$$F''_R|_{\omega_s = \omega_s + \Delta \omega} = \frac{V_g}{2} \left[ 1 + \left( \frac{\Delta \omega}{\omega_s} \right)^2 \right]$$

$$F''_R|_{\omega_c = \omega_s} \approx \frac{V_g}{2} \quad \text{if} \quad \frac{1}{2} \left( \frac{\Delta \omega}{\omega_s} \right)^2 \text{ is negligible compare to 1}$$

- Peak of $F''_R(t)$, $F''_{R(peak)}$, is more or less independent of grid frequency variation

$$F''_{R(peak)} \approx \frac{V_g}{2} \quad \text{for} \quad \omega_c = \omega_s$$
Mitigating the effect of grid frequency variation (contd.)

- The following trigonometric relation can be used to eliminate the phase angle \( \psi \)
  \[
  \sin(\omega_s t + \psi) V_g \cos \psi - \cos(\omega_s t + \psi) V_g \sin \psi = V_g \sin \omega_s t \\
  - \cos(\omega_s t + \psi) V_g \cos \psi - \sin(\omega_s t + \psi) V_g \sin \psi = -V_g \cos \omega_s t
  \]

- \( V_g \cos \psi = 2F'' \)

- Solve the following mathematical relation
  \[
  2F'_{R(\text{peak})} - V_g = \frac{2V_g \omega_c^2}{\omega_s^2 + \omega_c^2} - V_g = V_g \left( \frac{\omega_c^2 - \omega_s^2}{\omega_s^2 + \omega_c^2} \right)
  \]
  \[
  2F'_{R(\text{peak})} - V_g = V_g \sin \psi
  \]
  \[
  V_g \sin \psi \cong 2F'_{R(\text{peak})} - 2F''_{R(\text{peak})}
  \]

- \[
  F''_R(t) = \frac{V_g \omega_c \omega_s}{\omega_s^2 + \omega_c^2} \sin(\omega_s t + \psi)
  \]
- \[
  F''_{R(\text{peak})} = \frac{V_g \omega_c \omega_s}{\omega_c^2 + \omega_s^2}
  \]
- \[
  F''_{R(\text{peak})} \approx \frac{V_g}{2} \text{ for } \omega_c = \omega_s
  \]
- \[
  F'_R(t) = -\frac{V_g \omega_c^2}{\omega_s^2 + \omega_c^2} \cos(\omega_s t + \psi)
  \]
- \[
  F'_{R(\text{peak})} = \frac{V_g \omega_c^2}{\omega_c^2 + \omega_s^2}
  \]}
Mitigating the effect of grid frequency variation (contd.)

✓ Rewriting the relation

\[ \sin(\omega_s t + \psi)V_g \cos \psi - \cos(\omega_s t + \psi)V_g \sin \psi = V_g \sin \omega_s t \]

✓ Substituting

\[ V_g \cos \psi = 2F''_{R(peak)} \]

\[ V_g \sin \psi \approx 2F'_{R(peak)} - 2F''_{R(peak)} \]

\[ V_g \sin \omega_s t \approx U_1 = \sin(\omega_s t + \psi)(2F''_{R(peak)}) - \cos(\omega_s t + \psi)(2F'_{R(peak)} - 2F''_{R(peak)}) \]

\[ U_1 = 2F''_{R(t)} + 2F'_{R(t)} + 2F''_{R(peak)} \cos(\omega_s t + \psi) \]

\[ U_1 = \frac{2V_g \omega_c}{\sqrt{\omega_s^2 + (\omega_c - \omega_s)^2}} \frac{\sin(\omega_s t + \psi - \theta)}{\left(\frac{\omega_s^2}{\omega_s^2 + \omega_c^2}\right)} \text{ and } \tan \theta = \frac{(\omega_c - \omega_s)}{\omega_s} \]
constructing unit vector (contd.)

Mitigating the effect of grid frequency variation (contd.)

