QUANTITY DISCOUNTS

- Suppliers offer lower unit price on order for larger quantities as an incentive to buyers to purchase in large lot sizes

Two types of quantity discount

- All-unit discount
- Incremental discount
  - All-unit discount – lower unit price for the entire lot
  - Incremental discount – lower unit price only to units purchased above a specified quantity
  - Price-break quantity – quantities at which price changes

All-Units Quantity Discounts

The unit purchase cost is defined as

\[
P_i = \begin{cases} 
  P_0 & \text{for } U_0 \leq Q < U_i \\
  P_1 & \text{for } U_1 \leq Q < U_2 \\
  \vdots \\
  P_j & \text{for } U_j \leq Q < U_{j+1} 
\end{cases}
\]

Where \( U_0 < U_1 < \cdots < U_j \) and \( P_0 > P_1 \cdots > P_j \)

Fig. 5 Inventory costs: All unit quantity discounts
Fig. 6 Total cost variation in different cases under all unit discounts

Decision Rule: Order the quantity with the lowest cost. *Case 1:* Order > $U_1$; *Case 2:* Order = $U_1$; *Case 3:* Order < $U_1$

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**Fig. 5 All-units quantity discount logic**

- Calculate the EOQ for the lowest unit cost
- **Is the EOQ valid?**
  - Yes → Select the EOQ as the order
  - No → Calculate the EOQ for the next higher unit cost
- **Is the EOQ valid?**
  - No → Calculate the total cost for the valid EOQ and all larger price-break quantities
  - Yes → Select the order quantities with the lowest total cost
(Refer order quantity vs cost graphs and quantity vs total cost graph)

**Incremental Quantity Discounts**

The price schedule is

\[
P_i = \begin{cases} 
P_0 & \text{for each } U_0 \text{ to } U_1 - 1 \\
P_1 & \text{for each of the next } U_1 \text{ to } U_2 - 1 \\
\vdots \\
P_j & \text{for each of the next } U_j \text{ to } U_{j+1} \end{cases}
\]

Where \( U_0 < U_1 < \cdots < U_j \) and \( P_0 > P_1 \cdots > P_j \)

Unit purchase price is not constant for a lot size \( Q \) where \( U_i \leq Q < U_{i+1} \)

The purchase cost for a lot size of \( Q \) units:

\[
M_i = D_i + P_i Q
\]

Purchase cost per unit is

\[
\frac{M_i}{Q} = \frac{D_i}{Q} + p_i
\]

The total cost per year for a lot size of \( Q \) units is

\[
TC(Q) = \left[ P_i + \frac{D_i}{Q} \right] R + \frac{CR}{Q} + \frac{FQ}{2} \left[ P_i + \frac{D_i}{Q} \right]
\]

\[
= P_i R + \frac{(C + D_i)R}{Q} + \frac{P_i FQ}{2} + \frac{FD_i}{2}
\]

The minimum cost lot size is

\[
Q_i = \sqrt{\frac{2R(C + D_i)}{P_i F}}
\]

**Procedure**

- Calculate the EOQ for each unit purchase cost
- Determine which EOQs are valid
- Calculate the total cost for each valid EOQ
- Select the valid EOQ with the lowest total cost

**BATCH-TYPE PRODUCTION SYSTEMS**

- Multiple products are produced on the same equipments in batches
- These products share and even compete for common production capacity as individual
items

- Planning batch production involves
  1. Determining the order of production
  2. Batch size

- Batch production system may not seek optimum production level, but allocate production capacity to items in relation to their demands, production rates and existing inventory

**Economic Production Quantity (EPQ) – Single items**

- Finite replenishment rate or no instantaneous supply
- Production cost
- Setup cost
- Size of production run (order) to be determined
- Production run that minimizes the total inventory cost is the economic production quantity

**Notations**

\[ R \] – annual demand in units
\[ P \] – unit production cost
\[ Q \] – size of production run
\[ p \] – production rate
\[ r \] – demand rate
\[ C \] – setup cost per production run
\[ H \] – holding cost per unit
\[ B \] – Reorder level

\[ \text{Maximum inventory} = (p - r)t_p \]
\[ t_p = \frac{Q}{p} \]

Average inventory = \( \frac{(p - r)Q}{2p} \)

\[ TC(Q) = PR + \frac{CR}{Q} + \frac{HQ(p - r)}{2p} \]