✓ Rewriting the relation

\[-\cos(\omega_s t + \psi)V_g \cos \psi - \sin(\omega_s t + \psi)V_g \sin \psi = -V_g \cos \omega_s t\]

➢ Substituting

✓ \(V_g \cos \psi = 2F''_{R(peak)}\)
✓ \(V_g \sin \psi \approx 2F'_{R(peak)} - 2F''_{R(peak)}\)

\[-V_g \cos \omega_s t \cong U_2 = -\cos(\omega_s t + \psi)(2F''_{R(peak)}) - \sin(\omega_s t + \psi)(2F'_{R(peak)} - 2F''_{R(peak)})\]

\[U_2 = 2F_{R''}(t) - 2\{F''_{R(peak)} \cos(\omega_s t + \psi)\} - 2\{F'_{R(peak)} \sin(\omega_s t + \psi)\}\]

\[U_2 = -\frac{2V_g \omega_c}{\omega_s^2 + \omega_c^2} \sqrt{\omega_s^2 + (\omega_c - \omega_s)^2} \cos(\omega_s t + \psi - \theta)\quad \text{and} \quad \tan \theta = \frac{(\omega_c - \omega_s)}{\omega_s}\]
Mitigating the effect of grid frequency variation (contd.)

- Comparing steady state term of $U_1(t)$ and $U_2(t)$

\[
U_1 = \frac{2V_s \omega_c \sqrt{\omega_s^2 + (\omega_c - \omega_s)^2}}{\omega_s^2 + \omega_c^2} \sin(\omega_s t + \psi - \theta)
\]

\[
U_2 = -\frac{2V_s \omega_c \sqrt{\omega_s^2 + (\omega_c - \omega_s)^2}}{\omega_s^2 + \omega_c^2} \cos(\omega_s t + \psi - \theta)
\]

- Inference from the above result
  - $U_1(t)$ and $U_2(t)$ are $90^0$ displaced
  - At the zero crossing of $U_1(t)$, $U_2(t)$ will be in peak and vice versa
  - Unit magnitude of $U_1(t)$ and $U_2(t)$ can be obtained by dividing the term by its own magnitude
  - $U_1(t)$ is phase shifted from grid voltage by angle $(\psi - \theta)$

\[V_R = V_g \sin(\omega_s t)\]
Mitigating the effect of grid frequency variation (contd.)

- Inference from the above result (contd.)
  - Fix the corner frequency of LPF ($\omega_c$) is equal to $\omega_s = 2\pi 50$ rad/sec, where grid frequency $f_s = 50$Hz
  - Let Grid frequency varies a maximum of $\pm 10\%$ (45Hz to 55Hz)
  - Give a $\Delta \omega$ variation and substitute $\omega_c = \omega_s$ in the phase angle ($\psi - \theta$)
    \[
    \tan(\psi - \theta) = \frac{\tan \psi - \tan \theta}{1 + \tan \psi \tan \theta}
    \]
    \[
    \tan(\psi - \theta)_{\omega_s=\omega_s+\Delta\omega} \simeq -\frac{1}{2\left(\frac{\Delta \omega}{\omega_s}\right)^2} \quad \text{if} \quad 2\left(\frac{\Delta \omega}{\omega_s}\right)^2 \quad \text{and} \quad \frac{1}{2}\left(\frac{\Delta \omega}{\omega_s}\right)^3 \quad \text{is negligible compared to 1}
    \]
Mitigating the effect of grid frequency variation (contd.)

- Phase error with variation in grid frequency
  - Phase shift, \((\psi-\theta) = 0.32^0\) for \(-10\%\) grid frequency variation

![Graph showing phase error vs. % ge of grid frequency variation]
constructing unit vector (contd.)

\[ F_1(t) = 2F''_R(t) + 2F'_R(t) + 2F''_{R(peak)} \cos(\omega_3 t + \psi) \]

\[ U_1 = 2F''_R(t) + 2F'_R(t) + 2F''_{R(peak)} \cos(\omega_3 t + \psi) - 2F'_{R(peak)} \sin(\omega_3 t + \psi) \]

\[ F_2(t) = \sin(\omega_3 t + \psi) \]

\[ U_2 = 2F''_R(t) - 2F'_{R(peak)} \cos(\omega_3 t + \psi) - 2F''_{R(peak)} \sin(\omega_3 t + \psi) \]
Overview of the presentation

- Constructing two unit magnitude $90^0$ displaced components
- Mitigating the effect of grid frequency variation
  - Approximation method
  - Rigorous method
Mitigating the effect of grid frequency variation (contd.)

The approximation made in the earlier derivation is:

\[ 2F_R'(peak) - V_g = V_g \sin \psi \]

\[ F_R''(\omega_c = \omega_s) \approx \frac{V_g}{2} \quad \text{if} \quad \frac{1}{2} \left( \frac{\Delta \omega}{\omega_s} \right)^2 \text{is negligible compare to 1} \]

The above method is Approximation method

Another method of finding out \( V_g \sin(\omega_s t) \) is:

\[ F_R'(peak) + F_R''(peak) = \frac{V_g \omega_C}{(\omega_s^2 + \omega_C^2)}(\omega_C + \omega_s) \quad \& \quad F_R'(peak) - F_R''(peak) = \frac{V_g \omega_C}{(\omega_s^2 + \omega_C^2)}(\omega_C - \omega_s) \]

\[ \frac{(F_R'(peak) + F_R''(peak))(F_R'(peak) - F_R''(peak))}{F_R'(peak)} = \frac{V_g^2 \omega_C^2}{(\omega_s^2 + \omega_C^2)^2} \left( \frac{\omega_C^2 - \omega_s^2}{\omega_s^2 + \omega_C^2} \right) = V_g \frac{\omega_C^2 - \omega_s^2}{(\omega_s^2 + \omega_C^2)} = V_g \sin \psi \]
Mitigating the effect of grid frequency variation (contd.)

- Rewriting the relation
  \[ \sin(\omega_S t + \psi)V_g \cos \psi - \cos(\omega_S t + \psi)V_g \sin \psi = V_g \sin \omega_S t \]
  \[ - \cos(\omega_S t + \psi)V_g \cos \psi - \sin(\omega_S t + \psi)V_g \sin \psi = -V_g \cos \omega_S t \]

- Substituting
  \( V_g \cos \psi = 2F''_{R(\text{peak})} \)
  \( V_g \sin \psi = \frac{\left( F'_{R(\text{peak})} \right)^2 - F''_{R(\text{peak})}^2}{F'_{R(\text{peak})}} \)

\[
U_1 = V_g \sin \omega_S t = 2F''_{R(t)} - \left\{ \frac{\left( F'_{R(\text{peak})} \right)^2 - F''_{R(\text{peak})}^2}{F'_{R(\text{peak})}} \right\} \cos(\omega_S t + \psi)
\]

\[
U_2 = -V_g \cos \omega_S t = -2F''_{R(\text{peak})} \cos(\omega_S t + \psi) - \left\{ \frac{\left( F'_{R(\text{peak})} \right)^2 - F''_{R(\text{peak})}^2}{F'_{R(\text{peak})}} \right\} \sin(\omega_S t + \psi)
\]
constructing unit vector (contd.)

Mitigating the effect of grid frequency variation (contd.)

- Phase error with variation in grid frequency
  - Phase shift, $(\psi - \theta) = 0^0$ for any grid frequency

![Graph showing phase error with grid frequency variation]

- 1ph with comp (rigorous)
constructing unit vector (contd.)

\[
U_1(t) = V_g \sin \omega_s t = 2F'_R(t) - \left\{ \frac{(F'_R(\text{peak})^2 - F''_R(\text{peak})^2)}{F'_R(\text{peak})} \right\} \cos(\omega_s t + \psi)
\]

\[
U_2(t) = -V_g \cos \omega_s t = -2F''_R(\text{peak}) \cos(\omega_s t + \psi) - \left\{ \frac{(F'_R(\text{peak})^2 - F''_R(\text{peak})^2)}{F'_R(\text{peak})} \right\} \sin(\omega_s t + \psi)
\]

\[
F_1(t) = -\cos(\omega_s t + \psi)
\]

\[
F_2(t) = \sin(\omega_s t + \psi)
\]
Mitigating the effect of grid frequency variation (contd.)

Test results

- Grid frequency (simulated using function generator) is varied at $t=t_1$ from 50Hz to 45Hz

Grid voltage and unit vector without compensation for grid frequency variation

Grid voltage and unit vector with compensation for grid frequency variation
Mitigating the effect of grid frequency variation (contd.)

- Test results
  - Steady state waveform at 25Hz and 75Hz with approximation method

Frequency = 25Hz
Frequency = 75Hz
constructing unit vector (contd.)

Mitigating the effect of grid frequency variation (contd.)

- Test results
  - Step change of frequency from 25Hz and 75Hz as well as from 75Hz to 25Hz

Grid voltage and unit vector without compensation for grid frequency variation

Grid voltage and unit vector with compensation for grid frequency variation
Conclusions

- Single grid voltage is considered for the construction of unit vector
  - Two unity magnitude fundamental sinusoidal quantities, which are displaced by 90° from each other
  - One of the unit vector is in phase with grid voltage irrespective of the grid frequency variation
Thank you