On minimization

\[ Q_0 = \sqrt{\frac{2CRp}{H(p - r)}} \]

Optimum length of production run = \( \frac{Q_0}{p} \)

**Economic Production Quantity (EPQ)-Multiple Item**

- The use of product EPQs for each product in the group implies that the equipment will be available when it is needed
- This may happen when equipments are highly under utilized
- Otherwise there may be scheduling problems and difficulties in meeting the requirements of a single product EPQs
- Multiple product scheduling problem should be solved by determining the number of annual cycles (m) that minimizes the total cost of entire family of items

Maximum inventory for a given item \( i = (p_i - r_i)t_{pi} \)

\[ Q_i = p_i t_{pi} = \frac{R_i}{m} \]

Average inventory for the given item \( i = \frac{(p_i - r_i)R_i}{2mp_i} \)

\[ TC(m) = \sum_{i=1}^{n} P_i R_i + m \sum_{i=1}^{n} C_i + \frac{1}{2m} \sum_{i=1}^{n} H_i R_i (p_i - r_i) \]

where \( n \) – no. of items

On minimization

\[ m_0 = \sqrt{\frac{\sum_{i=1}^{n} H_i R_i (p_i - r_i)}{2 \sum_{i=1}^{n} C_i}} \]

Production run size \( Q_i = \frac{R_i}{m_0} \)

- The calculation of optimum run size ignores capacity considerations
• It assumes there is sufficient capacity available to meet demand
• The model appropriate only if the number of annual operating days satisfies (N)

\[ N \geq \sum_{i=1}^{n} \frac{R_i}{p_i} \quad \text{or} \quad \frac{N}{m_0} \geq \sum_{i=1}^{n} \frac{Q_i}{p_i} \]

**FIXED ORDER INTERVAL SYSTEMS (PERIODIC INVENTORY SYSTEM)**
• Time based inventory system in which orders are placed at equally spaced, predetermined points in time
• Parameters – maximum inventory level \((E)\) and review period \((T)\)
• Maximum inventory level is based on usage during both the lead time and the order interval
• It is also known as T - system or P - system

**System operating procedure**
• After a fixed period time \((T)\) has passed, the stock position of the item is determined
• An order is placed to replenish the stock
• Order size is maximum inventory minus inventory position at the time of review
(Refer figure: Fixed order interval system)

**Economic Order Interval (EOI) – Single Item**
• EOI can be determined by the minimisation of the total annual cost

\[ TC(T) = PR + \frac{C}{T} + \frac{PFRT}{2} \]

Where T- order interval in years

\[ T_0 = \sqrt{\frac{2C}{PFR}} = \text{EOI in years} \]

\[ m = \frac{1}{T} = \text{number of orders or review per year} \]

➢ In deterministic system there is no difference between the Q-system and P-system

(Refer graphs: Time vs Quantity and order interval vs cost of fixed order interval system)

Order quantity in P-system is

\[ Q_0 = RT_0 = R \sqrt{\frac{2C}{PFR}} = \sqrt{\frac{2CR}{PF}} \]

• Maximum inventory level \(E\) must be large enough to satisfy demand during the subsequent order interval \(T\) and also during the lead time \(L\)

\[ E = RT + RL = R(T + L) = Q + B \]
Economic Order Interval (EOI) – Multiple Items

- When a supplier provides numerous items, joint orders are more economical.
- Jointly ordered item’s stock review can be coordinated.
- Joint ordering results in:
  - Monitoring of stock levels often can be less frequent.
  - It can be scheduled to fit comfortably with other organisational activities.
  - Logistic and transportation cost saving may also be possible.
- This involves determination of the time interval $T$ which minimise inventory cost for the group as a whole.

Notations

- $R_i$ – annual requirement for item $i$
- $P_i$ – purchase cost of item $i$
- $n$ – total number of joint order items
- $C$ – order cost for the joint order
- $c$ – order cost associated with each individual item
- $T$ – order interval in years
- $F$ – annual holding cost fraction

$$
TC (T) = \sum_{i=1}^{n} P_i R_i + \frac{C + nc}{T} + 2 \sum_{i=1}^{n} P_i R_i
$$

$$
T_0 = \sqrt{\frac{2(C + nc)}{F \sum_{i=1}^{n} P_i R_i}}
$$

$$
E_i = \frac{R_i (T + L)}{N}
$$

- $T$ – Order interval in days
- $L$ – Lead time in days
- $N$ – Operating days in a year